Mathematics

Grade 4

Teacher Guide

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Mathematics Teacher Guide Grade 4

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Grade 4 Term 1 Unit 1 Whole numbers

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
1.1 Number names and number symbols	Revising number names and symbols for numbers smaller than 100	3 to 6
1.2 Count in hundreds, tens and units	Number names and symbols for numbers up to 1 000	7 to 10
1.3 Building up and breaking down numbers	The concept of place value	11 to 12
1.4 Number names, expanded notation and number symbols	Consolidation of understanding of place value	13 to 15
1.5 Represent, order and compare numbers	Expressing numbers in different ways	16 to 17

CAPS time allocation	2 hours
CAPS page references	14 and 37 to 38

Mathematical background

Understanding place value means that we know that the number represented by the symbol 357 is not "three five seven" or "3 and 5 and 7", but 300 + 50 + 7. It is crucial that learners understand from the start that 357 is not "three five seven" or "3 and 5 and 7", but 300 + 50 + 7. Always ask learners to express numbers by saying the whole number name, for example three hundred and fifty-seven, and to avoid referring to the number as three five seven.

Language constructions such as "break down a number into its place value parts" (see Sections 1.1 and 1.3) and learning aids such as place value cards were invented and are prescribed to promote understanding of place value.

Resources

Two resources are absolutely critical for the work in this unit:

- Counting apparatus: wooden or plastic cubes, rods and blocks; or loose sticks, bundles of 10 sticks, and heaps of 10 bundles of 10 sticks each.
- Place value cards, all of the same colour.

Each learner should have their own set of counters (cubes/rods or sticks/bundles) and their own set of place value cards.

Templates for place value cards for learners and teachers are given in the Addendum (pages 416 to 432). Your place value cards for demonstration purposes should be at least A4 size.

Please note: *Page numbers in the text of this Teacher Guide always refer to the Learner Book, unless otherwise stated.*
Addendum page numbers refer to the Addendum at the back of this book.

1.1 Number names and number symbols

Mathematical notes

This section deals with number concept, number names and number symbols for numbers up to 100. This is consolidation of Grade 2 work (although in Grade 2 learners are only expected to work up to 99). The rest of the sections deal with numbers up to 999 as required towards the end of Grade 3 and the first term in Grade 4.

Remember that the concepts introduced here will form the basis of the rest of the number work in Grade 4. These concepts will be touched on repeatedly throughout the year. Try to keep the pace of work moving, so that you complete this unit within the 2 hours allocated to it.

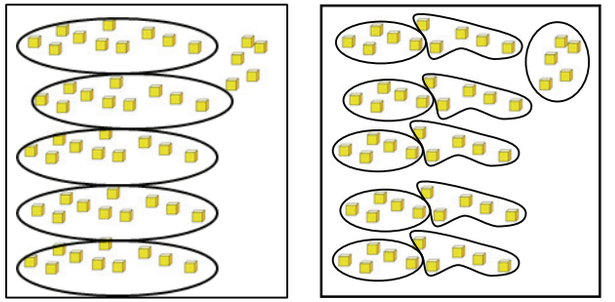
Teaching guidelines

Try to move quickly through Section 1.1: aim to cover this section in 30 minutes. One possibility is to use

- questions 1 and 2 as diagnostic assessment,
- page 4 and questions 3, 4 and 10 for concept development, and
- questions 5, 6, 7, 8, 9, 11, 12 and 13 for additional practice (e.g. as homework).

Questions 1 and 2 are diagnostic activities to help you to quickly assess the state of your new learners' knowledge of whole numbers.

Observe learners while they do question 2. Some learners may notice the groups of ten or groups of five, as shown alongside, and use this to produce the correct answer quickly.



Other learners may just count all the cubes one by one. This will take them much longer. It is a waste of time.

Once all learners have completed question 2, talk to them about how counting in groups can save them time and effort. Part of being good at mathematics is doing things in ways that are simple, smart and quick.

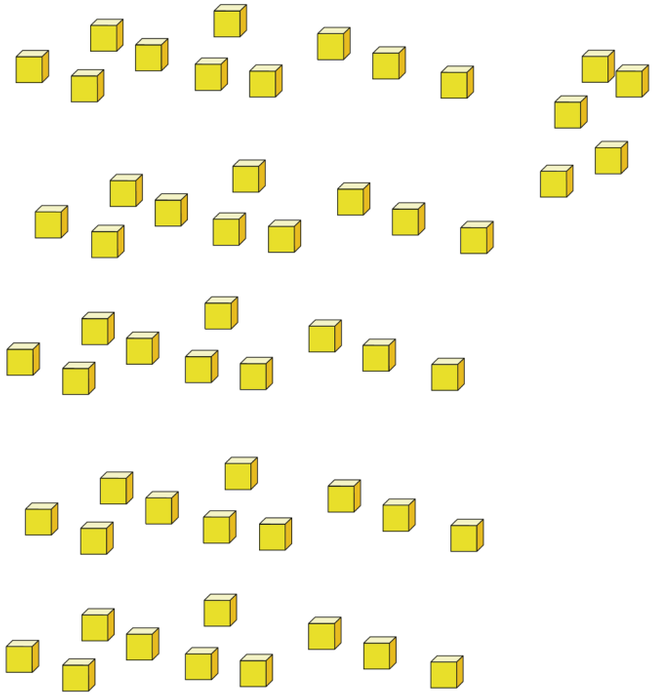
Answers

1. Learners count from 1 to 120.
2. fifty-five

UNIT 1 WHOLE NUMBERS

1.1 Number names and number symbols

1. Count softly or aloud in a group from *one* to *one hundred and twenty*.
2. How many cubes are shown below?
   Write your answer in words, for example *thirty-four*.

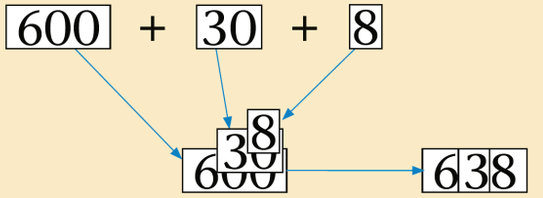


Mathematical notes

Page 4 states that the combined symbol 14 has two parts: 10 and 4. This concept is further explained in the summary bar and tinted passage on page 12, as shown alongside.

The cards that you use in school to show numbers can be called **place value cards**. This is because each card that you use to build a number shows one of the place value parts. Look at this example:

six hundred and thirty-eight is



Teaching guidelines

Hand out the cubes and rods or sticks and bundles to all learners, and ensure that they have paper or plastic bags in which to keep them safely.

You may let each learner pack out each of the numbers 1 to 9 with cubes or sticks, to form an array as shown on page 4.

Hand out the place value cards for units and tens to learners, and ensure that they have envelopes, plastic bags or small boxes in which to keep them safely. Learners must either store their place value cards and cubes or sticks in a safe place in class, or bring them to school every day.

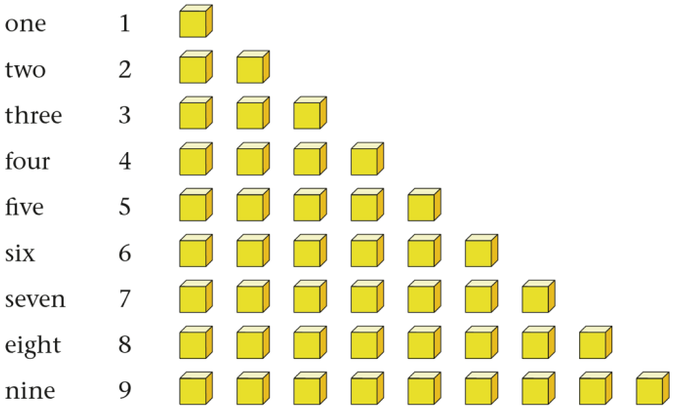
Demonstrate to learners how numbers can be built up with place value cards. Ask them to show numbers with their cards, for example 14. Ask them to show the two cards that they use: first separately, and then together.

The English number names 13 to 19 work in a slightly different way to other number names. Check that learners understand that "teen" stands for the tens part of these numbers. Check also that they understand that "thir" and "fif" stand for the unit parts three and five respectively.

Possible misconceptions

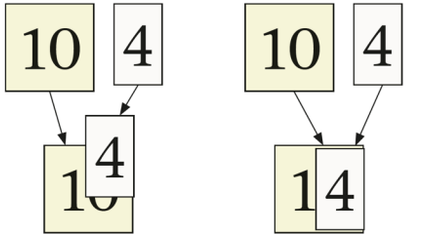
Talk with any learners who use the 1 card and the 4 card to show the number 14. Make sure they understand that 1 + 4 is 5, not 14, hence it is wrong to show 14 with the 1 card and the 4 card. Monitor learners for this error throughout this unit. Explain to all learners that they should place the units card, i.e. 4, over the zero of the tens card.

You already know that we have *single* symbols for the numbers one to nine, as shown below.



You also know that we do not have a single symbol for ten, or for numbers larger than ten.

For larger numbers we use *combined* symbols like 10, 11, 12, 17, 34, 68 and so on. Look at number 14, for example:



The combined symbol 14 shows that the number has two parts: 10 and 4. The 4 sits on top of the 0, as shown here.

The **number name** *fourteen* means *four and ten*.

Teaching guidelines

Let learners show the same 2-digit number in five different ways on their desks:

- by packing it out with cubes and rods or sticks and bundles
- by building it with place value cards
- by writing the number name
- by writing the number symbol
- by writing the expanded notation.

Notes on questions

Questions 3 and 4 have different focuses.

Question 3 focuses learners' attention on the meaning of the English number names, as explained in the summary bar at the bottom of page 4 of the Learner Book and page 5 of this Teacher Guide. This means that the order of the numbers in the answer to question 3 should be as it is in the name of the number, for example 5 and 10.

Question 4 focuses learners' attention on the place value parts of the numbers as explained with the place value cards on page 4. This means that the order of the numbers in the answer to question 4 should be in the place value order, for example 10 and 9.

Answers

3. (a) 5 and 10
   (b) 6 and 10
4. (a) 10 and 9
   (b) 19
5. (a) 40 and 8
   (b) 40 + 8
6. 47

3. (a) What does the number name *fifteen* mean?
   (b) What does the number name *sixteen* mean?

eighteen

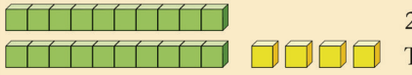
18



10 8

The number eighteen has two parts: ten and eight.
The **number symbol** for eighteen is 18.

4. (a) What are the parts of nineteen?
   (b) What is the number symbol for nineteen?



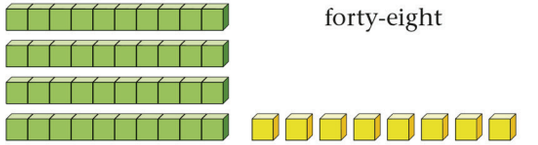
twenty-four
24 = 20 + 4
The number symbol is 24.



When you write this number as 20 + 4, you are using the **expanded notation**.

The English number name *twenty-four* tells you that the expanded notation is 20 + 4.

5. Look at number 48:

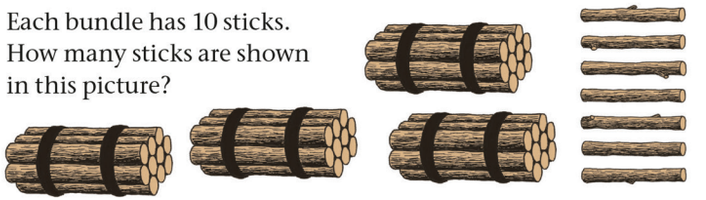


forty-eight



   (a) What are the parts of 48?
   (b) Write the expanded notation for 48.

6. Each bundle has 10 sticks.
   How many sticks are shown in this picture?



Teaching guidelines

To save time you can photocopy the following templates in the Addendum:

- for question 7, the 10 × 10 grid (page 435)
- for questions 10 to 13, the number lines (page 436).

Answers

7.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8. 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

9. 100 95 90 85 80 75 70 65 60 55 50 45 40 35 30 25 20 15 10 5 0

10. 0 10 20 30 40 50 60 70 80 90 100

11. 0 5 10 15 20 25 30 35 40 45 50

12. 0 2 4 6 8 10 12 14 16 18 20

13. 50 52 54 56 58 60 62 64 66 68 70

7. Write the numbers from 1 to 100, with ten numbers in each row as shown below.

1 2 3 4 5 6 7 8 9 10

11 12 13 . . . . . .

21 22 . . . . . .

31 32 and so on.

8. Count in fives by yourself, up to 100. Write the number symbols as you go along:

5 10 15 . . .

The numbers that you have to write in questions 8 and 9 are called **multiples of 5**.

9. Count backwards in fives from 100, and write the number symbols as you go along. Continue until you cannot go any further.

100 95 90 85 . . .

10. Draw a line in your book, and write the numbers 0, 10, 20, 30 . . . and so on up to 100 above the line. Leave more or less equal spaces between the numbers. Make a mark at each number.

0 10 20 100

This is called a **number line**.

You have shown the first ten multiples of 10 on your number line.

11. Draw a number line that starts at 0 and shows the multiples of 5 up to 50.

12. Draw a number line that starts at 0 and shows the multiples of 2 up to 20.

13. Draw a number line that starts at 50. Show the multiples of 2 from 50 up to 70.

1.2 Count in hundreds, tens and units

Mathematical notes

For the rest of this unit learners will be working with numbers up to 999.

Remember that all the concepts raised here will form the basis of the rest of the number work in Grade 4. These concepts will be touched on repeatedly throughout the year. Try to keep the pace of work moving, so that you can complete this unit within the 2 hours allocated to it.

Teaching guidelines

Try to move quickly through Section 1.2: aim to cover this section in 30 minutes. One possibility is to use

- the illustrations on page 7 and 8, questions 1(a), 2(a), the tinted passage on page 9 and the illustration in question 4 for concept development,
- questions 1(b) and (c), 2(b) and (c), 3(a) and (b), 5(a), 7, 8, 9, 10 and 11(e) to (h) for classwork, and
- questions 1(d) and (e), 2(d), 3(c) and (d), 5(b), 6 and 11(a) to (d) for additional practice.

Ask learners to write the following numbers in number symbols: *one hundred*, *one hundred and one*, *one hundred and two* and *one hundred and ten*.

Possible misconceptions

Some learners may write 100 1 instead of 101; 100 2 instead of 102; 100 10 instead of 110. This shows that learners have not yet mastered our way of writing numbers. They are writing the numbers as we say them. It also shows that learners have a good sense of the numbers beyond 100. These learners are writing the numbers in an unconventional expanded notation: they write 100 2 to mean 100 + 2 and 100 10 to mean 100 + 10. This is a good foundation on which to build the work that follows (see Section 1.4, page 13). Use place value cards and the bottom part of the tinted passage on page 9 to show learners how to "make" the numbers and write the symbols.

Answers

1. (a) 20 (b) 50 (c) 70 (d) 100 (e) 120

1.2 Count in hundreds, tens and units

Look at what this picture shows you:



*A heap of **hundred** sticks (10 bundles of 10 sticks each)*

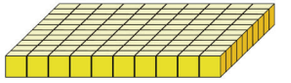


A bundle of ten sticks



One stick

A hundred, ten and one may also be represented in cubes as follows:



*One **hundred** cubes*



Ten cubes



One cube

1. (a) How many sticks are there in 2 bundles of 10 sticks each?
   (b) How many sticks are there in 5 bundles of 10 sticks each?
   (c) How many sticks are there in 7 bundles of 10 sticks each?
   (d) How many sticks are there in 10 bundles of 10 sticks each?
   (e) How many sticks are there in 12 bundles of 10 sticks each?

The symbol for **one hundred** is **100**.
The symbol for three hundred is 300.
The symbol for eight hundred is 800.

GRADE 4: MATHEMATICS [TERM 1] 7

Mathematical notes

According to the Curriculum and Assessment Policy Statement (CAPS), the focus of place value for Grade 4 learners is that they "should be able to break down numbers into hundreds, tens and units using:

- the number names (number words)
- place value cards
- expanded notation

Recommended apparatus: place value cards; Dienes blocks."

DBE (2011). *Curriculum and Assessment Policy Statement Grades 4–6 Mathematics*. Pretoria. Government Printer, page 38.

Place value cards are an indispensable tool to help learners to distinguish, in their own minds, between number symbols and the numbers themselves. It is important to use place value cards correctly. The basic place value card activity is to ask learners to "show" a number with cards. When learners are asked to show a number, for example 246, they should select and hold up the 200, 40 and 6 cards, not the 2, 4 and 6 cards.

Teaching guidelines

Let learners show the same 3-digit number in five different ways on their desks:

- by packing it out with cubes and rods or sticks and bundles
- by building it with place value cards
- by writing the number name
- by writing the number symbol
- by writing the expanded notation.

Answers

2. (a) 200 (b) 500 (c) 700 (d) 1 000
3. (a) 20 (b) 40 (c) 80 (d) 100

300 sticks are shown below, in three heaps of 100 each.



2. (a) How many sticks are there in 2 heaps of 10 bundles each?
   (b) How many sticks are there in 5 heaps of 10 bundles each?
   (c) How many sticks are there in 7 heaps of 10 bundles each?
   (d) How many sticks are there in 10 heaps of 10 bundles each?

10 hundreds is called **one thousand**.
The symbol for one thousand is **1 000**.

When we write the number symbol **1 000**, we usually leave a space between the 1 and the 000. This makes it easier to read the number.

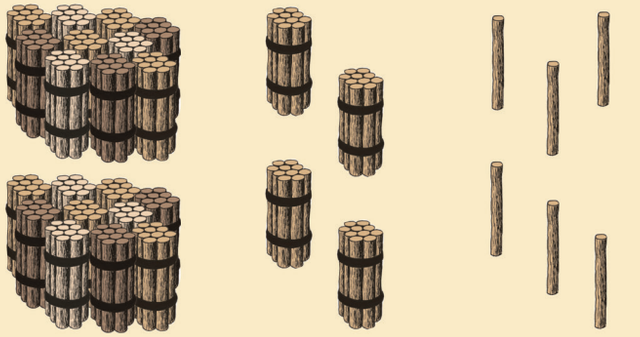
3. (a) How many bundles of 10 are there in 2 heaps of 100 sticks each?
   (b) How many bundles of 10 are there in 4 heaps of 100 sticks each?
   (c) How many bundles of 10 are there in 8 heaps of 100 sticks each?
   (d) How many bundles of 10 are there in 10 heaps of 100 sticks each?

Teaching guidelines

Demonstrate how to use place value cards to show numbers up to 999, that is:

- cover the zeros of the hundreds card with the tens card, and
- cover the zero of the tens card with the units card.

Two hundred and forty-six sticks are shown below.

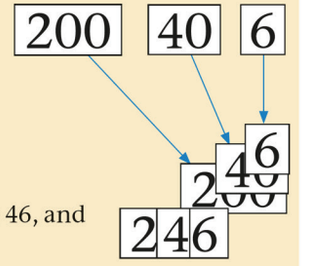


200 sticks *40 sticks* *6 sticks*

The number *two hundred and forty-six* is made up of three parts.
The three parts are:

- two hundred 200 This is the **hundreds part**.
- forty 40 This is the **tens part**.
- six 6 This is the **units part**.

The three parts can be combined to form the number symbol 246:



In the number symbol 246,
the 00 of the 200 is hidden behind the 46, and
the 0 of the 40 is hidden behind the 6.

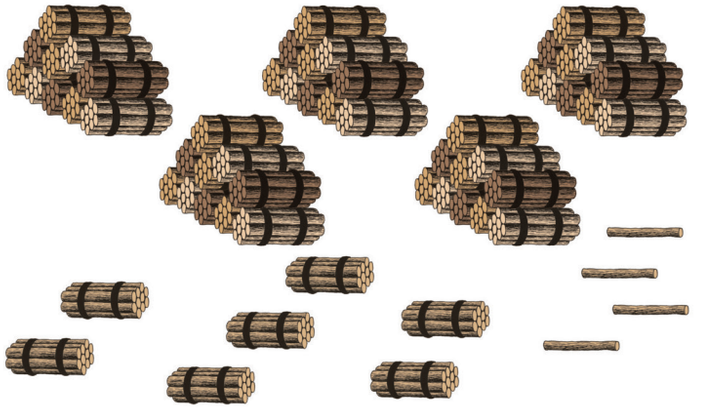
Teaching guidelines

In question 5, ask learners to explain their answers.

Answers

4. 574
5. (a) 501
   (b) 822
6. 475
7. (a) 3 heaps; 84 sticks remaining
   (b) 8 bundles
8. four hundred and sixty-two
   462
   400 + 60 + 2
9. 702
10. 600 and 7
11. (a) 600 40 7 (b) 700 40 6 (c) 400 70 6 (d) 400 60 7
    (e) 700 60 4 (f) 600 70 4 (g) 200 and 70 (h) 200 and 7

4. How many sticks are shown below?



5. Which is more?
   (a) 501 sticks or 389 sticks (b) 699 sticks or 822 sticks
6. How many sticks is 4 heaps, 7 bundles and 5 loose sticks?
7. (a) How many heaps of hundred can be made up from 384 sticks, and how many sticks will remain?
   (b) How many bundles of ten can be made from the sticks that remain?
8. Which number has the parts shown below?

   400 2 60

   Write the number name and the number symbol.
   Also write the number in expanded notation.
9. Which number has the parts 700 and 2?
10. Write down the parts of 607.
11. Write down the parts of each of these numbers.
    (a) 647 (b) 746 (c) 476 (d) 467
    (e) 764 (f) 674 (g) 270 (h) 207

1.3 Building up and breaking down numbers

Mathematical notes

Most methods of calculation with whole numbers involve:

- breaking numbers down into their place value parts,
- rearranging the parts as allowed by the associative, commutative and distributive properties of operations,
- combining parts by applying knowledge of addition, subtraction and multiplication facts, and
- building up the answers.

Teaching guidelines

Section 1.3 is a short section: aim to cover this section in 20 minutes. One possibility is to use

- page 11 and the summary bar and tinted passage on page 12 for concept development,
- questions 1, 3(a) and (c), and 4(a) for classwork, and
- questions 2(c) and (d), 3(b) and (d), and 4(b) and (c) for additional practice.

Choose any 2-digit number that has both a tens part and a units part: the example provided in the Learner Book is 34. Ask learners to write pairs of numbers (in an addition number sentence) that add up to this number. Then ask learners to use the bundles and loose sticks, and place value cards to show the number. Learners should also write down the number symbol in their exercise books, as well as its place value parts.

Possible misconceptions

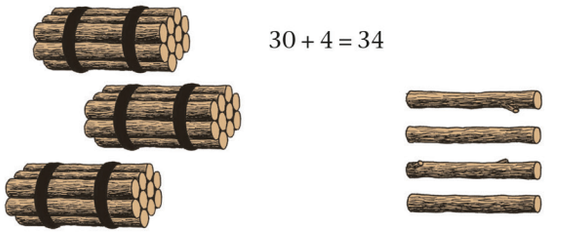
Check to see that all learners are using the correct place value cards (they need to use a tens card and a units card and cover the zero on the tens card with the units card). Check to see that they are writing the number symbol correctly.

Answers

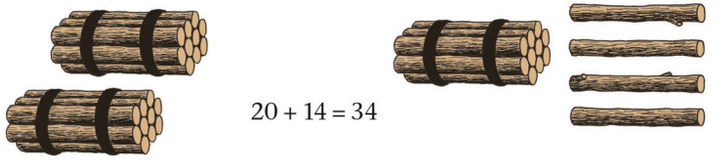
1. Various possibilities, e.g. 10 + 24; 7 + 27; 17 + 17

1.3 Building up and breaking down numbers

The numbers 30 and 4 can be combined to form the single number 34.



The number 34 can also be formed by combining two other numbers, for example 20 and 14.



There are many more ways in which 34 can be formed by combining two *other* numbers, for example:

18 + 16 = 34 25 + 9 = 34 28 + 6 = 34

1. Write down another three different ways in which 34 can be formed by combining two numbers.

You have seen that there are different ways in which 34 can be formed by combining two numbers. The same is true for any other number.

The English number name *thirty-four* and the number symbol *34* tell us *one* of the many ways in which 34 can be formed from two different numbers: they tell us that 34 = 30 + 4.

Answers

2. (a) forty-seven; 47
   (b) sixty-three; 63
   (c) thirty-eight; 38
   (d) eighty-four; 84
3. (a) 70 and 4
   (b) 50 and 9
   (c) 40 and 7
   (d) 80 and 3
4. (a) 500 20 7
   (b) 700 20 5
   (c) 500 70 2

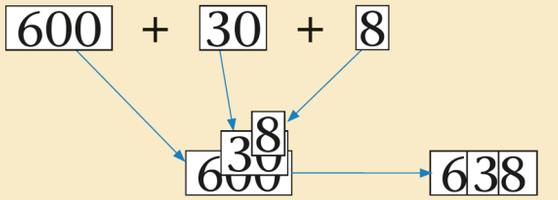
2. Look at the two parts given in each item below. Which number is formed by combining the two parts? Write the number name and the number symbol.
   (a) 40 and 7 (b) 60 and 3
   (c) 30 and 8 (d) 80 and 4

The parts that are mentioned in the name of a number are called the **place value parts**. For example, the place value parts of thirty-seven are 30 and 7.

3. What are the place value parts of each of the following numbers?
   (a) seventy-four (b) fifty-nine
   (c) forty-seven (d) 83

The cards that you use in school to show numbers can be called **place value cards**. This is because each card that you use to build a number shows one of the place value parts. Look at this example:

six hundred and thirty-eight is



4. (a) Which place value cards do you need to build 527?
   (b) Which place value cards do you need to build 725?
   (c) Which place value cards do you need to build 572?

1.4 Number names, expanded notation and number symbols

Mathematical notes

In the previous sections learners broke down numbers into place value parts, i.e. hundreds, tens and units. This lays the basis for what learners will do when they add, subtract and multiply. It also lays the basis for writing numbers in expanded notation.

Teaching guidelines

Section 1.4 is a short section: aim to cover this section in 20 minutes. One possibility is to use

- the tinted passages, questions 1, 2 and 3, and the explanatory text on page 14 for concept development,
- questions 4, 5, 6, 7 and 8 for classwork, and
- questions 9, 10 and 11 for additional practice.

Answers

1. 200, 80 and 3
2. 283
3. 200 + 80 + 3
4. eight hundred and thirty-six
5. 800 + 30 + 6
6. 800, 30 and 6

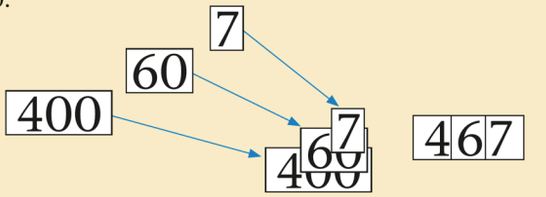
1.4 Number names, expanded notation and number symbols

400 + 60 + 7 is called the **expanded notation** for the number 467.

Four hundred and sixty-seven can be written **in symbols** like this:

400 + 60 + 7

In the short **number symbol** 467, the 60 is written on top of the two zeros of the 400, and the 7 is written on top of the zero of the 60:



1. Write down the place value parts that make up the number *two hundred and eighty-three.*
2. Write the number symbol for two hundred and eighty-three.
3. Write the expanded notation for the number two hundred and eighty-three.
4. Write the number name for 836.
5. Write 836 in expanded notation.
6. Write down the place value parts that make up the number 836.

The number symbol also tells us what the parts of the number are. Look at the number symbol 467, for example:



The "4" tells us that 400 is one of the parts of 467.

The symbol "6" tells us that 60 is one of the parts of 467.



GRADE 4: MATHEMATICS [TERM 1] 13

Mathematical notes

Sometimes learners are asked to state the value of a digit. Question 7 asks learners for the same information in a less abstract and less alienating way.

Sometimes learners are asked for the place value of a digit. Question 8 asks learners for this same information in a less abstract and less alienating way.

Teaching guidelines

Use the explanatory text on page 14 to do concept development before learners do questions 7 and 8.

Answers

7. (a) The 7 tells us that 70 is one of the parts of 573.
   (b) The 7 tells us that 7 is one of the parts of 357.
   (c) The 7 tells us that 700 is one of the parts of 735.
8. (a) Its position is in the tens place.
   (b) Its position is in the units place.
   (c) Its position is in the hundreds place.

7. What does the "7" in each of the following number symbols tell us about the number?
   (a) 573 (b) 357 (c) 735
8. (a) How do you know that the "7" in 573 means 70, and not 7 or 700?
   (b) How do you know that the "7" in 357 means 7, and not 70 or 700?
   (c) How do you know that the "7" in 735 means 700, and not 7 or 70?

The "9" in 298 is in the place where the number of tens is shown. The "9" in 298 tells us that there are 9 tens in 298.



298 = 200 + 90 + 8

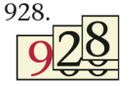
298 = 2 hundreds + 9 tens + 8 units

We can say the place value of the "9" in 298 is tens.

Place values

hundreds	tens	units
2	9	8

The "9" in 928 is in the place where the number of hundreds is shown. The "9" in 928 tells us that there are 9 hundreds in 928.



928 = 900 + 20 + 8

928 = 9 hundreds + 2 tens + 8 units

We can say the place value of the "9" in 928 is hundreds.

hundreds	tens	units
9	2	8

The "9" in 829 is in the place where the number of units is shown. The "9" in 829 tells us that there are 9 units in 829.



829 = 800 + 20 + 9

829 = 8 hundreds + 2 tens + 9 units

We can say the place value of the "9" in 829 is units.

hundreds	tens	units
8	2	9

Answers

9.

100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127	128	129
130	131	132	133	134	135	136	137	138	139
140	141	142	143	144	145	146	147	148	149
150									

10.

420	421	422	423	424	425	426	427	428	429
430	431	432	433	434	435	436	437	438	439
440	441	442	443	444	445	446	447	448	449
450	451	452	453	454	455	456	457	458	459
460									

11.

Number name	Number symbol	Expanded notation
six hundred and thirty-seven	637	600 + 30 + 7
six hundred and thirty-eight	638	600 + 30 + 8
six hundred and thirty-nine	639	600 + 30 + 9
six hundred and forty	640	600 + 40
six hundred and forty-one	641	600 + 40 + 1
six hundred and forty-two	642	600 + 40 + 2
six hundred and forty-three	643	600 + 40 + 3
six hundred and forty-four	644	600 + 40 + 4
six hundred and forty-five	645	600 + 40 + 5
six hundred and forty-six	646	600 + 40 + 6
six hundred and forty-seven	647	600 + 40 + 7
six hundred and forty-eight	648	600 + 40 + 8
six hundred and forty-nine	649	600 + 40 + 9
six hundred and fifty	650	600 + 50
six hundred and fifty-one	651	600 + 50 + 1
six hundred and fifty-two	652	600 + 50 + 2

Note the following:

- The number symbol for twenty-eight is 28 *not* 208.
  208 is the number symbol for two hundred and eight.
- The number symbol for one hundred and eighteen is 118 *not* 10018.
- The number symbol for one hundred and twenty-four is 124, *not* 10024 or 100204.
- The expanded notation for one hundred and twenty-four is 100 + 20 + 4.
- The number name *forty* does not have a letter u.
  We write *four* and *fourteen* but *forty*!

9. Write the number symbols for all the whole numbers from one hundred up to one hundred and fifty.
   Start like this:
   100 101 102 . . .

10. Write the number symbols for all the whole numbers from four hundred and twenty up to four hundred and sixty.

11. Draw three columns in your book.
    (a) Write the number names for all the whole numbers from 637 up to 652 in the first column.
    (b) In the second column, write the number symbols for these numbers.
    (c) In the third column, write the numbers in expanded notation.

1.5 Represent, order and compare numbers

Mathematical notes

When learners have a good understanding of place value and a strong number concept they will find it easy to order and compare numbers. That is why this section is at the end of the unit: it follows lots of work on counting groups of objects and building up and breaking down numbers into place value parts.

Teaching guidelines

Try to move quickly through Section 1.5: aim to cover this section in 30 minutes. One possibility is to use

- questions 1 and 8 as mental mathematics,
- question 10 for concept development,
- questions 2, 3 and 4 for classwork, and
- questions 5, 6, 7 and 9 for additional practice.

If you have already spent more than 2 hours on this topic, you could leave Section 1.5 until you do end-of-term revision or until Term 2.

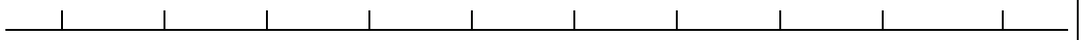
To save time for learners, you can make copies of the 10 × 10 grid and number lines provided in the Addendum on pages 435 and 436.

Answers

1. (a) 150; 153; 156; 159; 162; 165; 168; 171; 174; 177; 180; 183; 186; 189; 192; 195; 198; 201

   (b) 450; 447; 444; 441; 438; 435; 432; 429; 426; 423; 420; 417; 414; 411; 408; 405; 402; 399

2. 330 360 390 420 450 480 510 540 570 600



3. 183; 201; 479; 609; 685; 748; 989

4. 899; 810; 785; 775; 459; 309; 293

5. 450; 480; 510; 540; 570; 600; 630; 660; 690; 720; 750; 780; 810; 840; 870; 900

6. Answer on next page.

1.5 Represent, order and compare numbers

1. (a) Count in threes from 150 until you pass 200. Write down the number symbols as you go along.

   150; . . .; 156; . . .; . . .; 165; . . .; . . .; . . .; 177; . . . . . .

   (b) Count backwards in threes from 450 until you reach 399. Write down the number symbols as you go along.

   450; . . .; 444; . . .; . . .; 435; . . .; . . .; 426; 423; . . . . . .

2. Eight numbers are missing on this number line. Write the numbers from smallest to biggest in your book. You have to count in 30s to do this.



3. Arrange these numbers from smallest to biggest.
   (Hint: start by looking at the hundreds part of the numbers.)

   479 989 201 609 183 685 748

4. Arrange these numbers from biggest to smallest.

   810 775 309 899 785 459 293

5. Count in thirties from 450 until you reach 900.
   Write down the number symbols as you go along.

   450; . . .; 510; . . .; . . .; 600; . . .; . . .; . . .; 720; . . .; 780; . . .; . . .; . . .; 900

6. Count in 25s and complete the number grid.

125	150			
	275			
		425		
			575	
				725

Mathematical notes

The signs <, > and = show the relationships between numbers or expressions. They are called relationship signs.

Teaching guidelines

You can use the text in question 10 to explain to learners when to use the > and < signs.

Answers

6.

125	150	175	200	225
250	275	300	325	350
375	400	425	450	475
500	525	550	575	600
625	650	675	700	725

7. (a) 225 250 275 300 325 350 375 400 425 450 475 500 525

(b) 0 100 200 300 400 500 600 700 800 900 1 000

8. 900; 875; 850; 825; 800; 775; 750; 725; 700; 675; 650; 625; 600; 575; 550; 525; 500

9.

Number symbol	Number name	Expanded notation
634	six hundred and thirty-four	600 + 30 + 4
546	five hundred and forty-six	500 + 40 + 6
329	three hundred and twenty-nine	300 + 20 + 9
910	nine hundred and ten	900 + 10
734	seven hundred and thirty-four	700 + 30 + 4
204	two hundred and four	200 + 4
703	seven hundred and three	700 + 3
948	nine hundred and forty-eight	900 + 40 + 8

10. (a) 498 < 902 (b) 676 < 687 (c) 291 > 289 (d) 653 > 635

7. (a) Make a number line from 225 to 525.
Show 225, 250, 275 and so on on your number line. (Look at the example in question 2.)

(b) Make a number line from 0 to 1 000. Show 100, 200, 300 and so on.

8. Count backwards in 25s from 900 until you reach 500.
Write down the number symbols as you go along.
Start like this:

900; 875; 850; . . .

9. Copy and complete this table.

Number symbol	Number name	Expanded notation
	six hundred and thirty-four	
546		
		300 + 20 + 9
910		
		700 + 30 + 4
	two hundred and four	
703		
		900 + 40 + 8

10. In each case, decide which is the bigger of the two numbers.
Then represent them in this way:

- If the first number is bigger than the second then use >, for example 600 > 500.
- If the first number is smaller than the second then use <, for example 500 < 600.

Notice that the open part of the sign is always towards the bigger number.

(a) 498 and 902 (b) 676 and 687

(c) 291 and 289 (d) 653 and 635

Grade 4 Term 1 Unit 2 Number sentences

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 State addition and subtraction facts	True and false sentences in symbols and words	18 to 19
2.2 Equivalence	Calculations that give the same answer	20 to 21
2.3 Describe patterns with number sentences	Writing number sentences to describe patterns	22 to 23
2.4 Solve and complete number sentences	Making true number sentences	24 to 25

CAPS time allocation	3 hours
CAPS page references	20 and 39 to 42

Mathematical background

A number sentence is a statement about **numbers**, for example 3 × 12 + 5 × 12 = 8 × 12.

A number sentence is a **sentence**, the verb is =, "is equal to" or "is equivalent to".

3 × 12 + 5 × 12 is an expression. It can be called a **calculation plan**, a description of the intention to perform certain calculations.

A number sentence with expressions on both sides, such as 3 × 12 + 5 × 12 = 8 × 12, is a **statement of equivalence**.

It states that the two different calculation plans will produce the same number, which in this case is 96.

The equals sign means that what is on either side of the equals sign has the same value.

When you press the equals sign after entering an expression on the calculator the answer pops up. Many learners think that the equals sign (=) means "here comes the answer". If learners only work with calculations of the form 50 + 30 = □, it can lead them to develop this limited and limiting view of the equals sign. For this reason many of the number sentences in this unit do not take the form 8 + 4 = □. In this unit many of the number sentences have expressions on both sides of the equals sign. This is to help learners understand clearly that an equals sign only indicates that what is on either side of the equals sign has the same value (even if the expressions look different).

Most aspects of mathematics are connected. Sometimes learners do not connect different topics, or even bits of information from one question to another. This slows down their work rate. It also slows down their success rate in Mathematics. Much of this unit is designed to encourage learners to use the answer from one question in subsequent questions. It is also expected that they will use what they know from Unit 1 in *this* unit, and use what they learn in this unit in subsequent units. Encourage learners to link each question with what they have done previously and what they know from previous years. Help learners to develop the practice of thinking: *"What have I done before that can help me here?"*

2.1 State addition and subtraction facts

Mathematical notes

The fact that two **different** sets of calculations (calculation plans) with the same numbers produce the **same** answers, can be expressed in the form of a number sentence. For example, the fact that 3 × 40 + 3 × 8 and 3 × (40 + 8) give the same answers can be expressed with the number sentence:

$$3 \times (40 + 8) = 3 \times 40 + 3 \times 8$$

Such a number sentence is called a **statement of equivalence**.

Number sentences can be **true** or **false**. For example,
5 × (3 + 4) = 5 × 3 + 4 is false, but 5 × (3 + 4) = 5 × 3 + 5 × 4 is true.

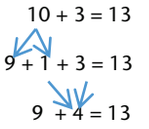
Teaching guidelines

Try to move quickly through Section 2.1: aim to cover this section in 45 minutes. One possibility is to use

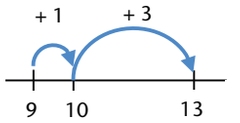
- question 1 for mental mathematics,
- the tinted passage on page 18 and questions 2(c) to (e), 5(d) and (e), and 6(a), (c) and (e) for concept development,
- questions 2(a) and (b), 3, 5(a), (b), (c) and (f), and 6(b) and (d) for classwork, and
- questions 4, 5(g) to (l) and 7 for additional practice.

You can begin by checking whether learners understand the meaning of true and false. You can use the example in the tinted passage.

Many learners will know that 10 + 3 = 13. They can use this number fact to calculate 9 + 4. Help learners to see that what they can do is to split the 10 into 9 + 1 and transfer the 1 to the 3. This will give them 9 + 4: see alongside.



Learners can also use this in reverse, by filling up to the nearest 10, as shown alongside.



Learners can apply versions of the strategies outlined above to use question 2(c) to get the answer to 2(d), and question 2(d) to get the answer to 2(e). They can use questions 2(b) and (c) to answer question 3. They can use questions 2(a) and (c) to answer question 4.

Answers

1. (a) True (b) False (c) True (d) True
2. (a) 10 (b) 9 (c) 10 (d) 12 (e) 12
3. False
4. True

UNIT 2

NUMBER SENTENCES

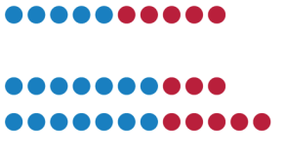
2.1 State addition and subtraction facts

If something is not true, we say it is false.
For example, this sentence is false:
A bird has eight legs.



Sentences such as the following are called **number sentences**:
If you add 3 to 10 you get the same as when you add 4 to 9.
We can also write number sentences in symbols:
9 + 4 = 10 + 3

1. Which of the following sentences are true, and which are false?
   (a) Four apples and three apples, altogether, is seven apples.
   (b) Six apples and three apples, altogether, is ten apples.
   (c) Six apples and one apple, altogether, is seven apples.
   (d) Five apples and two apples, altogether, is seven apples.
2. (a) How much is 5 + 5?
   (b) How much is 5 + 4?
   (c) How much is 7 + 3?
   (d) How much is 7 + 5?
   (e) How much is 8 + 4?



3. Nathi says 5 cubes + 4 cubes is the same number of cubes as 7 cubes + 3 cubes.
   He writes 5 + 4 = 7 + 3.
   Is this true or false?
4. Mpho says 5 + 5 is the same number as 7 + 3.
   She writes 5 + 5 = 7 + 3.
   Is this true or false?

Possible misconceptions

Sometimes we do not notice that we are using different words with the same meaning. Check that learners understand that *plus*, *add* and *the sum of* all mean the same thing. Similarly check that they understand that *subtract*, *minus* and *difference between* have the same meaning.

Sometimes a word or an expression can have one meaning in mathematics and another meaning in other aspects of life, or in different parts of mathematics. For example, when we ask "what is the difference between boys and girls," this *difference between* has a different meaning to "what is the difference between 8 and 4". Learners need to be alerted to the meaning of *difference between* in mathematics, otherwise they may say something like "eight is curvy and four is pointy".

Answers

5. (a) True (b) True (c) True
   (d) False; many possible answers, e.g. 80 + 70 = 100 + 50 or 80 + 70 = 90 + 60
   (e) False; many possible answers, e.g. 70 + 50 = 80 + 40 or 70 + 50 = 90 + 30 or 70 + 50 = 100 + 20
   (f) False; many possible answers, e.g. 19 – 5 = 20 – 6 or 19 – 5 = 18 – 4
   (g) True (h) True (i) True
   (j) True (k) True (l) True
6. (a) The sum of 7 and 9 is equal to the sum of 10 and 6.
   If you add 9 to 7 you will get the same answer as when you add 6 to 10.
   Seven plus nine is equal to ten plus six.
   (b) The sum of 13 and 7 is equal to the sum of 15 and 5.
   If you add 7 to 13 you will get the same answer as when you add 5 to 15.
   Thirteen plus seven is equal to fifteen plus five.
   (c) The difference between 19 and 5 is equal to the difference between 20 and 6.
   If you subtract 5 from 19 you will get the same answer as when you subtract 6 from 20.
   Nineteen minus five is equal to twenty minus six.
   (d) The sum of 5, 3 and 6 is equal to the sum of 6, 5 and 3.
   If you add 5 to 3 and 6 you will get the same answer as when you add 6 to 5 and 3.
   Five plus three plus six is equal to six plus five plus three.
   (e) The sum of six 4s is equal to the sum of four 6s.
   If you add six 4s you will get the same answer as when you add four 6s.
   Four plus four plus four plus four plus four plus four is equal to six plus six plus six plus six.
7. (a) 10 – 3 = 5 + 2 (b) 13 – 8 = 15 – 10 (c) 10 + 4 = 8 + 6

5. Decide which of the sentences below are false.
   If a sentence is false, make it true. Keep the left-hand side the same and change the right-hand side. Then write two more true number sentences with the same left-hand side.

   Example: 5 + 3 = 6 + 4 is false.
   But 5 + 3 = 6 + 2 is true.
   5 + 3 = 2 + 6 and 5 + 3 = 1 + 7 are also true number sentences.

   (a) 8 + 5 = 10 + 3 (b) 8 + 6 = 7 + 7
   (c) 2 + 9 = 9 + 2 (d) 80 + 70 = 100 + 20
   (e) 70 + 50 = 80 + 50 (f) 19 – 5 = 20 – 4
   (g) 13 – 7 = 14 – 8 (h) 13 – 7 = 15 – 9
   (i) 13 – 7 = 20 – 14 (j) 20 + 8 = 10 + 18
   (k) 10 + 6 = 20 – 4 (l) 30 + 17 = 40 + 7

The number sentence 7 + 5 = 9 + 3 can also be said in words, for example in any of the following ways:
The sum of 7 and 5 is equal to the sum of 9 and 3.
If you add 5 to 7 you will get the same answer as when you add 3 to 9.
Seven plus five is equal to nine plus three.

6. Write each of the following number sentences in words, in the three different ways shown above.
   (a) 7 + 9 = 10 + 6 (b) 13 + 7 = 15 + 5
   (c) 19 – 5 = 20 – 6 (d) 5 + 3 + 6 = 6 + 5 + 3
   (e) 4 + 4 + 4 + 4 + 4 + 4 = 6 + 6 + 6 + 6
7. Write each number sentence in symbols.
   (a) The difference between 10 and 3 is equal to the sum of 5 and 2.
   (b) If you subtract 8 from 13 you will get the same answer as when you subtract 10 from 15.
   (c) Ten plus four is equal to eight plus six.

2.2 Equivalence

Mathematical notes

This section is about the concept of equivalence.

To be able to write and interpret statements of equivalence, learners need to know certain basic conventions about the order of operations that are used in symbolic calculation plans. In this section in particular learners need to know the following:

- The order in which you add two numbers does not matter, for example 20 + 70 = 70 + 20. This is the commutative property of addition. If you don't know the answer to 20 + 70 immediately, then it is easier to start with 70 and add on 20.
- When you add many numbers you can group them in different ways without changing the answer, for example (5 + 27) + 3 = 5 + (27 + 3). This is the basis of the associative property of addition.

  Learners do *not* need to know the names of these properties.
- When part of a calculation appears in brackets, you do this first, for example 120 – (40 + 50) = 120 – 90 = 30. This is different to 120 – 40 + 50 = 80 + 50 = 130.

Without adhering to these conventions, different people may interpret the same calculation in different ways and confusion will result.

Teaching guidelines

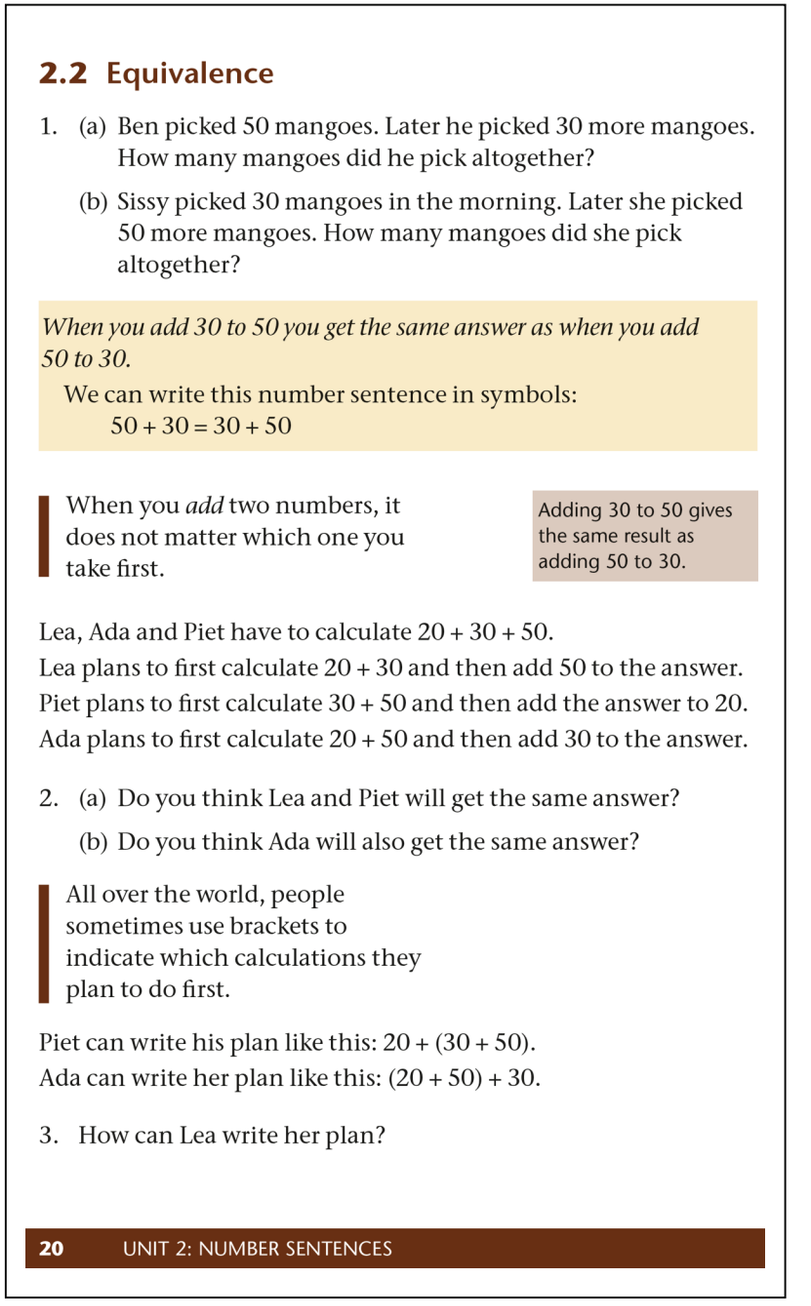
Aim to complete this section in 45 minutes. One possibility is to use

- page 20 and questions 5 and 6(a) for concept development,
- questions 4 and 6(b) to (f) for classwork, and
- question 7 for additional practice.

You can read questions 1(a) and (b) to learners and ask them to calculate the answers. Ask them what they notice about the numbers in the questions and the answers. Use number sentences to explain that when you add two numbers, you can change the order in which you add them without changing the answer. Read the text about Lea, Ada and Piet, and show learners how to write at least one of the calculation plans in a number sentence. Focus on the fact that we first calculate what is inside brackets. Ask learners to write the other calculation plans themselves. Consolidate the role of the brackets by modelling how you would do question 6(a).

Answers

1. (a) 80 mangoes (b) 80 mangoes
2. (a) Yes (b) Yes
3. (20 + 30) + 50



2.2 Equivalence

1. (a) Ben picked 50 mangoes. Later he picked 30 more mangoes. How many mangoes did he pick altogether?

   (b) Sissy picked 30 mangoes in the morning. Later she picked 50 more mangoes. How many mangoes did she pick altogether?

When you add 30 to 50 you get the same answer as when you add 50 to 30.

We can write this number sentence in symbols:

50 + 30 = 30 + 50

When you *add* two numbers, it does not matter which one you take first.

Adding 30 to 50 gives the same result as adding 50 to 30.

Lea, Ada and Piet have to calculate 20 + 30 + 50.
Lea plans to first calculate 20 + 30 and then add 50 to the answer.
Piet plans to first calculate 30 + 50 and then add the answer to 20.
Ada plans to first calculate 20 + 50 and then add 30 to the answer.

2. (a) Do you think Lea and Piet will get the same answer?

   (b) Do you think Ada will also get the same answer?

All over the world, people sometimes use brackets to indicate which calculations they plan to do first.

Piet can write his plan like this: 20 + (30 + 50).
Ada can write her plan like this: (20 + 50) + 30.

3. How can Lea write her plan?

20 UNIT 2: NUMBER SENTENCES

Mathematical notes

Many people think that the equals sign (=) means "here comes the answer". This is what happens when you do a calculation on a calculator: after you have pressed the equals sign the answer pops up. If learners only work with calculations of the form 50 + 30 = □, they can develop this limited and limiting view of the equals sign. This limited understanding of the equals sign is one reason why young learners sometimes struggle with questions of the form: 50 + □ = 80.

The equals sign only means that what is on either side of the equals sign has the same value.

Possible misconceptions

Sometimes teachers use the expression "is the same as" when they read a number sentence with an equals sign. This can be confusing to learners as the expressions on different sides of the equals sign may look different to them. For example, in the number sentence 70 + 50 = 80 + 40, the expressions on different sides of the equals sign *look* different *but have the same value*. It is important to be careful about what words we use when talking about the equals sign: "*has the same value as*" and "*is equal to*" are mathematically appropriate. You can emphasise that although expressions might look different they can have the same value.

Notes on questions

In question 5 learners are not expected to all come up with all possible answers. Write a list of all the answers that the class finds on the board. This will allow each learner to see a greater range of the answers.

Answers

4. (a) 13 + 7 = 20 (b) 8 + 12 = 20
   (c) 8 + 12 = 20 (d) 15 + 5 = 20

5. Learners' answers will vary; they can include:

(5 + 8) + 7	(5 + 7) + 8	5 + (7 + 8)	5 + (8 + 7)
(7 + 8) + 5	(7 + 5) + 8	7 + (8 + 5)	7 + (5 + 8)

6. (a) True (b) True
   (c) False; (20 – 8) – 5 = 20 – (8 + 5) (d) True
   (e) True (f) True

7. None are false (they are all true).

4. Below are four different plans to calculate 8 + 5 + 7.
   The brackets indicate which calculations must be done first.
   Do each calculation in the way the plan states.
   (a) (8 + 5) + 7 (b) 8 + (5 + 7)
   (c) 8 + (7 + 5) (d) (8 + 7) + 5

5. Which other plans can be followed to calculate 8 + 5 + 7?

Read this before you answer question 6:
In some cases in question 6, you may have to *do* the calculations to find out whether the two plans give the same result or not. The calculations given inside the brackets must be done first.

6. Which of the number sentences below are false?
   For each false number sentence, make a true number sentence by writing a different plan on the right-hand side.
   (a) (20 + 8) – 5 = 20 + (8 – 5)
   (b) (20 – 8) – 5 = (20 – 5) – 8
   (c) (20 – 8) – 5 = 20 – (8 – 5)
   (d) (8 + 2) + (7 + 3) = (8 + 3) + (7 + 2)
   (e) (8 + 7) + (2 + 3) = (8 + 3) + (7 + 2)
   (f) (20 + 4) + (10 + 2) = (20 + 10) + (4 + 2)

When you have to *add* many numbers, it does not matter where you start.

7. Which of the number sentences below are false?
   (a) 63 + 26 = (60 + 20) + (3 + 6)
   (b) 8 + 5 = (8 + 2) + 3
   (c) (20 + 7) – (10 + 4) = (20 – 10) + (7 – 4)
   (d) (30 + 6) – (10 + 8) = (20 – 10) + (16 – 8)

2.3 Describe patterns with number sentences

Teaching guidelines

Aim to complete this section in 45 minutes. One possibility is to use

- question 1 for concept development,
- questions 2, 3, 6 and 10 for classwork, and
- questions 4, 5, 7, 8 and 9 for additional practice.

You can start by focusing learners' attention on the number of cubes in each row and in each column in Diagram A. First explain what a row is and what a column is. Ask: *"How many cubes in each row in Diagram A?", "Are there the same number of cubes in each row?"* Repeat these questions for the columns in Diagram A. You can show learners the number sentences for the first and second row in Diagram A. Ask them to write number sentences for all the other rows as quickly as possible. You can then use the number sentences to show learners how each time 1 more is transferred from the group of red cubes to the group of yellow cubes. Furthermore, if they know that 10 + 1 = 11 they can use transfer to work out 7 + □ = 11.

Once learners have completed the number sentences for Diagram B and Diagram D, you can again model how you can use transfer and one of the easy number facts to generate all the other number facts represented by the number sentences.

Help learners to see how they can generalise this process to other number facts, e.g. if 250 + 50 = 300, what is the missing number in 241 + □ = 350 and 231+ □ = 350?

Answers

1. 8 + 3 = 11 7 + 4 = 11 6 + 5 = 11 5 + 6 = 11
   4 + 7 = 11 3 + 8 = 11 2 + 9 = 11 1 + 10 = 11
2. 8 + 2 = 10 7 + 3 = 10 6 + 4 = 10 5 + 5 = 10 4 + 6 = 10
   3 + 7 = 10 2 + 8 = 10 1 + 9 = 10 0 + 10 = 10
3. (a) 8 + 1 = 9 7 + 2 = 9 6 + 3 = 9 5 + 4 = 9
   4 + 5 = 9 3 + 6 = 9 2 + 7 = 9 1 + 8 = 9
   (b) 8 + 0 = 8 7 + 1 = 8 6 + 2 = 8 5 + 3 = 8
   4 + 4 = 4 3 + 5 = 8 2 + 6 = 8 1 + 7 = 8 0 + 8 = 8
4. Diagram A has 10 rows and 11 columns and Diagram B has 8 rows and 9 columns.
5. (a) 6 + 1 = 7 5 + 2 = 7 4 + 3 = 7
   3 + 4 = 7 2 + 5 = 7 1 + 6 = 7
   (b) 6 + 0 = 6 5 + 1 = 6 4 + 2 = 6
   3 + 3 = 6 2 + 4 = 6 1 + 5 = 6 0 + 6 = 6

2.3 Describe patterns with number sentences

In each **row** in Diagrams A, B and C, the total number of cubes = the number of red cubes + the number of yellow cubes.

The rows go from left to right.

1. For the first row in Diagram A, you can write the number sentence 10 + 1 = 11.

   For the second row you can write the number sentence 9 + 2 = 11.

   Write number sentences for all the other rows.

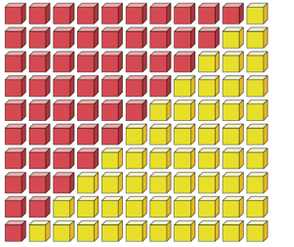


Diagram A

Now look at the **columns** in Diagram A. The columns go from top to bottom.

2. For the first column in Diagram A, you can write the number sentence 10 + 0 = 10.

   For the second column you can write the number sentence 9 + 1 = 10.

   Write number sentences for all columns.

3. (a) Write number sentences for all the rows in Diagram B.

   (b) Write number sentences for all the columns in Diagram B.

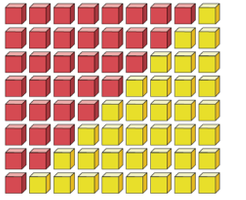


Diagram B

4. How do Diagrams A and B differ?
5. (a) Write number sentences for all the rows in Diagram C.

   (b) Write number sentences for all the columns in Diagram C.

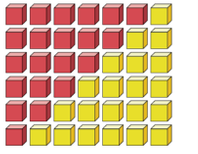


Diagram C

Answers

6. Bottom to top:

5 + 5 = 10	4 + 4 = 8	3 + 3 = 6	2 + 2 = 4	1 + 1 = 2

7. Top to bottom:

4 + 5 = 9	5 + 6 = 11	6 + 7 = 13	7 + 8 = 15
8 + 9 = 17	9 + 10 = 19	10 + 11 = 21	

8. Top to bottom:

4 + 6 = 10	5 + 7 = 12	6 + 8 = 14	7 + 9 = 16
8 + 10 = 18	9 + 11 = 20	10 + 12 = 22	

9. (a) 10 more red cubes
There are many ways that learners can get the answer. A quick way is to see that in Diagram E each row has 1 more red cube than in Diagram D. Learners can count the rows and see that there 10 rows. So there are 10 more red cubes.

(b) 10 more red cubes
Each row in Diagram F has 1 more red cube than in Diagram E.

10. Accept any of the following answers or any others that are true.

The same:

- They have the same number of cubes: 110.
- They both have red and yellow cubes.
- They have the same number of red and yellow cubes.
- In both diagrams both the red cubes and the yellow cubes are arranged in the shape of a triangle.
- The arrangement of the yellow cubes is the same in both diagrams.

Different:

- The overall shape of Diagram A (square) and Diagram D (triangle) is different.
- The red cubes are on the left in Diagram A; they are on the right in Diagram D.
- The position of the red cubes differs in the diagrams.
- In Diagram A the long sides of the triangles of cubes touch. In Diagram D the short sides of the triangles touch.



- In Diagram A all the rows have the same number of cubes. In Diagram D each row has a different number of cubes.
- In Diagram D, the number of red cubes is the same as the number of yellow cubes in each row. In Diagram A, no row has the same number of red cubes and yellow cubes.
- In Diagram A, each row has a total number of 10 cubes. In Diagram D, each row has a different total number of cubes – from one row to the next the total number of cubes increases by 2 as a red cube and a yellow cube is added.

6. We can write the number sentence 10 + 10 = 20 to describe the bottom row in Diagram D.

Write a number sentence for each of the other rows.

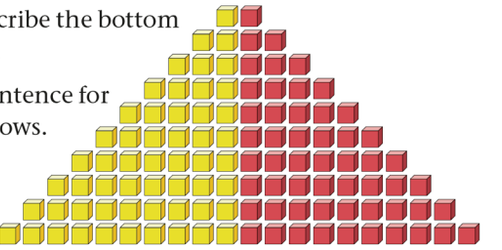


Diagram D

7. Write a number sentence for each row in Diagram E.

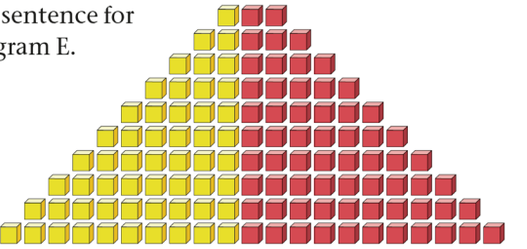


Diagram E

8. Write a number sentence for each row in Diagram F.

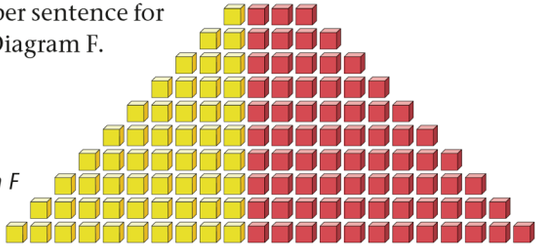


Diagram F

9. (a) There are more red cubes in Diagram E than in Diagram D. How many more?

(b) There are more red cubes in Diagram F than in Diagram E. How many more?

10. What is the same about Diagrams A and D, and what is different?

2.4 Solve and complete number sentences

Mathematical notes

Number sentences can be **open** or **closed**.

The sentence $3 \times (7 + 4) = 3 \times 7 + 3 \times 4$ is a **closed sentence**; all the numbers are given.

$73 + \ldots = 100$ or $73 + \square = 100$ is an **open sentence**; it is incomplete. It contains an **unknown**. It is actually a question: $73 + ? = 100$.

In algebra this is usually called an **equation**.

This section starts by introducing the idea of open number sentences, and completing them by finding the missing number.

Teaching guidelines

Aim to complete this section in 45 minutes. One possibility is to use

- questions 3(a) to (c) and (g) to (x) for mental mathematics,
- questions 1(c), (d), (e) and (h), 2, 3(d) to (f), and 5(a) to (l) for concept development,
- questions 1(a) and (b), and 5(m) to (x) for classwork, and
- questions 1(f), (g), (i) and (j), and question 4 for additional practice.

Notes on questions

In question 2 there are many possible answers. Take feedback on learners' answers. Write these on the board.

The number added to 8 must be 2 more than the number added to 10; this is because 10 is 2 more than 8. Learners in Grade 4 only work with positive numbers, so the numbers added to 8 need to be 2 or more. If 1 is added to 8, then the number added to 10 is –1: this is beyond the requirements for Grade 4 learners.

Answers

1. (a) 5 (b) 60 (c) 200 (d) 30
   (e) 6 (f) 25 (g) 500 (h) 80
   (i) 250 (j) 13

2. (a) to (c) Various possibilities; accept any answers that are true, for example
   $8 + 3 = 10 + 1$ $8 + 20 = 10 + 18$ $8 + 7 = 10 + 5$ $8 + 2 = 10 + 0$
   $8 + 102 = 10 + 100$,
   as long as the difference between the two numbers added is 2,
   with the bigger one added to 8 (the left-hand side of the equation).

2.4 Solve and complete number sentences

The number sentence below is incomplete.
One of the numbers is missing.

$4 + 6 = 2 + ?$

An incomplete number sentence is also called an **open number sentence**.

The number 8 will make the above number sentence true:

$4 + 6 = 2 + 8$

The sentence $4 + 6 = 2 + 8$ is called a **closed number sentence**.

Instead of a question mark, a little block □ or dots . . . or the word *number* may be used to write an open number sentence:

$4 + 6 = 2 + \ldots$ or $4 + 6 = 2 +$ *a number* or $4 + 6 = 2 + \square$

1. In each case, find the number that will make the number sentence true.
   (a) $7 + 3 = 5 + \ldots$ (b) $70 + 30 = 40 + \square$
   (c) $700 + 300 = 800 + \ldots$ (d) $80 + 50 = 80 + 20 + \square$
   (e) $7 + 9 = 10 + \ldots$ (f) $75 + \ldots = 100$
   (g) $\ldots + 500 = 1\,000$ (h) $120 + \ldots = 150 + 50$
   (i) $\ldots + 750 = 1\,000$ (j) $487 + \ldots = 500$

2. (a) Find two different numbers that will make this number sentence true:
   $8 +$ *a number* $= 10 +$ *a different number*
   (b) Find two other numbers that will also make the above number sentence true.
   (c) Find another two numbers that will make the above number sentence true.

Teaching guidelines

Help learners to see how they can use patterns to get to the answers. Perhaps start with questions 3(d), (e) and (f). Explain that if 3 + 6 = 9, then 3 tens + 6 tens = 9 tens, or 30 + 60 = 90, and similarly 3 hundreds + 6 hundreds = 9 hundreds, or 300 + 600 = 900.

Also help learners to see that they can use the answers in some rows to get the answers in the next row. For example, if they have worked out that 3 + 7 = 10 they can use this to solve 3 + 6 because 6 is one less than 7; the answer is one less than 10, i.e. 9.

Answers

3.	(a) 10	(b) 100	(c) 1 000
	(d) 9	(e) 90	(f) 900
	(g) 8	(h) 80	(i) 800
	(j) 10	(k) 100	(l) 1 000
	(m) 8	(n) 80	(o) 800
	(p) 7	(q) 70	(r) 700
	(s) 13	(t) 130	(u) 120
	(v) 13	(w) 130	(x) 110
4.	(a) 7	(b) 70	(c) 700
	(d) 6	(e) 60	(f) 600
	(g) 5	(h) 50	(i) 500
	(j) 4	(k) 40	(l) 400
	(m) 3	(n) 30	(o) 300
	(p) 4	(q) 40	(r) 400
	(s) 5	(t) 50	(u) 500
	(v) 6	(w) 60	(x) 600
5.	(a) 14	(b) 140	(c) 240
	(d) 9	(e) 90	(f) 190
	(g) 8	(h) 80	(i) 180
	(j) 380	(k) 370	(l) 360
	(m) 15	(n) 150	(o) 130
	(p) 7	(q) 70	(r) 670
	(s) 5	(t) 50	(u) 350
	(v) 5	(w) 50	(x) 60

3. Complete the number sentences:

(a) 3 + 7 = . . .	(b) 30 + 70 = . . .	(c) 300 + 700 = . . .
(d) 3 + 6 = . . .	(e) 30 + 60 = . . .	(f) 300 + 600 = . . .
(g) 2 + 6 = . . .	(h) 20 + 60 = . . .	(i) 200 + 600 = . . .
(j) 4 + 6 = . . .	(k) 40 + 60 = . . .	(l) 400 + 600 = . . .
(m) 3 + 5 = . . .	(n) 30 + 50 = . . .	(o) 300 + 500 = . . .
(p) 3 + 4 = . . .	(q) 30 + 40 = . . .	(r) 300 + 400 = . . .
(s) 9 + 4 = . . .	(t) 90 + 40 = . . .	(u) 80 + 40 = . . .
(v) 8 + 5 = . . .	(w) 80 + 50 = . . .	(x) 70 + 40 = . . .

4. Complete the number sentences:

(a) 10 – 3 = . . .	(b) 100 – 30 = . . .	(c) 1 000 – 300 = . . .
(d) 9 – 3 = . . .	(e) 90 – 30 = . . .	(f) 900 – 300 = . . .
(g) 8 – 3 = . . .	(h) 80 – 30 = . . .	(i) 800 – 300 = . . .
(j) 7 – 3 = . . .	(k) 70 – 30 = . . .	(l) 700 – 300 = . . .
(m) 7 – 4 = . . .	(n) 70 – 40 = . . .	(o) 700 – 400 = . . .
(p) 8 – 4 = . . .	(q) 80 – 40 = . . .	(r) 800 – 400 = . . .
(s) 9 – 4 = . . .	(t) 90 – 40 = . . .	(u) 900 – 400 = . . .
(v) 10 – 4 = . . .	(w) 100 – 40 = . . .	(x) 1 000 – 400 = . . .

5. Complete the number sentences:

(a) 9 + 5 = . . .	(b) 90 + 50 = . . .	(c) 190 + 50 = . . .
(d) 14 – 5 = . . .	(e) 140 – 50 = . . .	(f) 240 – 50 = . . .
(g) 13 – 5 = . . .	(h) 130 – 50 = . . .	(i) 230 – 50 = . . .
(j) 430 – 50 = . . .	(k) 430 – 60 = . . .	(l) 430 – 70 = . . .
(m) 8 + 7 = . . .	(n) 80 + 70 = . . .	(o) 60 + 70 = . . .
(p) 15 – 8 = . . .	(q) 150 – 80 = . . .	(r) 750 – 80 = . . .
(s) 13 – 8 = . . .	(t) 130 – 80 = . . .	(u) 430 – 80 = . . .
(v) 12 – 7 = . . .	(w) 120 – 70 = . . .	(x) 130 – 70 = . . .

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Adding on	Adding and subtracting with tens and units; practise using one known number fact to calculate answers to other questions	26 to 28
3.2 Add without counting	Number bonds to 20: using known number facts to do other calculations	28 to 31
3.3 Differences between numbers	Developing calculation strategies for subtraction using number bonds to 20	31 to 34
3.4 Addition facts for multiples of 10 and 100	Using bonds to 10 to develop strategies to add multiples of 10 and 100	34 to 37
3.5 Subtraction facts for multiples of 10 and 100	Using bonds to 10 to develop strategies to subtract multiples of 10 and 100	37 to 40
3.6 Add and subtract with multiples of 10	Using calculation strategies to add and subtract multiples of 10 and 100	40 to 43
3.7 Round off and estimate	Using rounding off and estimation to calculate multiples of 10 and 100	43 to 45
3.8 Break down and build up numbers	Practising calculation strategies	45 to 48
3.9 Solve problems	Using strategies developed in this unit to solve problems	49 to 50

CAPS time allocation	8 hours
CAPS page references	14 to 15 and 43 to 44

Mathematical background

Calculations with multi-digit numbers are done by breaking the task down into separate smaller tasks. For example, the single task 254 + 538 can be broken down into smaller tasks as follows:

Single task		**Three separate tasks**
254 + 538	**= (200 + 50 + 4) + (500 + 30 + 8)**	**= (200 + 500) + (50 + 30) + (4 + 8)**
	(The numbers are broken down into their place value parts.)	(The rearrangement can be done because addition is commutative and associative.)

Learners can only use breaking-down, rearranging and building-up methods effectively if they know the addition and subtraction bonds for units and for multiples of 10 and multiples of 100 well, or can quickly reconstruct these facts. Unfortunately most learners have poor knowledge of addition and subtraction bonds, and can only reconstruct addition and subtraction facts by drawing stripes and counting. Many learners do not even try to remember addition facts like 5 + 7 = 12, and adopt the habit of drawing stripes to count.

Resources

Learners still need sets of place value cards.

You will need two sets of large place value cards for teaching purposes.

Using place value cards when doing addition is a powerful way of consolidating understanding of the role of place value parts, and of the commutative and associative properties of addition.

3.1 Adding on

Critical knowledge

Some Grade 4 learners may still **count all** to add two small numbers, for example count as follows to calculate 8 + 5:

one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen

It is critical that learners progress beyond this way of adding, first by learning to **count on** from the larger number (for example *nine, ten, eleven, twelve, thirteen* in this case), and then to using strategies that are quicker than counting, for example **filling up to the nearest ten** in this case: 8 + 2 → 10 + 3 = 13. Practise these strategies during mental mathematics.

Mathematical notes

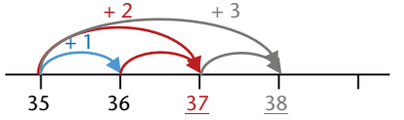
Because so many learners rely on counting to calculate, this unit builds calculation skills and strategies. It starts by encouraging counting-on skills, and then progresses to using known number facts to find other number facts. This is initially done in the Grade 2 calculation range (up to 99). By the end of the unit, learners work with the Grade 3 and Grade 4 Term 1 calculation range: adding and subtracting numbers up to 999. This is a long unit as it is building and consolidating Grade 2 and 3 skills. Encourage learners to work quickly and smartly. Aim to complete Section 3.1 in 45 minutes.

Notes on questions

The numbers in questions 1 to 4 were deliberately chosen to discourage counting.

Teaching guidelines

Talk with learners about mathematics being more than counting, and the importance of linking information and different parts of mathematics. Explain to learners that they should use the statement and answer in each question to quickly find the answer to the next question. For example, if you know 35 + 1 = 36, then 35 + 2 = 36 + 1 = 37. You can demonstrate question 1 using a number line.



Answers

1.	(a)	36	(b)	37	(c)	38	(d)	39	(e)	40	(f)	41
2.	(a)	80	(b)	90	(c)	100	(d)	110	(e)	120	(f)	130
3.	(a)	63	(b)	59	(c)	56	(d)	58	(e)	2		
4.	(a)	49	(b)	50	(c)	50	(d)	51	(e)	20	(f)	21
	(g)	22	(h)	23	(i)	25	(j)	26	(k)	27	(l)	28

UNIT 3

WHOLE NUMBERS: ADDITION AND SUBTRACTION

3.1 Adding on

1. How much is each of the following?
   (a) 35 + 1 (b) 35 + 2 (c) 35 + 3
   (d) 35 + 4 (e) 35 + 5 (f) 35 + 6
2. How much is each of the following?
   (a) 70 + 10 (b) 70 + 20 (c) 70 + 30
   (d) 70 + 40 (e) 70 + 50 (f) 70 + 60
3. (a) What number is seven more than 56?
   (b) What number is three more than 56?
   (c) What number is seven less than 63?
   (d) What number is five less than 63?
   (e) What is the difference between 56 and 58?

You can use one addition fact to find another addition fact.
You know that 11 is 1 more than 10.
So if you know that 5 + 5 = 10, you know that 5 + 6 = 11.
You know that 3 + 3 = 6.
Two more than 6 is 8, so 3 + 5 = 8.
If you always have to count, you will work so slowly that you will never perform well in Mathematics!

Do the questions below without counting on your fingers or other objects.

4. How much is each of the following?
   (a) 48 + 1 (b) 48 + 2 (c) 49 + 1 (d) 48 + 3
   (e) 10 + 10 (f) 10 + 11 (g) 10 + 12 (h) 10 + 13
   (i) 11 + 14 (j) 12 + 14 (k) 13 + 14 (l) 14 + 14

Possible misconceptions

Replacing a given computational task with a combination of separate tasks (as described on the introductory page and shown on page 27 of the Learner Book) only makes sense if the separate tasks are easy for learners to do. This means that learners should know or be able to quickly work out the answers to calculations like 200 + 500, 50 + 30 and 4 + 8.

Learners need to learn basic number facts and acquire skills to reconstruct basic number facts if they are to move beyond drawing stripes and counting. Sections 3.1 and 3.2 help learners to practise using given knowledge to form new knowledge and so avoid counting.

Teaching guidelines

The purpose of the tinted passage is to help learners understand why they have to learn basic addition and subtraction facts, and skills to make facts, namely in order to be able to do calculations with bigger numbers quickly and easily. Although learners do not do calculations with bigger numbers in this section, you may demonstrate the calculation of 543 + 236 (or other numbers) on the board. It works quite well to use large place value cards. The place value cards can be shifted around on the board to visually display the commutative and associative properties of addition.

Encourage learners to think: *"What have I done before that can help me here?"* and to use the information from previous questions in the question that follows. For example, in questions 5(b), (c) and (d) all learners need to do is add 1 to the previous answer:

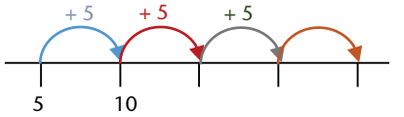
(a) 6 + 6 = 12

(b) 6 + 7 = 6 + 6 + 1 = 12 + 1 = 13

(c) 7 + 7 = 6 + 1 + 7 = 13 + 1 = 14

(d) 7 + 8 = 7 + 7 + 1 = 14 + 1 = 15

In questions 5(e), (f), (g) and (h) 5 is added to the previous answer. Learners can simply count in 5s to get their answers.



In questions 5(i), (j), (k) and (l) 5 is taken away from the previous answer. Learners can simply count back in 5s.

Answers

5.	(a) 12	(b) 13	(c) 14	(d) 15
	(e) 10	(f) 15	(g) 20	(h) 25
	(i) 30	(j) 25	(k) 20	(l) 15
	(m) 10	(n) 10	(o) 10	(p) 10
	(q) 7	(r) 3	(s) 6	(t) 4
	(u) 10	(v) 20	(w) 30	(x) 40

5. How much is each of the following?

(a) 6 + 6	(b) 6 + 7	(c) 7 + 7	(d) 7 + 8
(e) 5 + 5	(f) 10 + 5	(g) 15 + 5	(h) 20 + 5
(i) 35 – 5	(j) 30 – 5	(k) 25 – 5	(l) 20 – 5
(m) 7 + 3	(n) 6 + 4	(o) 5 + 5	(p) 4 + 6
(q) 10 – 3	(r) 10 – 7	(s) 10 – 4	(t) 10 – 6
(u) 8 + 2	(v) 16 + 4	(w) 26 + 4	(x) 36 + 4

To calculate 543 + 236 you can break the numbers down into their place value parts:

500 40 3 200 30 6

You can add the hundreds parts 500 + 200 = 700 **700**

and the tens parts 40 + 30 = 70 **70**

and the units parts. 3 + 6 = 9 **9**

You can then build the answer up: 700 + 70 + 9 = 779 **779**

To add numbers in this way you need to know facts like those below and many others very well.

8 + 5 = 13	80 + 50 = 130	50 + 80 = 130
13 – 8 = 5	130 – 50 = 80	6 + 4 = 10
60 + 40 = 100	70 + 30 = 100	800 – 300 = 500

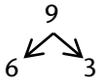
You need to know the answer immediately, or you must be able to work out the answers very quickly.

In this Unit, you will strengthen your knowledge of basic addition facts, and you will learn ways to quickly form facts that you do not know immediately.

Notes on questions

Learners may approach Question 7 in different ways. They do not need to work out how many eggs are in the basket. They can simply work with the numbers they are given.

 + 6 = 44  + 9 = ☐ Learners can split 9 into 6 + 3.



This means that the total number of eggs is 3 more than 44.

 + 9 =  + 6 + 3 = 44 + 3 = 47 eggs.

Question 7 provides a forced experience of finding a total without counting all, because counting all is impossible in this case. It is also an example of using given number facts to make new number facts.

Answers

6. (a) 3 + 7 = 10 (b) 3 + 8 = 11
   (c) 11 – 3 = 8 (d) 11 – 8 = 3
7. "9 eggs more" means 3 more than 6 eggs. She now has 44 + 3 = 47 eggs.

3.2 Add without counting

Teaching guidelines

This section continues the focus on helping learners to build calculation strategies and to make new number facts from given number facts.

Aim to complete this section in 45 minutes.

Think about whether you can use parts of Sections 3.1 to 3.3 for baseline assessment. These sections build basic calculation skills that can be used or consolidated in mental mathematics.

Answers

1. (a) 20 (b) 23
   (c) 20 (d) 20

6. Copy and complete the number sentences. Do not count!
   (a) 3 + 7 = . . . (b) 3 + 8 = . . .
   (c) 11 – 3 = . . . (d) 11 – 8 = . . .
7. Ma Susan is collecting her hens' eggs. She knows that if she finds 6 eggs more, she will have 44 eggs in total in her basket.

    + 6 = 44

   To her surprise Ma Susan finds 9 eggs more, not only 6. How many eggs does she now have in total?

3.2 Add without counting

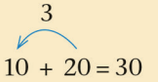
If you always have to count to find facts like 7 + 5 = 12, it will always take you very long to do calculations. *You have to learn to make addition and subtraction facts without counting!*

A good way to do that is to make new facts from facts that you already know. For example if you know that 35 + 2 = 37, it is easy to know that 35 + 3 = 38, because it is just 1 more.

In this section you will learn different ways to form new facts from facts that you already know.

1. You know that 10 + 10 = 20.
   Try to use this fact to quickly find the answers to the following. Start from 10 + 10 = 20 in each case.
   (a) 8 + 12 (b) 10 + 13
   (c) 9 + 11 (d) 7 + 13

An easy way to make a new addition fact from a fact that you know, is to **shift** part of one number to the other number, for example:



By shifting 3 from the 20 to the 10, you get the new fact 13 + 17 = 30.

Answers

2. Note that the order in which two numbers are added does no matter. For example, in (a) learners could write either 25 + 15 = 40 or 15 + 25 = 40. This applies in general to the order of addition and the answers below.

   (a) 25 + 15 = 40

   (b) 23 + 17 = 40

   (c) Any three from: 19 + 21 = 40; 18 + 22 = 40; 16 + 24 = 40; 14 + 26 = 40; 13 + 27 = 40; 12 + 28 = 40; 11 + 29 = 40; 10 + 30 = 40; 9 + 31 = 40; 8 + 32 = 40; 7 + 33 = 40; 6 + 34 = 40, etc.

3. (a) Any three from: 11 + 4 = 15; 13 + 2 = 15; 14 + 1 = 15; 9 + 6 = 15; 8 + 7 = 15

   (b) Any three from: 9 + 8 = 17; 11 + 6 = 17; 12 + 5 = 17; 13 + 4 = 17; 14 + 3 = 17; 15 + 2 = 17; 16 + 1 = 17

4. Many possibilities, e.g.

   If you add 2 to 20, the answer is increased by 2: (20 + 2) + 10 = (30 + 2) → 22 + 10 = 32

   If you subtract 2 from 20 and 1 from 10, the answer is 3 less:

   (20 – 2) + (10 – 1) = (30 – 3) → 18 + 9 = 27

   If you add 2 to 20 and subtract 4 from 10, you must add 2 and subtract 4 from the answer, i.e. (20 + 2) + (10 – 4) = 30 + 2 – 4 → 22 + 6 = 32 – 4 = 28

   (20 + 5) + (10 + 2) = (30 + 7) → 25 + 12 = 37

   (20 – 3) + (10 + 4) = (30 +1) → 17 + 14 = 31

   (20 + 2) + (10 + 4) = (30 + 6) → 22 + 14 = 36

   (20 – 1) + (10 – 4) = (30 – 5) → 19 + 6 = 25

   (20 + 6) + (10 – 4) = (30 + 2) → 26 + 6 = 32

   (20 + 3) + (10 – 2) = (30 + 1) → 23 + 8 = 31

   (20 + 7) + (10 – 3) = (30 + 4) → 27 + 7 = 34

5. 8 + 8 = 16. Shifting 1 from the first 8 to the second 8 → (8 – 1) + (8 + 1) i.e. 7 + 9 = 16
   Subtracting 1 from both 7 and 16 keeps the number sentence correct.
   → (7 – 1) + 9 = (16 – 1) i.e. 6 + 9 = 15

6. 16 + 9 → 16 + 4 → 20 + 5 = 25
   28 + 7 → 28 + 2 → 30 + 5 = 35

2. (a) Shift 5 from the one 20 to the other 20 to make a new fact from 20 + 20 = 40.

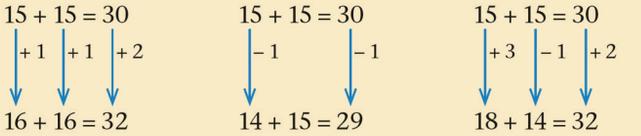
   (b) Shift 3 to make another fact from 20 + 20 = 40.

   (c) Make three more facts from 20 + 20 = 40.

3. (a) Make three different facts from 10 + 5 = 15.

   (b) Make three different facts from 10 + 7 = 17.

The examples below show another way how you can use an addition fact that you already know to build other addition facts.

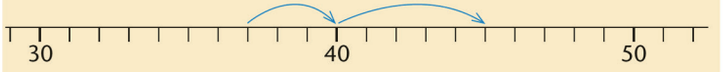


4. Start with 20 + 10 = 30 and make ten other addition facts, all with answers different than 30.

5. Zweli knows that 8 + 8 = 16.
   How can he use this knowledge to find out how much 6 + 9 is?

Here is a different way to find addition facts.

To find out how much 37 + 8 is, you can fill up to 40 and then add what remains of the 8:



You can describe your thinking by using an arrow:

37 + 3 → 40 + 5 = 45

By first adding 3 and then adding 5 you have added 8 in total.

6. Use arrows to show how 16 + 9 and 28 + 7 can be calculated by filling up to 20 and 30.

Answers

7. $15 + 8 \rightarrow 15 + 5 \rightarrow 20 + 3 = 23$
   $35 + 8 \rightarrow 35 + 5 \rightarrow 40 + 3 = 43$
   $17 + 7 \rightarrow 17 + 3 \rightarrow 20 + 4 = 24$
   $46 + 9 \rightarrow 46 + 4 \rightarrow 50 + 5 = 55$
8. (a) $8 + 7 \rightarrow 8 + 2 + 5 \rightarrow 10 + 5 = 15$
   (b) $6 + 7 \rightarrow 6 + 4 + 3 \rightarrow 10 + 3 = 13$
   (c) $8 + 9 \rightarrow 8 + 2 + 7 \rightarrow 10 + 7 = 17$
9. (a) $7 + 5 = 12$ $12 - 5 = 7$ $7 + 3 \rightarrow 10 + 2 = 12$ $12 - 2 \rightarrow 10 - 3 = 7$
   (b) $6 + 9 = 15$ $15 - 9 = 6$ $6 + 4 \rightarrow 10 + 5 = 15$ $15 - 5 \rightarrow 10 - 4 = 6$

Notes on questions

We have stressed the importance of learners knowing certain addition–subtraction facts (number bonds) and learning calculation strategies that allow them to make one number fact from another. One method to learn number facts like bonds and tables is used in question 10 on page 31. This has been used successfully in South Africa and other countries. This is a mental mathematics activity in which learners identify the number facts they can recall at speed. Mental mathematics has two aspects: the rapid recall of number facts and the development of calculation strategies that can be applied quickly.

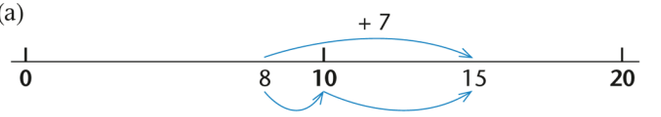
The purpose of question 10 is not to assess learners by giving them marks for those number facts they know. The purpose is for them to think about strategies that they can use to find the answers to questions they do not automatically know the answers for. Learners then practise these strategies with these number bonds until they are able to calculate them at speed and can rapidly recall these number facts. This consolidates the work done on calculating strategies in this unit.

Teaching guidelines

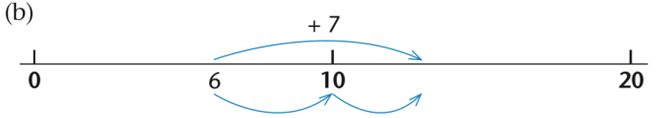
Ask learners to write the letters (a) to (x) in their exercise books. Then ask them to write the answers to questions (a) to (x) as you read them out. They should also have their textbooks open so that they can see the patterns in the structuring of the questions. Reading the questions to learners will help them to keep up the pace. Learners should not spend time working out the answers. Once you have finished reading out the questions, ask learners to turn to the back of their exercise books and write a log. Here they should record the questions they could not answer. They should use strategies they have learnt in this unit to work out the answers. Explain that they should then learn these calculation strategies.

7. Show how 15 + 8, 35 + 8, 17 + 7 and 46 + 9 can be calculated by filling up to tens.
8. The number line diagrams below show how you could be thinking when you are calculating 8 + 7, 6 + 7 and 8 + 9. Use arrows as shown on the previous page to show the same thinking.

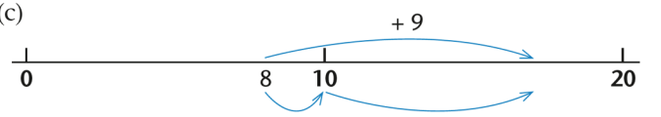
(a)



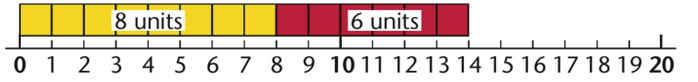
(b)



(c)



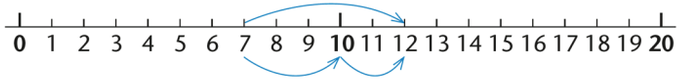
You can see on this number line that 8 + 6 = 14.



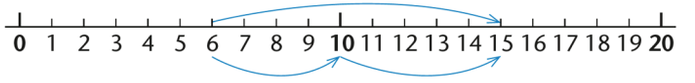
You can also see that 14 – 8 = 6, and 14 – 6 = 8.

9. Which addition fact, and which two subtraction facts can you see on each number line below?

(a)



(b)



It is important for learners to take responsibility for their own learning from an early age rather than always relying on the teacher or parents to tell them how well they are doing or what they still need to learn and do. We suggest that learners themselves identify which calculations they need to work on. Successful learners take responsibility for their own learning.

Answers

10. (a) 10 + 7 = 17 (b) 7 + 8 = 15 (c) 7 + 9 = 16
    (d) 9 + 7 = 16 (e) 7 + 7 = 14 (f) 7 + 6 = 13
    (g) 7 + 10 = 17 (h) 7 + 4 = 11 (i) 7 + 3 = 10
    (j) 10 + 9 = 19 (k) 9 + 8 = 17 (l) 9 + 2 = 11
    (m) 9 + 10 = 19 (n) 9 + 4 = 13 (o) 9 + 3 = 12
    (p) 10 + 6 = 16 (q) 6 + 8 = 14 (r) 6 + 7 = 13
    (s) 6 + 9 = 15 (t) 6 + 6 = 12 (u) 6 + 5 = 11
    (v) 6 + 10 = 16 (w) 6 + 4 = 10 (x) 6 + 3 = 9
11. See above.

3.3 Differences between numbers

Mathematical notes

Subtraction has different meanings. We can also say subtraction relates to different types of situations (see the second row on page 120 in the Intermediate Phase Mathematics CAPS):

- To decrease (take away):
  If I have R50 and I spend R8, I can calculate 50 – 8 to find out how much money I have left.
- To determine a shortfall or missing part of a sum:
  If an item costs R80 and I have only R65, I can calculate 80 – 65 to find out how much money I am short.
- To determine a difference (sometimes called comparison by difference):
  If Mr Setati is 64 years old and Mrs Setati is 28 years old, I can calculate 64 – 28 to establish the difference in their ages.

Answers

1. 7 m
2. (a) 7 (b) 16 (c) 17 (d) 9
   (e) 8 (f) 15 (g) 6 (h) 9
3. 5 cm

10. Copy the number sentences for which you *cannot* find the answers quickly.

    (a) 10 + 7 = ... (b) 7 + 8 = ... (c) 7 + 9 = ...
    (d) 9 + 7 = ... (e) 7 + 7 = ... (f) 7 + 6 = ...
    (g) 7 + 10 = ... (h) 7 + 4 = ... (i) 7 + 3 = ...
    (j) 10 + 9 = ... (k) 9 + 8 = ... (l) 9 + 2 = ...
    (m) 9 + 10 = ... (n) 9 + 4 = ... (o) 9 + 3 = ...
    (p) 10 + 6 = ... (q) 6 + 8 = ... (r) 6 + 7 = ...
    (s) 6 + 9 = ... (t) 6 + 6 = ... (u) 6 + 5 = ...
    (v) 6 + 10 = ... (w) 6 + 4 = ... (x) 6 + 3 = ...

11. Now complete the sentences you have copied in question 10. You may work from the facts that you know, or fill up to 10, or work in any other way you prefer.

3.3 Differences between numbers

The red candle below is 19 units long and the purple candle is 14 units long.

The **difference** between the lengths is 5 units, or

19 – 14 = 5



1. A certain tree is 16 m tall, and another tree is 9 m tall. What is the difference between the heights of the two trees?
2. Calculate each of the following.
   (a) 16 – 9 (b) 9 + 7
   (c) 8 + 9 (d) 17 – 8
   (e) 17 – 9 (f) 6 + 9
   (g) 15 – 9 (h) 15 – 6
3. The blue line is 7 cm long, and the red line is 12 cm long.

   By how much do the lengths of the two lines differ?

Notes on questions

Questions 1, 3, 4(a) and 12 demonstrate the use of subtraction to find a difference.

Questions 5 and 10 demonstrate subtraction as taking away.

Questions 9 and 11 demonstrate subtraction to find a shortfall or missing part of a sum.

Teaching guidelines

Aim to complete Section 3.3 in 1 hour. One possibility is to use

- the tinted passage on page 31, questions 5, 9, 10 and 13 for concept development,
- questions 1, 3, 4, 6, 14 and 15 for classwork, and
- questions 2, 7, 8, 11, 12 and 16 for additional practice.

Note that learners should do the parts of question 17 that relate to questions 14 and 15 as classwork.

Instead of counting to get answers, learners should think: "*What have I done before that can help me here?*", particularly from question 6 onwards.

Answers

4. (a) 8 units (b) 22 units

5. The candle burns down 5 cm every half an hour. For question (c), learners will need to know that $1\frac{1}{2}$ hours = $(\frac{1}{2}+\frac{1}{2}+\frac{1}{2})$ hour, or $3 \times \frac{1}{2}$ hour, so it burns down

   5 cm + 5 cm + 5 cm, or 3×5 cm.

   (a) 13 cm (b) 8 cm (c) 3 cm

6. (a) 13 (b) 8 (c) 3 (d) 7 (e) 10 (f) 11
   (g) 12 (h) 13 (i) 14

7. (a) 5 (b) 7 (c) 5 (d) 6 (e) 13 (f) 4

8. (a) 6 + 4 = 10 (b) 10 – 6 = 4 (c) 10 – 4 = 6
   (d) 10 – 5 = 5 (e) 8 + 4 = 12 (f) 12 – 4 = 8
   (g) 8 + 10 = 18 (h) 18 – 8 = 10 (i) 8 + 9 = 17
   (j) 17 – 8 = 9 (k) 8 + 7 = 15 (l) 15 – 8 = 7

9. 8 + 2 → 10 + 4 = 14; you have to add 6

10. 13 – 3 → 10 – 5 → 5; 13 – 8 = 5

4. One candle is 15 units long and another candle is 7 units long.
   (a) What is the difference between the two lengths?
   (b) How long are these two candles together?

5. This candle was 18 cm long. It burned for half an hour and then it was 5 cm shorter.



   (a) How long is the candle after half an hour of burning?
   (b) In the next half hour of burning, the candle loses another 5 cm in length. How long is it now?
   (c) How long do you think the candle will be after one and a half hours of burning?

6. How much is each of the following?
   (a) 18 – 5 (b) 13 – 5 (c) 8 – 5
   (d) 15 – 8 (e) 20 – 10 (f) 20 – 9
   (g) 20 – 8 (h) 20 – 7 (i) 20 – 6

7. How much is each of the following?
   (a) 9 – 4 (b) 12 – 5 (c) 12 – 7
   (d) 15 – 9 (e) 5 + 8 (f) 11 – 7

8. Write the missing number in each of these number sentences.
   (a) 6 + . . . = 10 (b) 10 – 6 = . . . (c) 10 – 4 = . . .
   (d) 10 – 5 = . . . (e) 8 + . . . = 12 (f) 12 – 4 = . . .
   (g) 8 + . . . = 18 (h) 18 – 8 = . . . (i) 8 + . . . = 17
   (j) 17 – 8 = . . . (k) 8 + . . . = 15 (l) 15 – 8 = . . .

9. What should you add to 8 to get 14?



10. How much is 13 – 8?



Mathematical notes

Most aspects of mathematics are connected. Sometimes learners do not connect different topics, or even bits of information from one question to another. This slows down their work and success rate in Mathematics. Much of this unit is designed to encourage learners to use the answer from one question in subsequent questions.

Notes on questions

Question 11 is the reverse of question 10. Learners should use what they know from one sub-question to get the answers to the other sub-questions in questions 13, 14, 15 and 16. Refer to the "Notes on questions" in Section 3.2, which talks about question 10, for how to handle questions 14, 15 and 16.

Answers

11. $8 + 2 \rightarrow 10 + 3 = 13$; you must add 5.

12. 6 m

13. (a) $14 + \underline{6} \rightarrow 20 + \underline{10} \rightarrow 30 + \underline{2} = 32$

    (b) $6 + 10 + 2 = 18$ (c) 32 (from (a) and (b)) (d) 14 (e) 18

14. Learners copy the number sentences for which they cannot find the answers quickly.

(a) $10 - 5 = 5$	(b) $15 - 8 = 7$	(c) $15 - 7 = 8$
(d) $15 - 9 = 6$	(e) $15 - 5 = 10$	(f) $15 - 6 = 9$
(g) $15 - 10 = 5$	(h) $15 - 4 = 11$	(i) $5 - 3 = 2$
(j) $10 - 7 = 3$	(k) $17 - 8 = 9$	(l) $17 - 9 = 8$
(m) $9 - 7 = 2$	(n) $17 - 7 = 10$	(o) $17 - 6 = 11$
(p) $17 - 10 = 7$	(q) $7 - 4 = 3$	(r) $7 - 3 = 4$
(s) $10 - 4 = 6$	(t) $14 - 8 = 6$	(u) $14 - 7 = 7$
(v) $14 - 9 = 5$	(w) $14 - 4 = 10$	(x) $14 - 6 = 8$

15. Learners copy the number sentences for which they cannot find the answers quickly.

(a) $14 - 10 = 4$	(b) $14 - 5 = 9$	(c) $14 - 3 = 11$
(d) $10 - 9 = 1$	(e) $19 - 8 = 11$	(f) $19 - 7 = 12$
(g) $19 - 9 = 10$	(h) $19 - 2 = 17$	(i) $19 - 6 = 13$
(j) $19 - 10 = 9$	(k) $19 - 4 = 15$	(l) $19 - 3 = 16$
(m) $10 - 6 = 4$	(n) $16 - 8 = 8$	(o) $16 - 7 = 9$
(p) $16 - 9 = 7$	(q) $16 - 6 = 10$	(r) $16 - 5 = 11$

11. What must you add to 8 to get 13?

12. What is the difference between 15 m and 9 m?



13. (a) Copy and complete these calculations:

    $14 + 6 \rightarrow \ldots + 10 \rightarrow \ldots + 2 = \ldots$

    (b) How much did you add on to 14 in total?

    (c) How much is 14 + 18?

    (d) How much is 32 – 18?

    (e) How much is 32 – 14?

Subtraction is the **inverse** of addition. Addition is the **inverse** of subtraction.

14. Copy the number sentences for which you *cannot* find the answers quickly.

(a) $10 - 5 = \ldots$	(b) $15 - 8 = \ldots$	(c) $15 - 7 = \ldots$
(d) $15 - 9 = \ldots$	(e) $15 - 5 = \ldots$	(f) $15 - 6 = \ldots$
(g) $15 - 10 = \ldots$	(h) $15 - 4 = \ldots$	(i) $5 - 3 = \ldots$
(j) $10 - 7 = \ldots$	(k) $17 - 8 = \ldots$	(l) $17 - 9 = \ldots$
(m) $9 - 7 = \ldots$	(n) $17 - 7 = \ldots$	(o) $17 - 6 = \ldots$
(p) $17 - 10 = \ldots$	(q) $7 - 4 = \ldots$	(r) $7 - 3 = \ldots$
(s) $10 - 4 = \ldots$	(t) $14 - 8 = \ldots$	(u) $14 - 7 = \ldots$
(v) $14 - 9 = \ldots$	(w) $14 - 4 = \ldots$	(x) $14 - 6 = \ldots$

15. Copy the number sentences for which you *cannot* find the answers quickly.

(a) $14 - 10 = \ldots$	(b) $14 - 5 = \ldots$	(c) $14 - 3 = \ldots$
(d) $10 - 9 = \ldots$	(e) $19 - 8 = \ldots$	(f) $19 - 7 = \ldots$
(g) $19 - 9 = \ldots$	(h) $19 - 2 = \ldots$	(i) $19 - 6 = \ldots$
(j) $19 - 10 = \ldots$	(k) $19 - 4 = \ldots$	(l) $19 - 3 = \ldots$
(m) $10 - 6 = \ldots$	(n) $16 - 8 = \ldots$	(o) $16 - 7 = \ldots$
(p) $16 - 9 = \ldots$	(q) $16 - 6 = \ldots$	(r) $16 - 5 = \ldots$

Answers

16. Learners copy the number sentences for which they cannot find the answers quickly.

(a) 18 – 9 = 9	(b) 18 – 8 = 10	(c) 18 – 6 = 12
(d) 18 – 10 = 8	(e) 8 – 4 = 4	(f) 8 – 3 = 5
(g) 10 – 3 = 7	(h) 13 – 8 = 5	(i) 13 – 7 = 6
(j) 13 – 9 = 4	(k) 13 – 3 = 10	(l) 13 – 6 = 7
(m) 13 – 10 = 3	(n) 13 – 4 = 9	(o) 13 – 5 = 8
(p) 10 – 2 = 8	(q) 12 – 8 = 4	(r) 12 – 7 = 5
(s) 12 – 9 = 3	(t) 12 – 4 = 8	(u) 12 – 6 = 6

17. See answers to questions 14 to 16 above.

3.4 Addition facts for multiples of 10 and 100

Teaching guidelines

You can use question 1 as a baseline assessment. If learners cannot answer all the questions very quickly, then explain (as in the first tinted passage on page 34) that they know that 4 + 5 = 9, so 4 *tens* + 5 *tens* = 9 *tens*, or 40 + 50 = 90, and 4 *hundreds* + 5 *hundreds* = 9 *hundreds*, or 400 + 500 = 900. Then demonstrate how to use a number line first to add on in tens and then to add on multiples of ten, building up to the nearest 100 (see the tinted passages at the bottom of page 34 and top of page 35).

Aim to complete Section 3.4 in 1 hour. One possibility is to use

- question 1 on page 34 as a baseline assessment,
- the tinted passages on pages 34, 35 and 36 for concept development,
- questions 2(a) to (e), 3(a) and (b), 4(a) and (b), 5(a) and (b), 6, 7, 8 and 9 for classwork, and
- questions 2(f) to (i), 3(c) to (f), 4(c) and (d), 5(c), 10 and 11 for additional practice.

Answers

1. (a) 150 (b) 900 (c) 120 (d) 900 (e) 50
   (f) 400 (g) 160 (h) 60 (i) 90 (j) 1 400

16. Copy the number sentences for which you *cannot* find the answers quickly.

(a) 18 – 9 = ...	(b) 18 – 8 = ...	(c) 18 – 6 = ...
(d) 18 – 10 = ...	(e) 8 – 4 = ...	(f) 8 – 3 = ...
(g) 10 – 3 = ...	(h) 13 – 8 = ...	(i) 13 – 7 = ...
(j) 13 – 9 = ...	(k) 13 – 3 = ...	(l) 13 – 6 = ...
(m) 13 – 10 = ...	(n) 13 – 4 = ...	(o) 13 – 5 = ...
(p) 10 – 2 = ...	(q) 12 – 8 = ...	(r) 12 – 7 = ...
(s) 12 – 9 = ...	(t) 12 – 4 = ...	(u) 12 – 6 = ...

17. Now complete the sentences that you copied in questions 14, 15 and 16. You may work from the facts that you know, or fill up to tens, or work in any other way you prefer.

3.4 Addition facts for multiples of 10 and 100

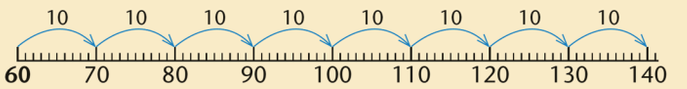
30 + 50 is 3 tens plus 5 tens.
300 + 500 is 3 hundreds plus 5 hundreds.
So, if you know that 3 + 5 = 8,
you can also know that 30 + 50 = 80 and 300 + 500 = 800.

1. How much is each of the following?

   If you don't know the answers immediately, read the text below and on the next page first. Then try again.

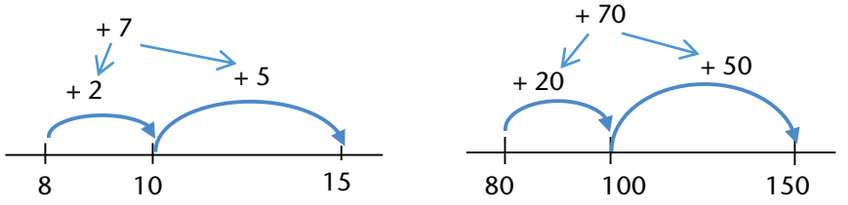
(a) 80 + 70	(b) 700 + 200
(c) 30 + 90	(d) 500 + 400
(e) 80 – 30	(f) 900 – 500
(g) 70 + 90	(h) 140 – 80
(i) 130 – 40	(j) 900 + 500

You can think of **movements on a number line** when you count on in tens to do addition:



Teaching guidelines

You can show learners how to add on in multiples of ten, bridging to the next 100, using the number line. Encourage learners to use an empty number line.



At first you can demonstrate this with units, for example 8 + 7 = □. Then repeat it with multiples of ten, for example 80 + 70 = □. Learners can either do question 2 mentally or draw rough empty number lines.

Ask learners to do question 3 mentally. Show them how to use transfer to make more number facts from doubles facts: see the second tinted passage on page 35.

Answers

2. (a) 150 (b) 130 (c) 130
   (d) 60 (e) 120 (f) 170
   (g) 160 (h) 70 (i) 150
3. (a) 40 (b) 400 (c) 60
   (d) 600 (e) 80 (f) 800
4. (a) 40 + 60 = 100; 30 + 70 = 100; 20 + 80 = 100; 10 + 90 = 100, or any other correct facts.

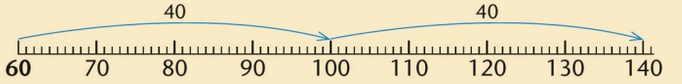
   Some learners may generalise the strategy from the example and offer other solutions, such as 45 + 55 = 100.

   (b) 400 + 600 = 1 000; 300 + 700 = 1 000; 200 + 800 = 1 000; 100 + 900 = 1 000, or any other correct facts.

   (c) 30 + 50 = 80; 20 + 60 = 80; 10 + 70 = 80, or any other correct facts.

   (d) 300 + 500 = 800; 200 + 600 = 800; 100 + 700 = 800, or any other correct facts.

Instead of eight steps of 10 each, you may calculate 60 + 80 in two bigger steps of 40 and 40, by filling up to 100:



You do not have to draw a number line to think like this. You can just think of a number line and write as follows:
60 + 40 → 100 + 40 = 140

You can also use **doubling** to calculate 60 + 80:
Start with 60 + 60 = 120; then add 20 to make 60 + 80 = 140.

2. How much is each of the following?
   (a) 80 + 70 (b) 80 + 50 (c) 70 + 60
   (d) 100 – 40 (e) 50 + 70 (f) 90 + 80
   (g) 70 + 90 (h) 160 – 90 (i) 70 + 80
3. What is the sum in each case?
   (a) 20 + 20 (b) 200 + 200 (c) 30 + 30
   (d) 300 + 300 (e) 40 + 40 (f) 400 + 400

A double can be used to make other facts.
For example, you can **transfer** 100 from the one 300 in 300 + 300 = 600 to the other 300:

100
300 + 300 = 600 ⟶ 200 + 400 = 600

You can make a different fact by transferring 200:

200
300 + 300 = 600 ⟶ 100 + 500 = 600

4. (a) Use 50 + 50 = 100 to make four other facts.
   (b) Use 500 + 500 = 1 000 to make four other facts.
   (c) Use 40 + 40 = 80 to make three other facts.
   (d) Use 400 + 400 = 800 to make three other facts.

Answers

5. (a) Examples: (20 +10) + 30 = (50 + 10) → 30 + 30 = 60
   (20 +10) + (30 +10) = (50 + 20) → 30 + 40 = 70
   20 + (30 +10) = (50 + 10) → 20 + 40 = 60
   (20 + 20) + (30 + 20) = (50 + 40) → 40 + 50 = 90
   (20 + 20) + (30 + 10) = (50 + 30) → 40 + 40 = 80

   (b) Examples: (200 +200) + 300 = (500 + 200) → 400 + 300 = 700
   (200 + 200) + (300 +100) = (500 + 300) → 400 + 400 = 800
   200 + (300 + 200) = (500 + 200) → 200 + 500 = 700
   (200 +100) + (300 + 300) = (500 + 400) → 300 + 600 = 900
   (200 + 300) + (300 +100) = (500 + 400) → 500 + 400 = 900

   (c) Examples: (300 + 200) + 400 = (700 + 200) → 500 + 400 = 900
   (300 + 100) + (400 + 200) = (700 + 300) → 400 + 600 = 1 000
   300 + (400 + 100) = (700 + 100) → 300 + 500 = 800
   (300 + 100) + (400 + 100) = (700 + 200) → 400 + 500 = 900
   (300 + 100) + 400 = (700 + 100) → 400 + 400 = 800

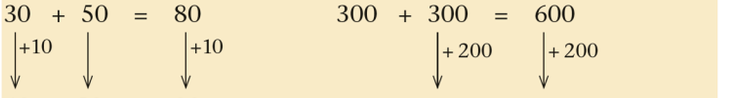
6. (a) 80 + 90 = 170 (b) 80 + 80 = 160 (c) 80 + 60 = 140
   (d) 80 + 10 = 90 (e) 80 + 40 = 120 (f) 80 + 30 = 110
   (g) 10 + 30 = 40 (h) 30 + 80 = 110 (i) 30 + 70 = 100
   (j) 30 + 90 = 120 (k) 30 + 30 = 60 (l) 30 + 60 = 90
   (m) 30 + 10 = 40 (n) 30 + 40 = 70 (o) 30 + 50 = 80

7. See above.

8. (a) 100 + 700 = 800 (b) 700 + 200 = 900
   (b) 900 + 100 = 1 000 (d) 300 + 700 = 1 000
   (e) 800 + 900 = 1 700 (f) 600 + 400 = 1 000
   (g) 100 + 400 = 500 (h) 400 + 300 = 700
   (i) 400 + 200 = 600 (j) 200 + 300 = 500

9. See above.

You can also **add to numbers** in facts that you know, to make new facts about multiples of ten and hundred:



5. (a) Make five other facts from 20 + 30 = 50.
   (b) Make five other facts from 200 + 300 = 500.
   (c) Make five other facts from 300 + 400 = 700.

6. Copy the number sentences for which you *cannot* find the answers quickly.

   (a) 80 + 90 = ... (b) 80 + 80 = ... (c) 80 + 60 = ...
   (d) 80 + 10 = ... (e) 80 + 40 = ... (f) 80 + 30 = ...
   (g) 10 + 30 = ... (h) 30 + 80 = ... (i) 30 + 70 = ...
   (j) 30 + 90 = ... (k) 30 + 30 = ... (l) 30 + 60 = ...
   (m) 30 + 10 = ... (n) 30 + 40 = ... (o) 30 + 50 = ...

7. Now complete the number sentences that you have copied in question 6. You may work from the facts that you know, or fill up to hundreds, or work in any other way you prefer.

8. Copy the number sentences for which you *cannot* find the answers quickly.

   (a) 100 + 700 = ... (b) 700 + 200 = ...
   (c) 900 + 100 = ... (d) 300 + 700 = ...
   (e) 800 + 900 = ... (f) 600 + 400 = ...
   (g) 100 + 400 = ... (h) 400 + 300 = ...
   (i) 400 + 200 = ... (j) 200 + 300 = ...

9. Now complete the sentences that you copied in question 8. You may work from the facts that you know, or work in any other way you prefer.

Answers

10. (a) 600 (b) 600
    (c) 100 (d) 400
    (e) 600 (f) 700
    (g) 900 (h) 30
    (i) 90 (j) 70
    (k) 40 (l) 60
    (m) 770 (n) 495
11. (a) 780 (b) 900
    (c) 950 (d) 650

3.5 Subtraction facts for multiples of 10 and 100

Mathematical notes

This section focuses on subtraction facts for multiples of 10 and 100. It focuses learners' attention on the connection between addition and subtraction.

Teaching guidelines

Aim to complete Section 3.5 in 1 hour. One possibility is to use

- the tinted passages on pages 38 and 40, questions 6(a) and (b), 9(b) and 10(a) for concept development,
- questions 1, 2, 3, 4, 5, 6(c) and (d), 7, 8, 9(a), (c) and (d), 10(b), (c) and (d), 11, 12 and 13 for classwork, and
- questions 14 and 15 for additional practice. Explain to learners that they should first read the first tinted passage on page 40.

Let learners start by working through questions 1 to 5 as quickly as they can. Then use the tinted passage at the top of page 38 to show learners the connection between addition and subtraction. They ought to have used this in questions 1 to 5. They also should have seen that questions 1, 2 and 3 were connected by the numbers and operations. Similarly questions 4 and 5 are connected by the numbers and operations.

Answers

1. (a) 500 chickens (b) 700 chickens
2. (a) 700 (b) 500 3. 500
4. 500 sheep 5. (a) 300 (b) 200

10. Find the numbers that are missing from the number sentences below. Write the answers only.
    (a) 400 + . . . = 1 000 (b) 1 000 – 400 = . . .
    (c) 40 + 60 = . . . (d) 340 + 60 = . . .
    (e) 570 + 30 = . . . (f) 570 + 130 = . . .
    (g) 570 + 330 = . . . (h) 270 + . . . = 300
    (i) 210 + . . . = 300 (j) 530 + . . . = 600
    (k) 160 + . . . = 200 (l) 740 + . . . = 800
    (m) 400 + 370 = . . . (n) 300 + 195 = . . .

11. How much is each of the following?
    (a) 300 + 400 + 20 + 60 (b) 200 + 300 + 400
    (c) 500 + 30 + 400 + 20 (d) 200 + 300 + 80 + 70

3.5 Subtraction facts for multiples of 10 and 100

1. Farmer Nhlapo has 700 chickens. His wife has no chickens.
   (a) Farmer Nhlapo gives 200 of his chickens to his wife. How many chickens does he have left?
   (b) How many chickens do Farmer Nhlapo and his wife have altogether?
2. (a) Calculate 200 + 500.
   (b) Calculate 700 – 200.
3. What number is missing from this number sentence?
   □ – 300 = 200
4. Farmer Marais has many sheep. He gives 300 sheep to his brother. Now he has 200 sheep left.
   How many sheep did farmer Marais have, before he gave 300 sheep to his brother?
5. (a) How much is 500 – 200?
   (b) How much is 500 – 300?

Mathematical notes

In this section learners work with the relationship between addition and subtraction. If the same numbers are used, then subtraction undoes what addition does, for example 3 + 4 = 7 implies that 7 – 4 = 3 and also that 7 – 3 = 4. Similarly, addition undoes what subtraction does: if 8 – 3 = 5, then 3 + 5 = 8 and 5 + 3 = 8. This means that learners can use addition to check their subtraction calculations and use subtraction to check their addition calculations. It also means that learners can use addition to do subtraction calculations. This is the focus of this section. More particularly, learners use "filling up" to 100, as they did in Section 3.4.

In Unit 2 Section 2.2 of this guide we discussed the commutative property of addition.

- The order in which you add two numbers does not matter. For example, 200 + 700 = 700 + 200. This is the commutative property of addition. If you don't know the answer to 200 + 700 immediately, then it is easier to start with 700 and add on 200.

We mentioned that learners do *not* need to know the name of this property, but that they should use it to make calculations more manageable.

Teaching guidelines

This section starts with word problems. Learners can use any method they like to solve the problems, but do discourage counting in units. The questions have been designed to highlight the relationship between addition and subtraction. In the Intermediate Phase CAPS addition and subtraction are always discussed together. This is because learners can use addition to do subtraction calculations.

Focus learners' attention on question 6. Suggest to them that they can rewrite the number sentences to begin with the biggest number, for example 200 + 700 = 700 + 200. It is far more efficient to count on from 700. This is dealt with again in Section 3.6, question 4, page 41.

Use the tinted passages for concept development and especially for explaining the "filling up" method. You might like to refer to the "Teaching guidelines" for Term 2 Unit 2 Section 2.2, which outline a suggested role play to explain using adding on to subtract.

Answers

6. (a) 90; 90 – 50 = 40 and 90 – 40 = 50

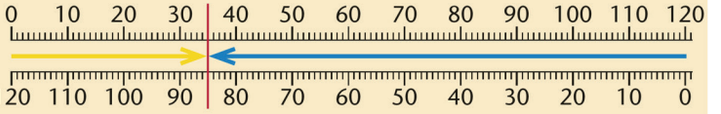
   (b) 900; 900 – 300 = 600 and 900 – 600 = 300

   (c) 80; 80 – 30 = 50 and 80 – 50 = 30

   (d) 900; 900 – 200 = 700 and 900 – 700 = 200

When you know an addition fact, you can easily find two subtraction facts.

For example if you know that 35 + 85 = 120, you also know that 120 – 35 = 85 and 120 – 85 = 35.



6. In each case give the answer, and write two subtraction facts. Follow the above example of 35 + 85 = 120.

   (a) 40 + 50 (b) 300 + 600

   (c) 30 + 50 (d) 200 + 700

One way to subtract is to ask yourself what must be added to the smaller number to make up the difference.

For example, to calculate 160 – 70 you may ask yourself what must be added to 70 to get to 160, and first fill up to 100. It is like finding the missing numbers in number sentences:

70 + *what number?* → 100 + *what number?* = 160

When you have found the missing numbers, you can add them to get the final answer.

70 + **30** → 100 + **60** = 160

So, 160 – 70 = 30 + 60 which is 90.

So 160 – 70 = 90

You can also subtract piece by piece, filling up to 100 first.

For example, to calculate 130 – 80 you may think like this:

130 – 30 = 100. *Now there is 50 more to subtract.*

100 – 50 = 50

So, 130 – 80 = 50

The above thinking can also be described as follows:

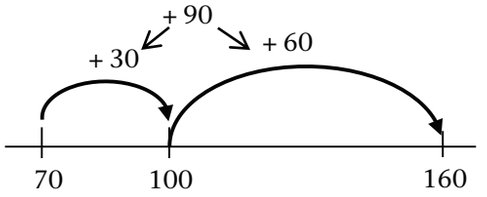
130 – **30** → 100 – **50** = 50, so 130 – 80 = 50

Notes on questions

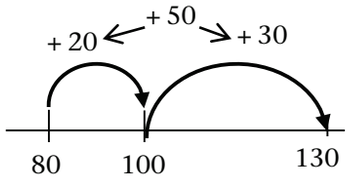
In question 12 learners practise and apply what they have learnt in this section. It is important that learners show addition number sentences for each subtraction number sentence. This movement between addition and subtraction will encourage the development of mental calculation strategies and efficiency with number bonds. Learners should also take note of the connections between sub-questions, e.g. sub-questions (b), (c), (e) and (f) use transfer to get the answers quickly; i.e., if 170 – 80 = 90, then 170 – 90 = 80. They do not have to work out the second calculation.

Answers

7. 160 – 70 = 30 + 60 = 90



8. 130 – 80 = 20 + 30 = 50



9. (a) 70 **+ 30** → 100 **+ 20** = 120, so 120 – 70 = **30 + 20 = 50**
   (b) 50 **+ 50** → 100 **+ 40** = 140, so 140 – 50 = **50 + 40 = 90**
   (c) 70 **+ 30** → 100 **+ 60** = 160, so 160 – 70 = **30 + 60 = 90**
   (d) 50 **+ 50** → 100 **+ 30** = 130, so 130 – 50 = **50 + 30 = 80**
10. (a) 120 **– 20** → 100 **– 60** = 40, so 120 **– 80** = **40**
    (b) 140 **– 40** → 100 **– 30** = 70, so 140 **– 70** = **70**
    (c) 150 **– 50** → 100 **– 30** = 70, so 150 **– 80** = **70**
    (d) 130 **– 30** → 100 **– 50** = 50, so 130 **– 80** = **50**
11. Learners check their answers by doing addition, e.g. question 9(a): 70 + 50 = 120
12.

(a)	100 – 70	= 30	(b)	170 – 80	= 90	(c)	170 – 90	= 80
(d)	130 – 90	= 40	(e)	170 – 70	= 100	(f)	170 – 60	= 110
(g)	170 – 10	= 160	(h)	130 – 60	= 70	(i)	160 – 50	= 110
(j)	100 – 40	= 60	(k)	140 – 80	= 60	(l)	140 – 70	= 70
(m)	140 – 90	= 50	(n)	140 – 40	= 100	(o)	140 – 60	= 80
(p)	100 – 90	= 10	(q)	190 – 80	= 110	(r)	190 – 70	= 120
(s)	100 – 60	= 40	(t)	160 – 80	= 80	(u)	160 – 70	= 90
(v)	160 – 90	= 70	(w)	160 – 60	= 100	(x)	150 – 70	= 80

13. See above.

7. Make a rough drawing of a number line to show how 160 – 70 can be calculated by adding on to 70 as shown on the previous page.
8. Make a rough drawing of a number line to show how 130 – 80 can be calculated by subtracting 80 piece by piece.
9. Use arrows as you did before to show how the following can be calculated by filling up to 100.
   (a) 120 – 70 (b) 140 – 50
   (c) 160 – 70 (d) 130 – 50
10. Show how the following can be calculated by subtracting piece by piece.
    (a) 120 – 80 (b) 140 – 70
    (c) 150 – 80 (d) 130 – 80
11. Check each of your answers for questions 9 and 10 by doing addition.
12. Write the answers for the questions that you can do quickly, and show with addition that your answers are right. Copy the number sentences for which you *cannot* find the answers quickly.

(a) 100 – 70 = . . .	(b) 170 – 80 = . . .	(c) 170 – 90 = . . .
(d) 130 – 90 = . . .	(e) 170 – 70 = . . .	(f) 170 – 60 = . . .
(g) 170 – 10 = . . .	(h) 130 – 60 = . . .	(i) 160 – 50 = . . .
(j) 100 – 40 = . . .	(k) 140 – 80 = . . .	(l) 140 – 70 = . . .
(m) 140 – 90 = . . .	(n) 140 – 40 = . . .	(o) 140 – 60 = . . .
(p) 100 – 90 = . . .	(q) 190 – 80 = . . .	(r) 190 – 70 = . . .
(s) 100 – 60 = . . .	(t) 160 – 80 = . . .	(u) 160 – 70 = . . .
(v) 160 – 90 = . . .	(w) 160 – 60 =	(x) 150 – 70 = . . .

13. Now complete the sentences you have copied in question 12. You may work in any way you prefer.

Answers

14. 900 – 100 = 800 900 – 800 = 100
    900 – 500 = 400 900 – 400 = 500
    900 – 700 = 200 900 – 200 = 700

15. (a) 500 (b) 300 (c) 400 (d) 600
    (e) 500 (f) 400 (g) 700 (h) 400

3.6 Add and subtract with multiples of 10

Mathematical notes

This section consolidates the work done in Sections 3.4 and 3.5. Learners continue to use the following calculation strategies:

- breaking down numbers into hundreds and tens (this is further extended in Section 3.8)
- filling up to a multiple of hundred (this is further extended in Section 3.8)
- adding on
- rounding off and compensating (this is further extended in Section 3.7)
- thinking of number line images to help when calculating.

New methods are continuously introduced to learners so that they have a range of methods they can use. This will allow them to be flexible in their calculation methods.

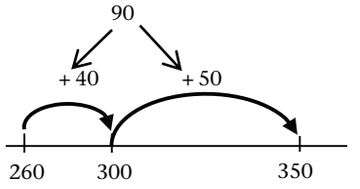
Teaching guidelines

Aim to complete Section 3.6 in 1 hour. One possibility is to use

- the tinted passages on pages 41 and 42, and question 3 for concept development,
- questions 1, 2, 4(a) to (e), 9 and 10 for classwork, and
- questions 4(f) to (j), 5, 6, 7 and 8 for additional practice.

Answers

1. (a) 350 (b) 350 (c) 230 (d) 820
2. (a) 350 because 270 + 30 → 300 + 50 → 350
   (b) 540 (c) 650 (d) 440



There are eight different ways to form 900 by adding two multiples of hundred:

100 + 800 = 900 200 + 700 = 900
300 + 600 = 900 400 + 500 = 900
500 + 400 = 900 600 + 300 = 900
700 + 200 = 900 800 + 100 = 900

If you know that 300 + 600 = 900, you also know two subtraction facts, namely 900 – 600 = 300 and 900 – 300 = 600.

14. Write all the other subtraction facts that you can know if you know the different addition facts given above.
15. How much is each of the following?
    (a) 900 – 400 (b) 900 – 600
    (c) 1 000 – 600 (d) 1 000 – 400
    (e) 800 – 300 (f) 700 – 300
    (g) 1 000 – 300 (h) 600 – 200

3.6 Add and subtract with multiples of 10

1. Calculate each of the following. You may think of movements on a number line and filling up to the nearest 100 to help you to do this.
   (a) 260 + 90 (b) 280 + 70
   (c) 170 + 60 (d) 670 + 150

You need not draw a number line when you add *by filling up*. You can just write as follows:

280 + 20 → **300** + 70 = 370, so 280 + 90 = 370

2. Calculate.
   (a) 270 + 80 (b) 480 + 60
   (c) 560 + 90 (d) 370 + 70

Notes on questions

In question 4 learners can use any method. Please take note of learners' preferred method of calculation and the methods they choose *not* to use. It is important that for question 4(d) and (h) learners recognise that they can "switch" the numbers around and place the bigger number first, i.e. (d) 230 + 60 and (h) 380 + 70. This will allow for a far more efficient calculation.

Answers

3. (a) 200 + 70 + 70 → 200 + 140 → 200 + 100 + 40 → 300 + 40 = 340
   270 + 30 → 300 + 40 = 340

   (b) 600 + 60 + 80 → 600 + 140 → 600 + 100 + 40 → 700 + 40 = 740
   660 + 40 → 700 + 40 = 740

   (c) 300 + 50 + 200 + 80 → 300 + 200 + 130 → 500 + 100 + 30 = 630
   350 + 50 → 400 + 230 → 400 + 200 + 30 → 600 + 30 = 630

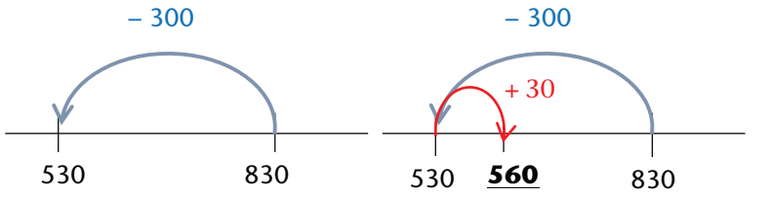
4. (a) 810 (b) 810 (c) 530 (d) 290
   (e) 720 (f) 760 (g) 820 (h) 450
   (i) 830 (j) 890

Teaching guidelines

The second tinted passage demonstrates two methods that learners can use to do addition and subtraction calculations.

Learners have used rounding off in Grade 3. Here they are using rounding off as part of a calculation strategy. You might like to look at the "Teaching guidelines" on rounding off in Section 3.7. You can use the tinted passage at the bottom of page 43 to explain rounding off and compensating, but it is also useful to illustrate this on a number line. For example, 830 – 270 can be shown in stages like this: 830 – 300 → 530 + 30 = 560

When learners do rounding off and compensating it is important that they remember to put back the numbers. It is important that they practise this method often in mental mathematics, especially with a lower number range.



A different method to add multiples of 10 is to break numbers down into hundreds parts and tens parts, for example:

260 + 90 = 200 + 60 + 90
= 200 + 150
= 200 + 100 + 50
= 300 + 50
= 350

You need not write down all the steps when you work like this.

3. Calculate each of the following in two ways.
   Do it by breaking numbers down into hundreds and tens.
   Also do it by filling up to a multiple of hundred.
   Show your thinking clearly in writing in each case.
   (a) 270 + 70 (b) 660 + 80 (c) 350 + 280
4. Calculate each of the following in the way that you prefer.
   You may prefer different methods for different calculations.
   (a) 780 + 30 (b) 580 + 230
   (c) 470 + 60 (d) 60 + 230
   (e) 630 + 90 (f) 420 + 340
   (g) 480 + 340 (h) 70 + 380
   (i) 730 + 100 (j) 730 + 160

It sometimes makes addition easier if you switch the numbers around.

Addition can be used to check whether subtraction was done correctly.

You can calculate 940 – 380 by **adding on** to 380 until you reach 940, as shown here:
380 + **20** → 400 + **540** = 940
So 940 – 380 = 20 + 540, which is 560.

You can also subtract by **rounding off and compensating**.
For example, you can calculate 830 – 270 by first subtracting 300:
830 – 300 = 530 but you have now subtracted 30 too much.
So you have to compensate by putting the 30 back:
530 + 30 = 560, so 830 – 270 = 560.

Answers

5. (a) 490; 460 (b) 490; 460 (c) 360 + 490 = 850; 270 + 460 = 730

6. (a) 260 (b) 240 (c) 270 (d) 480

   (e) 340 (f) 80 (g) 140 (h) 590

   Check answers with addition.

Teaching guidelines

Learners often break down numbers into place value parts. In some subtraction examples this does not make the calculation easier.

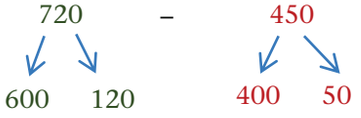
You can use the tinted passage on page 42 to explain to learners how to break down numbers and then build them up to make the subtraction more manageable. Learners can say to themselves: "*How can I split and recombine **these particular numbers** to make the calculation easier?*" You can also use arrow diagrams like the ones below to help learners recognise ways of breaking the numbers down.



700 – 400 *This is easy to do.*

20 – 50 *This is NOT easy to do.*

Let's try another way to break it down:



600 – 400 = 200 *This is easy to do.*

120 – 50 = 70 *This is manageable.*

Focus learners' attention on the recording strategy suggested in the tinted passage.

Possible misconceptions

Some learners may assume that because they can swap the order of numbers when they add, they can do the same with subtraction. Learners sometimes swap the order of numbers they subtract, especially in examples like 20 – 50. However, subtraction is *not* commutative. This needs to be stressed through examples and be explicitly stated: if you swap the order of numbers when you subtract you are doing a different calculation and will get a different answer.

5. (a) Calculate 850 – 360 and 730 – 270 in any way you prefer.

   (b) Check your answers by doing the same calculations in a different way.

   (c) Also check your answers by doing addition.

6. Calculate in any way you prefer, and check each answer by doing addition.

   (a) 780 – 520 (b) 520 – 280

   (c) 430 – 160 (d) 760 – 280

   (e) 630 – 290 (f) 420 – 340

   (g) 480 – 340 (h) 970 – 380

Another way to subtract is to **break the numbers down** and **work with the parts**, then **build the answer up**.

For example, to calculate 720 – 450 you can think like this:

720 is 700 and 20 and

450 is 400 and 50.

It is easy to subtract 400 from 700 but to subtract 50 from 20 creates a problem.

To make it easier, you can replace 700 + 20 with 600 + 120.

Then you can subtract 50 from 120 and 400 from 600.

So, 720 = 600 + 120 and 450 = 400 + 50.

Hence $720 - 450 = \underbrace{600 - 400}_{200} + \underbrace{120 - 50}_{70}$

$= 200 + 70$

$= 270$

Instead of the curly signs ⏟ we may use brackets to indicate the plan to calculate 600 – 400 and 120 – 50 separately:

720 – 450 = (600 – 400) + (120 – 50)

= 200 + 70

= 270

Answers

7. (a) 270 (b) 480 (c) 340 (d) 80
8. Learners compare their answers with the ones they had in question 6.
9. (a) 590 (b) 590
   (c) 300 (d) 170
   (e) 270 (f) 540
10. Learners write a short paragraph on Lerato's method.

3.7 Round off and estimate

Mathematical notes

In this section learners use rounding off and estimating as a calculation strategy. Learners will first round off numbers to the nearest ten and then round off the *same* numbers to the nearest hundred, and then calculate.

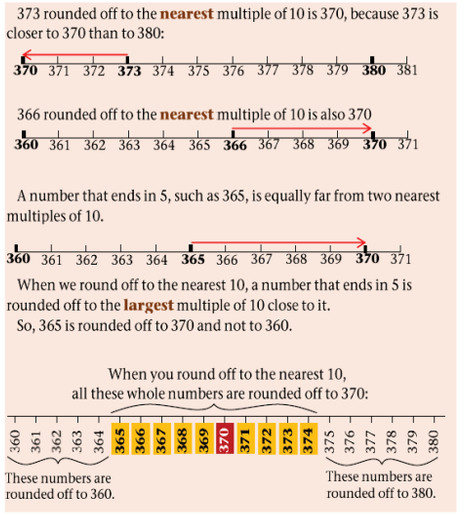
Teaching guidelines

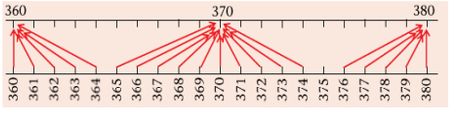
Aim to complete Section 3.7 in 1 hour.
One possibility is to use

- the tinted passage on page 43 to 44 and the one from Grade 6 alongside, questions 1(a) and (b), 2(a) and (b), and 6 (and related parts of questions 7 and 9) for concept development,
- questions 1(c) to (h), 3(a), (b) and (c), 4(a) and (b), and 8 (and related parts of questions 7 and 9) for classwork, and
- questions 3(d), (e) and f), 4(c) and (d), and 5 (and related parts of questions 7 and 9) for additional practice.

You can use a number line to help learners to see the connection between rounding to the *nearest* 10 and seeing the *closest* 10.

Pages 14 to 16 of the Grade 6 Learner Book illustrate how you can use a number line to show rounding off.





7. Break down, work with parts and build up the answer to calculate the following.
   (a) 430 – 160 (b) 760 – 280
   (c) 630 – 290 (d) 420 – 340
8. Check whether you got the same answers for the above calculations as when you did them in question 6.
9. Use any method that you prefer to calculate the following. Check all your answers by doing addition.
   (a) 810 – 220 (b) 710 – 120
   (c) 630 – 330 (d) 630 – 460
   (e) 640 – 370 (f) 820 – 280
10. Lerato calculated 840 – 550 like this:

    840 – 540 = 300 so 840 – 550 is 10 less than 300, and that is 290.

    Do you think Lerato was clever to work like this? Write a short paragraph about it.

3.7 Round off and estimate

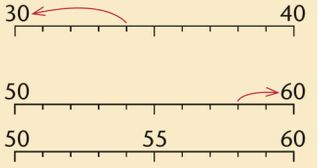
It is often useful to quickly make approximate answers for calculations. Imagine you buy items for R34 and R58 and do not have time to accurately calculate R34 + R58.

It can help you to know that you need to pay about R30 + R60, which is R90.

34 can be **rounded off** to the **nearest multiple of ten**, which is 30.

58 can be rounded off to 60.

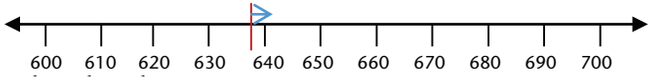
55 is equally far from 50 and 60.



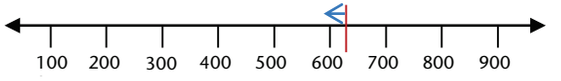
People all over the world have agreed to round off "upwards" in a case like this, so 55 is normally rounded off to 60.

A commonly taught procedure is to underline the tens part of the number and to circle the units part. For example, if rounding 34, you would do this: 3④. Learners are then told that if the number in the circle is 5 or more, you increase the underlined number, but if the number in the circle is less than 5 the underlined number stays the same. It is best to *avoid* teaching this procedure. It does *not* help learners to understand the meaning of rounding off. It causes problems when learners need to round to 5, from Grade 5 onwards.

Rather stick with number lines that show the multiples and which are closest or nearest. When using number lines you can see that 638 is more than halfway between 630 and 640, so it is rounded up to 640.



However, 638 is less than halfway between 600 and 700, so it is rounded down to 600.



Critical knowledge

Rounding off to the nearest 10 means seeing which multiple of 10 is closest to the number being rounded. Fives are halfway between tens, they are rounded up.

Rounding off to the nearest 100 means seeing which multiple of 100 is closest to the number being rounded. Fifties are halfway between hundreds, they are rounded up.

Learners need to understand how rounding off can help them in Mathematics and in the world.

Answers

1. (a) 390 (b) 480 (c) 190 (d) 590
   (e) 250 (f) 260 (g) 250 (h) 60
2. (a) 400 (b) 500 (c) 200 (d) 600
   (e) 200 (f) 300 (g) 300 (h) 100
3. (a) 40 + 50 = 90 (b) 50 + 50 = 100 (c) 50 + 60 = 110
   (d) 70 – 40 = 30 (e) 80 – 40 = 40 (f) 80 – 30 = 50
4. (a) 300 + 500 = 800 (b) 900 – 400 = 500
   (c) 300 + 500 = 800 (d) 900 – 300 = 600
5. There are approximately 100 + 400 + 300 = 800 houses.

678 rounded off to the nearest multiple of ten is 680.
678 can also be rounded off to the **nearest multiple of hundred**, which is 700.

634 rounded off to the nearest multiple of hundred is 600.
634 rounded off to the nearest multiple of ten is 630.

1. Round off each number to the nearest ten.
   (a) 387 (b) 484 (c) 185 (d) 594
   (e) 249 (f) 255 (g) 250 (h) 63
2. Round off the numbers in question 1 to the nearest hundred.
3. In each case round off to the nearest ten, then calculate the sum or difference of the rounded numbers.
   (a) 43 + 52 (b) 46 + 52 (c) 46 + 57
   (d) 74 – 35 (e) 76 – 35 (f) 76 – 34
4. In each case round off to the nearest hundred, then calculate the sum or difference of the rounded numbers.
   (a) 267 + 466 (b) 932 – 437
   (c) 343 + 549 (d) 886 – 278

Rounding off can be used to **estimate** the answers for calculations.

Example

A traveller has driven 268 kilometres of the 859 kilometres to his destination. Approximately how far does he still have to drive?

268 and 859 rounded off to the nearest hundred are 300 and 900.
The traveller still has to drive approximately 900 – 300 kilometres, that is 600 kilometres.

Round off to *the nearest hundred* when you do questions 5 and 6.

5. There are 108 houses in one town, 362 houses in another town and 269 houses in a third town. Approximately how many houses are there in the three towns together?

Notes on questions

Once learners have completed question 8, make time to talk about the usefulness of rounding in the context of the word sum. Ask learners how rounding off helps Mrs Tshabalala, or anyone else, when shopping.

Answers

6. Approximately 700 – 600 = 100 cows are left.
7. Question 5: 110 + 360 + 270 = 740 Question 6: 730 – 570 = 160
8. 50 + 130 + 30 + 70 + 60 + 60 + 200 + 70 = R670
9. Question 5: Rounding: 100 + 400 + 300 = 800 houses
   Compensating: 800 + 8 – 38 – 31 = 739 houses
   Question 6: Rounding: 700 – 600 = 100 cows
   Compensating: 100 + 34 + 32 = 166 cows

Learners may find the compensating section of the question above challenging. They rounded 734 down to 700, so they add back 34. They rounded 568 up to 600. They subtracted more than they should have, so they have to *add* 32 back.

Question 8: Rounding: 50 + 130 + 30 + 70 + 60 + 60 + 200 + 70 = R670
Compensating: 670 – 3 + 2 + 4 + 4 – 2 – 2 – 2 – 4 = R667

3.8 Break down and build up numbers

Teaching guidelines

This is a critically important section. It provides for consolidation of the break-down, rearrange and build-up method of addition and subtraction (that learners will eventually, in Grade 5, document in the vertical column notation).

Demonstrate the calculation of 364 + 588 as outlined in the tinted passage, but let learners complete the last step, as asked in question 1.

Question 1 (and 8) address an important subskill, namely transfer, which means to convert the sum obtained when different place value parts are added or subtracted in the break-down and build-up method to the standard expanded notation for a number, e.g.

800 + 140 + 12 = 800 + (100 + 40) + (10 + 2) = (800 + 100) + (40 + 10) + 2 = 900 + 50 + 2

(Learners need not express it in this way with brackets.)

Mathematical notes

In this and other sections, calculation strategies are named. Learners are *not* expected to remember and use the names of the strategies. Learners are *never* given the name of a strategy in a test or exam and asked to use the named strategy. Learners are only required to know how to use the range of strategies. It is also expected that they should be able to decide which calculation strategy will suit them best in any specific situation.

6. A farmer has 734 cows. A veldfire breaks out and 568 of the cows are killed. Approximately how many cows are left?
7. Do questions 5 and 6 again, but this time round off the numbers to *the nearest ten* before you calculate.
8. Mrs Tshabalala wrote the prices of items she bought on a paper slip.
   Round off each price to the nearest ten rand.
   Then estimate the total by adding the rounded-off prices.

   47
   132
   34
   74
   58
   58
   198
   66

To calculate 78 + 64 you can first **round off** to the nearest ten and get an approximate answer: 80 + 60 = 140.
Then you can **compensate** to get the accurate answer: 140 – 2 + 4 = 142.

9. Compensate to get the accurate answers for questions 5, 6 and 8.

3.8 Break down and build up numbers

To add numbers, it helps to break them down into place value parts first. For example, when you have to calculate 364 + 588, the numbers may be taken apart like this:

364: 4 300 60

588: 80 8 500

The hundreds can be added to the hundreds, the tens to the tens and the ones to the ones:

300 + 500 = 800 60 + 80 = 140 4 + 8 = 12

So, 364 + 588 = 800 + 140 + 12

1. Write 800 + 140 + 12 as a single number.

Teaching guidelines

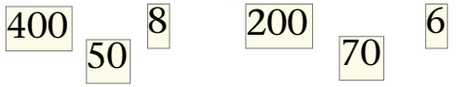
Allow learners to engage with questions 2 and 3 on their own. When they have engaged with question 3 for at least 5 minutes, demonstrate the break-down, rearrange and build-up method on the board, using the example in the tinted passage.

You can use your large demonstration place value cards on the board, as shown below, to demonstrate how to break down numbers into place value parts, rearrange the parts, combine parts and then build up the answer. You can work with the cards on the left-hand side of the board, and document the work in writing on the right-hand side of the board.



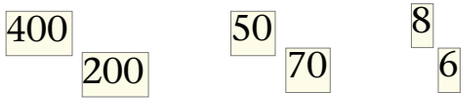
458 + 276

Now separate the cards to produce a new display on the board:



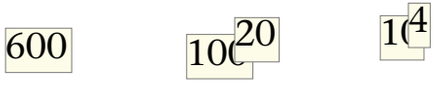
458 + 276
= 400 + 50 + 8 + 200 + 70 + 6

Rearrange the cards to produce another new display on the board:



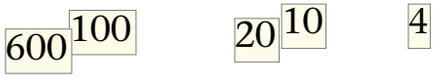
458 + 276
= 400 + 50 + 8 + 200 + 70 + 6
= 400 + 200 + 50 + 70 + 8 + 6

Ask learners how much 400 + 200, 50 + 70 and 8 + 6 is, then replace the hundreds cards (400 and 200) on the board with the 600 card, the tens cards with a 100 card and a 20 card, and the units cards with a 10 card and a 4 card:



458 + 276
= 400 + 50 + 8 + 200 + 70 + 6
= 400 + 200 + 50 + 70 + 8 + 6
= 600 + 120 + 14

Rearrange the cards so that the display is now as below:



458 + 276
= 400 + 50 + 8 + 200 + 70 + 6
= 400 + 200 + 50 + 70 + 8 + 6
= 600 + 120 + 14
= 700 + 30 + 4
= 734

Allow learners to use place value cards to support their work for question 4 if they wish to, but allow them to decide individually whether they do so or not. (While aids like the place value cards can be very useful, learners should also press along to learn to calculate without aids as soon as they can.)

Answers

1. 952 2. 787 3. 1 042 4. (a) 695 (b) 675 (c) 814 (d) 766

2. Two numbers are given in expanded notation below. Add the two numbers and write the answer as a single number.

   The numbers are 400 + 30 + 4 and 300 + 50 + 3.

3. Calculate 364 + 678.

To calculate 458 + 276 you may **break down the numbers** and **work with the parts**, then **build up the answer**:

Break down:

458 = 400 + 50 + 8 and 276 = 200 + 70 + 6

Work with the parts:

400 + 200 = 600 50 + 70 = 120 8 + 6 = 14

Build up the answer:

600 + 120 + 14
(100)
= 700 + 20 + 14
(10)
= 700 + 30 + 4
= 734

4. Calculate.
   (a) 239 + 456 (b) 387 + 288
   (c) 368 + 446 (d) 532 + 234

Two different ways to calculate 734 – 568 are shown below.

A. The filling-up or add-on method

To calculate how much 734 – 568 is you may ask yourself how much you must add to 568 to reach 734.

You may then do this in steps as shown below.

568 + **32** → 600 + **134** = 734

You had to add 32 and 134 which is a total of 166.

So 734 – 568 = 166.

Answers

5. (a) 900 – 600 + 30 – 20 + 7 – 4 = 300 + 10 + 3 = 313
   (b) 800 – 400 + 70 – 40 + 6 – 5 = 400 + 30 + 1 = 431
   (c) 300 + 40 – 30 + 8 – 3 = 300 + 10 + 5 = 315
   (d) 900 – 300 + 80 – 50 + 7 – 2 = 600 + 30 + 5 = 635

6. (a) 624 + 76 → 700 + 200 → 900 + 37 = 937,
   and 76 + 200 + 37 → 200 + 70 + 30 + 6 + 7 = 313
   so, 937 – 624 = 313

   (b) 445 + 55 → 500 + 300 → 800 + 76 = 876,
   and 55 + 300 + 76 → 300 + 50 + 70 + 5 + 6 = 431
   so, 876 – 445 = 431

   (c) 315
   (d) 635

7. 924 = 800 + 110 + 14 and 637 = 600 + 30 + 7
   800 – 600 = 200; 110 – 30 = 80; 14 – 7 = 7
   so, 924 – 637 = 287

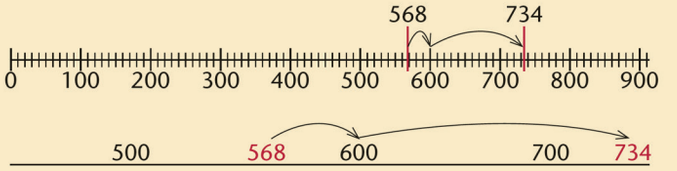
8. (a) 843 (b) 843
   (c) 689 (d) 699

Notes on questions

In question 7 it is not helpful to simply break down both numbers into their place value parts. Learners will have to remember the method that was outlined in Section 3.6 on page 42. For those learners who do not remember this method, you can use the tinted passage on page 48 to help them. You can also refer back to the arrow diagrams that were shown in this guide in Section 3.5.

In question 8 learners build up the number from parts and write it as a single number. Questions 8(b), (c) and (d) are important because learners will break down numbers in this way when doing subtraction using the breaking down method. Question 8 prepares learners for question 9 and other instances where they will be using the breaking down method.

Some people think of movements on a number line to help them to keep track of their thinking when they use the above method of subtraction:



B. The break down and build up method

In a case such as 786 – 523, the *break down and build up method* is quite easy:

786 = 700 + 80 + 6 or **700** and **80** and **6**.
523 = 500 + 20 + 3 or **500** and **20** and **3**.

500 can be taken from 700 to give 200: **700 – 500 = 200**
20 can be taken from 80 to give 60: **80 – 20 = 60**
3 can be taken from 6 to give 3: **6 – 3 = 3**
Hence 786 – 523 = 200 + 60 + 3, which is 263.

5. Use the *break down and build up method* to calculate each of the following.
   (a) 937 – 624 (b) 876 – 445
   (c) 348 – 33 (d) 987 – 352

6. Do the calculations in question 5 with the *filling-up method*. Then compare the answers that you got with the two methods to check whether you worked correctly.

7. Try to calculate 924 – 637 using the *break down and build up method*.

8. Write each of the following as a single number.
   (a) 800 + 40 + 3 (b) 700 + 130 + 13
   (c) 600 + 70 + 19 (d) 400 + 260 + 39

Answers

9. (a) 258 (b) 376
   (c) 183 (d) 565
10. Different method, but same answers as for question 9.
11. (a) 531 (b) 534
    (c) 258 (d) 237
12. (a) No (b) Own investigation

Notes on questions

Question 11 allows learners to practise the relationship between addition and subtraction. By the end of this unit they should know that a subtraction calculation can be checked by doing addition, and vice versa.

Question 12 focuses learners' attention on the fact that the subtraction sign only applies to the number immediately behind it: for example $8 - 2 + 3 \neq 8 - 5$. If you want the subtraction sign to apply to all the numbers that follow, you have to insert brackets: for example $8 - (2 + 3) = 8 - 5$.

Possible misconceptions

In question 12 learners can investigate by doing both Janice's and Zain's calculations. They will see that they get different answers. They can then try calculations with similar structures but different (smaller) numbers, for example $10 - 3 + 4$. They can also swap the order of the numbers and operations, for example $10 + 4 - 3$. They should try to make some general statements. They can try using several more sets of small numbers. It is important that there is discussion after this calculation.

When we have to calculate 843 – 385 and we **break down** 843 and 385, we find that the parts are not convenient for calculating 843 – 385:

843 = 800 + 40 + 3
385 = 300 + 80 + 5

Fortunately we can make a plan:
We can replace 800 + 40 + 3 with 700 + 130 + 13.

843 = 700 + 130 + 13
385 = 300 + 80 + 5

It is now easy to **work with the parts**, that is to subtract the hundreds parts, the tens parts and the units parts:

700 – 300 = 400 130 – 80 = 50 13 – 5 = 8

Now you can **build up the answer**:

843 – 385 = 400 + 50 + 8, which is 458.

9. Use the *break down and build up method* to calculate these.
   (a) 934 – 676 (b) 845 – 469
   (c) 348 – 165 (d) 952 – 387
10. Do the calculations in question 9 by using the *filling-up method*. Then compare the answers that you got with the two methods to check whether you worked correctly.
11. Use any method that you prefer to do each calculation below. Use addition to check your answers.
    (a) 876 – 345 (b) 766 – 232
    (c) 734 – 476 (d) 624 – 387
12. Janice and Zain both calculate 760 – 340 + 260.
    Janice first calculates 760 – 340; then she adds 260.
    Zain first calculates 340 + 260; then he subtracts from 760.
    (a) Do you expect them to get the same answer?
    (b) Investigate whether your expectation is right.

3.9 Solve problems

Teaching guidelines

Learners have learnt and practised a range of calculating strategies in this unit. These include adding on, working from known number facts, using doubles and near doubles, transferring amounts from one number to another to make the calculation easier, building up to tens/hundreds, rounding off and compensating, and breaking down and building up numbers. They can use any of these strategies when they solve these problems.

There are many calculations you could do to solve most of these problems. If learners use a different strategy or calculation plan from the one you expect, check to see whether it is also valid. For example, in question 1(a) learners may add on to get an answer, i.e. do 78 + **2 + 20 + 53** = 153 (the shorter stick is 75 cm), where you might have subtracted to get the answer 153 cm – 78 cm = 75 cm. The only strategy that should be discouraged is counting in ones to get the answer.

It can build learners' confidence if they approach problems with the mindset that they have the tools to solve the problem. This is why we have suggested that you encourage learners to think: *"What have I done before that can help me here?"*

Aim to complete Section 3.9 in $1\frac{1}{2}$ hours. One possibility is to use

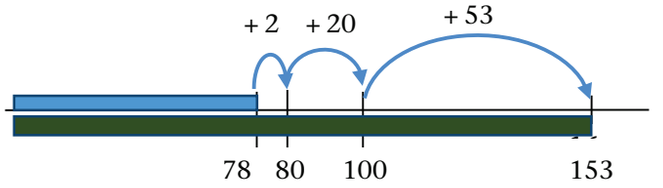
- questions 1(a), 2, 7, 8, 12 and 13 for concept development,
- questions 1(b), 4, 5 and 9 for classwork, and
- questions 1(c), 3, 6, 10 and 11 for additional practice.

Notes on questions

In question 2(a) the difference between349 m and 276 m is less than 100 m. This means that it is not sensible to round to 100 m.

Answers

1. (a) 153 cm – 78 cm = 75 cm
   153 cm + 75 cm = 228 cm
   (b) 364 cm – 118 cm = 246 cm
   364 cm + 118 cm = 482 cm
   (c) 387 cm – 185 cm = 202 cm
   387 cm + 202 cm = 589 cm



2. (a) 300 m – 300 m = 0 m (b) 350 m – 280 m = 70 m
   (c) 73 m (d) 73 m (e) 3 m

3.9 Solve problems

1. In each question below you will compare two sticks. You can make rough drawings to help you to think about the questions.
   (a) One stick is 153 cm long and the other stick is 78 cm shorter.
   How long is the shorter stick?
   How long are the two sticks together?
   (b) One stick is 364 cm long and the other stick is 118 cm long.
   What is the difference between the lengths?
   How long are the two sticks together?
   (c) The longer of two sticks is 387 cm long and the difference between the lengths of the two sticks is 185 cm.
   How long is the shorter stick?
   How long are the two sticks together?

2. 276 m of the road from the school gate to the office is tarred. The total distance is 349 m.
   (a) Estimate, to the nearest 100 m, how much of the road still needs to be tarred.
   (b) Estimate, to the nearest 10 m, how much of the road still needs to be tarred.
   (c) Calculate exactly how much of the road still needs to be tarred.
   (d) How far out was your estimate in (a)?
   (e) How far out was your estimate in (b)?

You can sometimes make rough drawings to help you to understand questions.

Teaching guidelines

Remember that learners need to estimate their answers before calculating. You can help learners to estimate by asking them to choose the best estimate from a choice of three to five options. For example, in question 3 you could ask them to choose the best estimate from

(i) 600 (ii) 900 (iii) 300 (iv) 6 000 (v) 9 000

They should also explain their choice.

There are a range of different ways in which learners could add or subtract to solve these problems. For example, in question 4 learners should estimate that the answer will be about 400. They could add on and compensate: 507 **+ 400** → 907 **– 5** = 902. They will then get 400 girls – 5 girls = 395 girls. Or they could subtract 902 – 507. This could be done in stages: 902 – 500 → 402 – 2 → 400 – 5 = 395.

Notes on questions

Questions 4 and 5 have the same answer because question 5 is one possible number sentence that could be used to solve question 4. This is also true for questions 7 and 8. Questions 5 and 7 prepare learners for how they could approach question 9, but also questions 10, 12 and 13.

Answers

3. 839 fruit trees
4. 395 girls
5. 395
6. R910
7. 547
8. 547 trees
9. □ + 324 = 713
10. R703
11. 610 hamburgers
12. R166
13. 632 pumpkins

In all the questions below, make an estimate of the answer before you do the accurate calculations. Write your estimates down.

3. Johannes has 168 peach trees, 392 plum trees and 279 pear trees in his orchard. How many fruit trees are there altogether?
4. Out of the 902 children in a school, 507 are boys. How many girls are there in the school?
5. What number is missing from this number sentence?

   902 = 507 + □
6. John saved R374 for a bike in one year. The next year he saved another R536. How much did he save over the two years?
7. What number is missing from 308 + □ = 855?
8. A farmer has already pruned 308 of her 855 peach trees. How many trees must she still prune?
9. Write a number sentence, such as in question 5 or 7, that will have the same answer as the question below.

   During the year, 324 houses were built in a housing development. At the end of the year there were 713 houses. How many houses were there at the beginning of the year?
10. After Rallai had withdrawn R275 of his savings, there was still R428 left. How much money did he originally have in his savings account?
11. Shop A sold 462 hamburgers and Shop B sold 148 hamburgers more. How many hamburgers did Shop B sell?
12. Jan has saved R568. He wants to buy a jacket that costs R734. How much more must he save?
13. Katharina grows pumpkins on her plot. Last month she harvested 867 pumpkins and sold 235 to shops in her village. She sent the rest to the fresh produce market in Durban. How many pumpkins did she send to the market?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Seeing patterns	Pattern is what remains the same while other things change	51
4.2 Making patterns	Developing the basic notions of sequences	52 to 53
4.3 Describing patterns	A constant difference relationship	54 to 55
4.4 Recognising and describing patterns	A simple direct proportion relationship; introducing flow diagrams	56 to 57
4.5 Tables or multiples	Multiples (times table) as sequences	58 to 59

CAPS time allocation	4 hours
CAPS page references	18 to 19 and 46 to 51

Mathematical background

Numeric patterns, as part of the Content Area "Patterns, Functions and Algebra", should serve as building blocks to develop the basic concepts of Algebra in the Senior and FET phases. The study of numeric patterns should develop the idea of a relationship between two variable quantities, for example:

One variable quantity (the "input numbers")	1	2	3	4	5	6	7	8	9	10	11
Another variable quantity (the "output numbers")	4	7	10	13	16	19	22	25	28	31	34

The word pattern means that something is repeated. In the above case, the sequence 4, 7, 10, 13, 16, ... can be formed by repeatedly adding 3.

This pattern in the sequence is formed by performing the same calculation each time, to move from one number to the next.

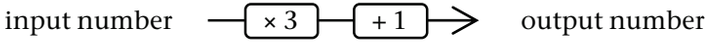
Such a pattern is called a **recursive pattern**. The word "recur" means that something occurs repeatedly or repeats itself.

The above sequence of output numbers can also be formed by multiplying each input number by 3 and adding 1:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$3 \times \mathbf{1} + 1$	$3 \times \mathbf{2} + 1$	$3 \times \mathbf{3} + 1$	$3 \times \mathbf{4} + 1$	$3 \times \mathbf{5} + 1$	$3 \times \mathbf{6} + 1$	$3 \times \mathbf{7} + 1$	$3 \times \mathbf{8} + 1$	$3 \times \mathbf{9} + 1$	$3 \times \mathbf{10} + 1$	$3 \times \mathbf{11} + 1$
4	**7**	**10**	**13**	**16**	**19**	**22**	**25**	**28**	**31**	**34**

A relationship between two variable quantities, in which each value of the second quantity is uniquely determined by the corresponding value of the first quantity, is called a **function** – the middle word in the CAPS title for this content area.

In the above case, the link between the input and output numbers (also called the independent and dependent variables) is given by the calculation plan (rule) "multiply the input number by 3 and add 1", which can also be represented as $3 \times \square + 1$, or with this flow diagram:



4.1 Seeing patterns

Mathematical notes

This introduction to the notion of patterns develops the underlying big idea that to see patterns means to look for the aspect that remains unchanged (constant) as all other aspects change (vary).

Critical knowledge

Learners have to develop the understanding of looking for the one aspect that remains unchanged as all other things change. This defines the notion of "pattern".

Teaching guidelines

We suggest that instead of letting learners read and complete the questions in their exercise books, you rather teach this section live in class through questions and answers (with different content if needed).

During such whole-class teaching you can take input from learners, and let learners explain their reasoning so that they can learn from each other.

Learners should find the introduction (identifying the one that does not fit) relatively easy and you should help build their confidence that working with patterns is in fact common sense and easy.

Answers

1. (a) 56; not a multiple of 5
   (b) 23; the only odd number
   (c) *aca*; no *b*
   (d) E; only shape with five sides
2. The answers and reasons are all correct.

UNIT 4 NUMERIC PATTERNS

4.1 Seeing patterns

The more patterns you can see in mathematics, the better you are at mathematics!

How good are you at seeing patterns?



1. In each case, say which item does not fit with the others. Explain why you say so.
   (a) 5, 10, 30, 45, 56, 25, 55, 20, 35
   (b) 12, 36, 10, 48, 23, 32, 40, 66, 24, 98
   (c) *abc*, *bca*, *cab*, *bac*, *aca*, *acb*, *cba*
   (d) 

2. Here are Lindi's answers for question 1. Do you agree with her? Or did you see different patterns?
   (a) The numbers are all different. But what is the same is that they are all multiples of 5; except 56, which is not.
   (b) The numbers are all different. But what is the same is that they are all even numbers; except 23, which is an odd number.
   (c) The "words" are all different. But what is the same is that they all have the letters *a*, *b* and *c*, in any order; except *aca* which does not have a *b*.
   (d) The figures are all different. But what is the same is that they all have 4 sides; except Figure E, which has 5 sides.

Different, but the same!
To see a **pattern**, we look for what is the *same* in ALL the *different* things.

4.2 Making patterns

Mathematical notes

Our approach to introducing **sequences** is to connect it to the familiar work of counting in multiples (e.g. 5, 10, 15, ...) and counting on in multiples (e.g. 1, 6, 11, 16, ...).

It is not really new work; it is only different in representation. When counting, we usually do it *verbally*, but in our work with sequences we have to *write* it down.

Of course our focus is also different. Whereas our counting activities are mostly aimed at number concept development and mental fluency, our work with numeric patterns studies the relationships between the numbers in the sequences we produce.

And we ask different questions, for example:

If Sally would continue counting 5, 10, 15, 20, ...

- *What is the 100th number that she will count?*
- *Will she count 436?*

Our work on numeric patterns must develop the knowledge that will enable Sally to *calculate* the 100th number *instead of actually counting* all the way up to the 100th number, and to *reason* whether 436 is in the sequence without actually having to count past 436.

Teaching guidelines

We suggest that instead of letting learners read the tinted passage, you rather teach it live in class through questions and answers (with different content if needed). That is, that you ask individual learners to "count in fives" and to "count on in fives", etc. and that you then write it down while introducing necessary *vocabulary* and *notations*.

Also, you should make sure that learners know how to read values from a table representation. The best way to do this is to ask them specific questions, for example: "*By just looking at the table and without counting, if the Position no. is 5, what is the number in the sequence*?" and "*In what position is 20 in the sequence*?" and "*Complete the table.*"

You should note that representation in a table is a social convention, and learners can only learn how to read and handle tables by learning it from others. In the beginning learners often complete missing values as below, i.e. by continuing the sequence without considering the corresponding position number:

Position no.	1	2	3	4	5	6	7	100
Sequence	5	10	15	20	25	30	35	40 ✗

+5 +5 +5 +5

We introduced the zigzag notation in an effort to prevent this, to show learners that there are some missing values. You should discuss the problem and the notation with learners.

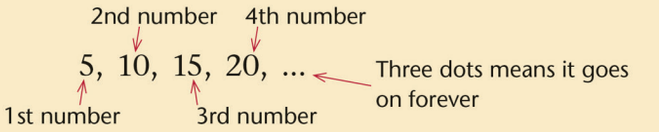
4.2 Making patterns

A row of numbers that follow each other in a pattern, is called a **sequence**. Each number is calculated from the previous number in the same way.

Here are two examples of sequences that you already know:

- When the teacher asks Sally to "count in fives", she counts like this by adding 5 each time:
  5, 10, 15, 20, 25, ...
- For the instruction "start at 1 and count on in fives", Sally counts like this:
  1, 6, 11, 16, 21, ...

We use the following words and notation to write and to talk about sequences. For example, in the case of the first sequence above:



We can also write the sequence in a table and indicate the **position number**, like this:

Position number	1	2	3	4	5	6	7	100
Sequence	5	10	15	20	25			

Teaching guidelines

Learners are often confused by the instruction "count on in fives" and will probably ask: "But when should we stop?"

This highlights the difference between a **finite sequence** and an **infinite sequence**:

- A finite sequence has a last number. For example, the sequence 1, 6, 11, 16, 21 has only five numbers and the last number is 21.
- An infinite sequence has no last number and goes on forever. For example, the sequence 1, 6, 11, 16, 21, ... goes on forever.

So you can answer learners that they should "not stop", but should write down only the first few numbers and then use the notation of three dots introduced on page 52 to indicate that it goes on forever.

Finding the 100th number

Methods for finding a rule in order to calculate the 100th number is only developed later (see pages 58 to 59 for multiples and pages 265 to 266 for non-multiples). Therefore you should not teach it now. Rather, it is an opportunity for learners to think about the problem and to begin to formulate their attempts to answer the question.

Answers

1. (a) 5, 10, 15, 20, 25, ... The units digit is only 5 or 0; the pattern 5, 0 is repeated.
   100th number = 100 × 5 = 500 Learners will probably find this easy.
   (b) 1, 6, 11, 16, 21, ... The units digit is only 1 or 6; the pattern 1, 6 is repeated.
   100th number = 496 One can reason that it is 4 less than the multiples of 5.
   (c) 2, 7, 12, 17, 22, ... The units digit is only 2 or 7; the pattern 2, 7 is repeated.
   100th number = 497
   (d) 3, 8, 13, 18, 23, ... The units digit is only 3 or 8; the pattern 3, 8 is repeated.
   100th number = 498
   (e) 4, 9, 14, 19, 24, ... The units digit is only 4 or 9; the pattern 4, 9 is repeated.
   100th number = 499
   (f) See (a)
2. Different: they all start with different numbers (except (a) and (f)).
   Same: you add on 5 to the previous number in all the sequences.
3. Let learners explain their instructions and then decide if the generated sequence is correct.

When learning about sequences, you will learn how to *recognise* patterns, *describe* patterns, and *continue* patterns.

You will learn how to answer questions such as these. You can try to answer them now, if you like.

- If Sally continues counting in the same way, what are the next two numbers she will count in each sequence?
- What will be the 100th number Sally will count in each sequence?
- If Sally continues and continues, will the number 325 be in her sequences or not? How do you know?

1. For each of the instructions in (a) to (f) below:
   - write down the sequence according to the instruction.
   - describe in your own words what is different and what is the same for all the numbers in the sequence.
   - write down the 100th number in the sequence.

   (a) Count in fives.
   (b) Start at 1 and count on in fives.
   (c) Start at 2 and count on in fives.
   (d) Start at 3 and count on in fives.
   (e) Start at 4 and count on in fives.
   (f) Start at 5 and count on in fives.
2. What is different, and what is the same for *all* the sequences in question 1?
3. Write down your own instruction for a sequence, and then write down the sequence.

4.3 Describing patterns

Teaching guidelines

The critical knowledge for learners to gain from this section is the central idea of patterns, namely to describe what remains constant as all other things change.

In this specific context, although the ages of the father and son are not the same and changing all the time, there is something that remains unchanged (**constant**) all the time, and that is the *difference* between their ages, namely 30 years.

You can help learners understand this idea of searching for what remains constant while other things change by writing a series of equivalent statements, for example:

30 – 0 = 30
31 – 1 = 30
32 – 2 = 30
35 – 5 = 30
50 – 20 = 30

Once we have identified this essence of the situation, it of course becomes a model or method to solve problems in the context, for example to find unknown ages of the father or the son in question 3:

□ – 12 = 30
□ – 20 = 30

The rest of the activity introduces or develops different representations for the situation: how to say it in words, write it in a table, put it in a flow diagram, and write a calculation plan (rule).

You should especially make sure that learners understand how the flow diagram representation works, because it may be new to them. Probably the best strategy is to ask them to explain how they got their answers, for example in questions 4 to 7 on the next page. See also the discussion of flow diagrams with two operators on page 187.

Answers

1. 40 years
2. Their ages change, but the difference between their ages remains the same (30 years).
3. (a) When the son is 12 years, the father will be 30 years older: 12 + 30 = 42 years.
   (b) When the son is 20 years, the father will be 30 years older: 20 + 30 = 50 years.

4.3 Describing patterns

1. When a son was born, his father was 30 years old. How old is the father when the son is 10 years old?

Over the years, the son's mother wrote down the father's and the son's ages in a table, like this:

Age of son (years)	0	1	2	3	4	5	12	20
Age of father (years)	30	31	32	33	34	35		

2. Describe any patterns that you can see in the table. (What is different and what remains the same?)
3. Complete the table. This means:
   (a) Work out how old the father will be when the son is 12.
   (b) Work out how old the father will be when the son is 20.

We can describe the father and son's ages in different ways:

- In **words**:
  The father is always 30 years older than the son.
- As a **calculation plan (rule)**:
  Father's age = *Son's age* + 30

  Although their ages change all the time, the calculation plan remains the same for all ages:

Son's age	1	2	3	4	...	10	11
	↓+30	↓+30	↓+30	↓+30		↓+30	↓+30
Father's age	31	32	33	34	...	40	41

- With a **flow diagram**, such as the one on the next page.

Teaching guidelines

Questions 4, 5 and 6 do not specify which representation (words, tables, calculation plans, flow diagrams) learners should use to answer the questions. You could carefully note which representations learners are actually using, which they find easy; where and why they make mistakes.

It is also important to make connections with other concepts in mathematics. For example, the problem to find the son's age in question 6 can be transformed to the equivalent mathematical problem to solve an open number sentence:

For which value of ☐ is ☐ + 30 = 46

This can then be solved by using the **inverse operation**: ☐ = 46 – 30 = 16.

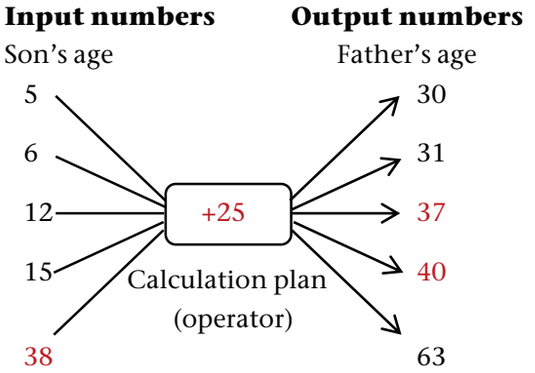
Answers

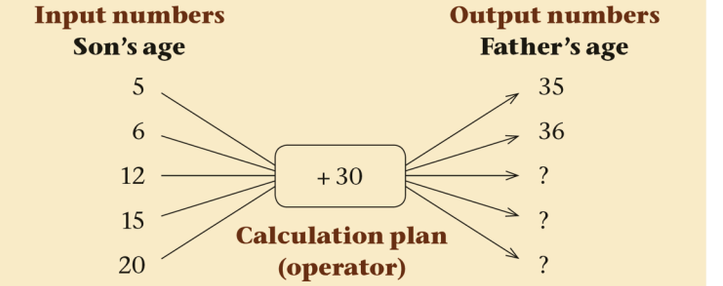
4. 12 + 30 = 42 15 + 30 = 45 20 + 30 = 50

5. 60 – 30 = 30

6. 37 – 30 = 7 40 – 30 = 10 43 – 30 = 13
   46 – 30 = 16 50 – 30 = 20

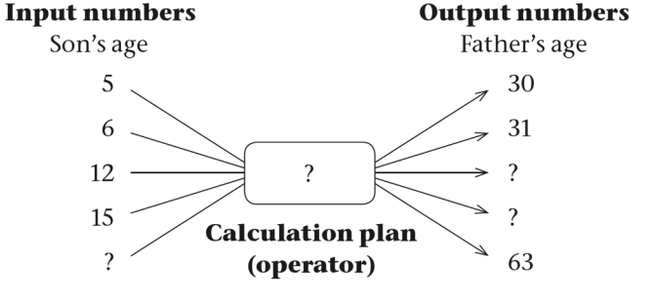
7.





This flow diagram shows that we must, for example, take the **input number** 5 and add 30 to it to give the **output number** 35.

4. Find the three missing output numbers; this means the father's age when the son was 12, 15 and 20.
5. How do you calculate the input number for the output number 60? This is the same as finding the missing number in the open number sentence ☐ + 30 = 60. Calculate it.
6. Find the son's age when the father is 37, 40, 43, 46 and 50.
7. This flow diagram shows the ages of another father and son.



(a) What is the calculation plan (operator) connecting the input and output numbers?

(b) Find the missing input and output numbers.

4.4 Recognising and describing patterns

Critical knowledge

The critical knowledge for learners to gain from this section is the central idea of patterns, namely that although the number of candles and the cost are changing, there is something that remains unchanged (**constant**) all the time, and that is the **rate** of cost per candle (R4).

It is therefore the same ideas as the father–son ages in the previous section, but here we have a different **model**, namely multiplication or division instead of addition and subtraction. Again, you can help learners understand this by writing a series of equivalent statements showing the constant quotient of 4. Recognising this constant of course also helps with a solution strategy to solve problems. For example:

$4 \div 1 = 4$ $\quad$ $16 \div 4 = 4$ $\quad$ $\square \div 20 = 4$

$8 \div 2 = 4$ $\quad$ $20 \div 5 = 4$ $\quad$ $100 \div \square = 4$

$12 \div 3 = 4$ $\quad$ $\square \div 6 = 4$

Although learners should be able to solve these by inspection, you could also draw their attention to the use of the **inverse operation**, for example:
$\square \div 20 = 4$, so $\square = 4 \times 20 = 80$.

Answers

1. (a)

No. of candles	1	2	3	4	5	6	10	20	25
Cost (rands)	4	8	12	16	20	24	40	80	100

(b) Learners may already be using Theo's method or Nadia's method.

2. (a) Yes, but Theo may struggle to calculate the cost of a large number of candles.
   (b) Theo added R4 to the cost for each candle added.
   (c) Cost of 6 candles = R20 + R4 = R24; Cost of 7 candles = R24 + R4 = R28
   Cost of 20 candles = R4 + 19 × R4 = 80; or R28 + 13 × R4 = R80

3. (a) Yes
   (b) *Cost* = *No. of candles* × 4
   (c) Cost of 6 candles = 6 × 4 = R24
   Cost of 7 candles = 7 × 4 = R28
   Cost of 20 candles = 20 × 4 = R80

4. Nadia's method is easier/quicker/convenient when you have to work out the cost of large numbers of candles.

4.4 Recognising and describing patterns

1. Theo's mother often sends him to buy candles for their home. The candles cost R4 each.
   (a) Complete this table to calculate the cost of different numbers of candles.
   (b) Describe your method.



No. of candles	1	2	3	4	5	6	10	20	
Cost (rands)	4								100

2. This is Theo's method:

No. of candles	1	2	3	4	5	6	7	20
Cost (rands)	4	8	12	16	20			
		+4	+4	+4	+4			

   (a) Is Theo's method correct?
   (b) Describe his method in words.
   (c) Use Theo's method to calculate the cost of 6, 7 and 20 candles.

3. This is Nadia's method:

No. of candles	1	2	3	4	5	6	7	20
	×4	×4	×4	×4	×4	×4		
Cost (rands)	4	8	12	16	20			

   (a) Is Nadia's method correct?
   (b) Describe her method in words.
   (c) Use Nadia's method to calculate the cost of 6, 7 and 20 candles.

4. Which method do you prefer, Theo's method or Nadia's method? Why? Discuss.

Note on finding input numbers

To find unknown input numbers for known output numbers in the flow diagram is equivalent to solving an open number sentence.

In question 5, for the given 68 as output number, the problem is to find the unknown ☐ in ☐ × 4 = 68. Learners will mostly use one of the following two solution strategies:

Trial and improvement, working from left to right (forward)*:*

Is ☐ = 5? No, 5 × 4 = 20, not 68. Try a bigger number.

Is ☐ = 12? No, 12 × 4 = 48, which is still too small. Try a bigger number.

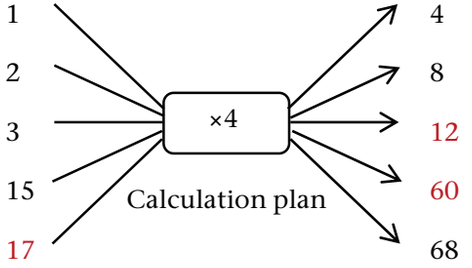
Is ☐ = 17? Yes, 17 × 4 = 68!

Using inverse operations, i.e. working from right to left (backwards):

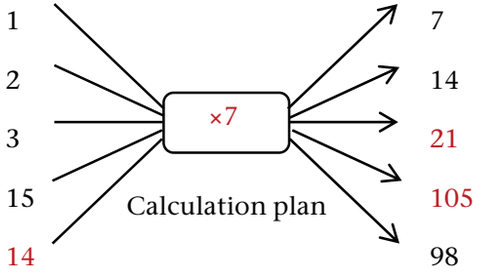
If ☐ × 4 = 68, then ☐ = 68 ÷ 4 = 17

Answers

5.

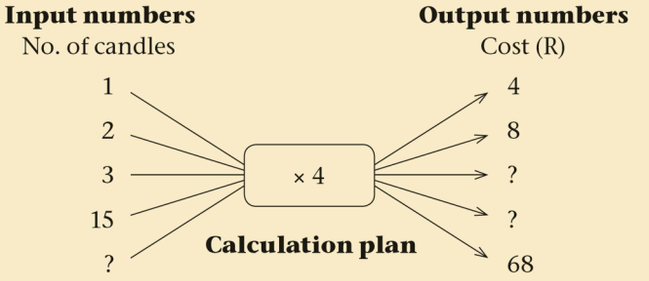


6.

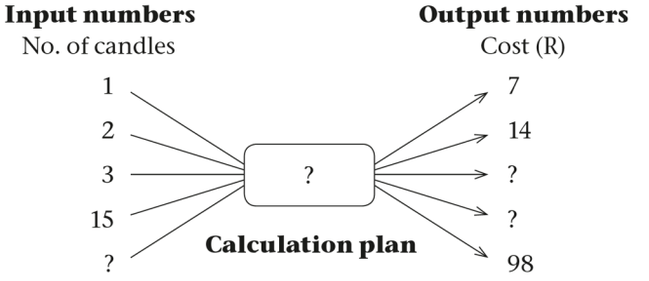


We can describe the cost of the candles in different ways:

- In **words**:
  The candles cost R4 each.
- As a **calculation plan (rule)**:
  Cost of candles = Number of candles × 4
- With this **flow diagram**:



5. Calculate the missing numbers in the flow diagram.
6. This flow diagram shows the cost of candles at another shop.



(a) What is the calculation plan connecting the input and output numbers?

(b) Find the missing input and output numbers.

4.5 Tables or multiples

Mathematical notes

We thoroughly develop and reinforce sequences of multiples by connecting it to the familiar work of "the times table". Help your learners to make the connections between the two contexts. The sequences of multiples (e.g. 5, 10, 15, 20, ...) will in turn become the building block to study other sequences such as 6, 11, 16, 21, ... (one more than a multiple of 5).

Critical knowledge

All learners should understand, know and be able to apply the knowledge common to all sequences of multiples:

- The multiples of k all have a constant difference of $+k$ between consecutive numbers (the "horizontal" pattern).
- The multiples of k all have a calculation plan of the form $\times k$ (the "vertical" pattern).

Teaching guidelines

Learners often are very unproductive and waste much time by first having to draw a neat table. It may be a good idea for you to make copies of the table provided in the Addendum (page 437) for each learner – this should save time and ensure that learners are productive.

Answers

1.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70

2. Let learners discuss and share their methods so that they can learn from each other.
3. Let learners discuss and share the patterns they see so that they learn from each other.

4.5 Tables or multiples

Here is part of the multiplication table.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15					
4	4	8	12							
5	5	10	15	20	25					
6	6	12	18							
7	7	14	21							

1. Complete the table.
2. Which methods did you use to complete the table? Discuss.
3. Discuss what patterns you see in the table, and how that helps you to "remember" the tables.

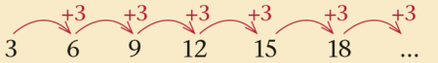
The multiplication table consists of sequences of **multiples**. For example:

2, 4, 6, 8, 10, 12, ... are the **multiples of 2** (or even numbers).
3, 6, 9, 12, 15, ... are the **multiples of 3**.

These sequences are all different, but they are the same in two ways:

- In all the "tables" or sequences of multiples we add the same number (the multiple) to get the next number in the sequence. For example:





Teaching guidelines

It is important that you encourage learners to use both recursive (here *counting on*) and functional reasoning (here *the multiplication tables*).

You should also help learners to understand their usefulness: counting on is easy to continue the sequence for a few numbers, while vertical multiplication (using a rule) will be easier to find further-lying larger output numbers.

Note on finding the 100th number

You should try to help learners to connect new work with previous, known ideas. That gives meaning to the new work and makes it easy. Let's take as example the new work to find a rule to calculate the 100th number in sequences of multiples (question 4). Have we seen this before? Now look at this simple word problem:

> *At the market there are many boxes with 5 apples in each box. How many apples are there in 6 boxes? How many apples are there in 100 boxes?*

If we would count the apples in 6 boxes, we will count 5, 10, 15, 20, 25, 30.

But we also know the number of apples is 5 + 5 + 5 + 5 + 5 + 5, and we know this is 6 × 5 = 30.

Comparing these two methods, but now thinking about sequences, we should realise that the 6th number in the sequence 5, 10, 15, 20, ... is 6 × 5.

Similarly: To find the number of apples in 100 boxes we could count 5, 10, 15, 20, ...

But we also know that the number of apples in 100 boxes is 100 × 5.

Therefore, the 100th number in the sequence 5, 10, 15, 20, ... is 100 × 5.

Answers

4. (a)

A	..., 18, 20, 22, 24, 26	100th:	200
B	..., 24, 27, 30, 33, 36	100th:	300
C	..., 40, 45, 50, 55, 60	100th:	500
D	..., 49, 56, 63, 70, 77	100th:	700
E	..., 72, 81, 90, 99, 108	100th:	900
F	..., 80, 90, 100, 110, 120	100th:	1 000

(b) In each sequence the numbers change, but the difference between consecutive numbers is the same. In the different sequences the constant differences are different, but they all have a constant difference.

We can, for example, describe the sequence 3, 6, 9, 12, ... by saying that we add 3 to each number to get the next number. So, we can also use it as a calculation plan.

- The sequences of multiples all have the same kind of calculation plan (rule), namely multiplication only:

Position no.	1	2	3	4	5	6	...
	↓×2	↓×2	↓×2	↓×2	↓×2	↓×2	
Multiple no.	2	4	6	8	10	12	...

Position no.	1	2	3	4	5	6	...
	↓×3	↓×3	↓×3	↓×3	↓×3	↓×3	
Multiple no.	3	6	9	12	15	18	...

So, we can describe the sequence 3, 6, 9, 12, ... with a calculation plan:

Multiple number = *Position number* × 3

So: multiple 100 = 100 × 3 = 300

4. (a) Calculate the next five numbers and the 100th number in each sequence. Describe your methods.

Sequence A: 2, 4, 6, 8, 10, 12, 14, 16, ...

Sequence B: 3, 6, 9, 12, 15, 18, 21, ...

Sequence C: 5, 10, 15, 20, 25, 30, 35, ...

Sequence D: 7, 14, 21, 28, 35, 42, 49, ...

Sequence E: 9, 18, 27, 36, 45, 54, 63, ...

Sequence F: 10, 20, 30, 40, 50, 60, 70, ...

(b) What is the same and what is different in each and in all of these sequences?

Grade 4 Term 1 Unit 5 Whole numbers: Multiplication and division

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 Multiplication	The concept of multiplication	60
5.2 Count objects	Mental mathematics	61 to 63
5.3 Learn more multiplication facts	Mental mathematics	64 to 66
5.4 Multiply by 7, 8 and 9	Mental mathematics	67 to 68
5.5 Multiplication in reverse	Division as the inverse of multiplication	69 to 70
5.6 Multiplying in steps	Breaking numbers into factors to create new multiplication facts from known facts	71 to 72

CAPS time allocation	4 hours
CAPS page references	13 to 15 and 52 to 55

Mathematical background

Multiplication can be understood:

- as repeated addition (see the illustrations on pages 61, 63 and 64),
- in terms of rectangular arrays of objects (objects arranged in rows and columns – see the illustrations on pages 6o, 67 and 68), and
- in terms of the ratio between two quantities.

Division can be understood in the same ways as multiplication.

Division and multiplication are linked. For example, if we want to know 36 ÷ 9 = ☐ it is the same as finding out 9 × ☐ = 36. Division is done to find out by what a number must be multiplied to obtain a given result (see Section 5.5 of this unit). This is why division is called the inverse of multiplication.

In previous units we have mentioned that many South African learners solve all kinds of calculations by drawing dots or stripes and/or counting in ones. Unit 5 continues to develop learners' calculation strategies so that they stop counting everything in ones. In this unit learners will work with multiplication and division represented in different ways: words, pictures, arrays, flow diagrams, tables, multiplication grids, and number sentences with multiplication and division signs.

This unit draws on the work done on place value in previous units, as well as the work done on multiplication in Unit 4. It builds learners' knowledge and conceptual understanding of the multiplication tables. The multiplication tables are important number facts for learners to know but, equally important, are strategies for quickly calculating those multiplication facts that they do not know off by heart.

This is a long unit. Encourage learners to work quickly and smartly. Further guidance on time allocation is provided in the discussions for each section.

5.1 Multiplication

Mathematical notes

This short section motivates the need for multiplication. It shows that counting in ones, and even skip counting is inefficient once numbers get beyond a certain size.

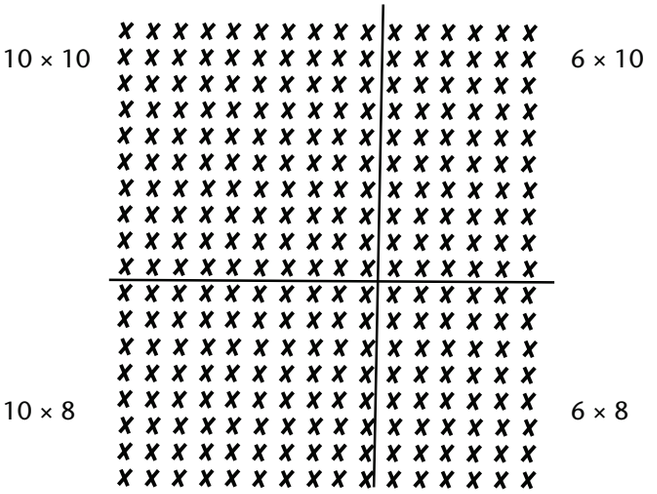
This section also gives a glimpse of what learners will do in later terms, i.e. breaking down numbers into place value parts and multiplying each of the parts. The aim here is not for them to learn this method, but merely to see that multiplication is helpful and efficient.

Teaching guidelines

Aim to complete this section in 20 minutes.

Ask learners if they can find the number of red squares on the page without counting them all. While they work, you may draw a similar array on the board. It has 18 rows, with 16 squares (or crosses) in each row.

Once learners have done the work and compared their answers, you may demonstrate that the answer can easily be found by dividing the array into four sections as shown below, provided one knows the answers for 10 × 10, 6 × 10, 10 × 8 and 6 × 8. You can then tell learners that they will learn facts like these in the days ahead, so that they can quickly and easily do calculations such as 16 × 18 later.



Answers

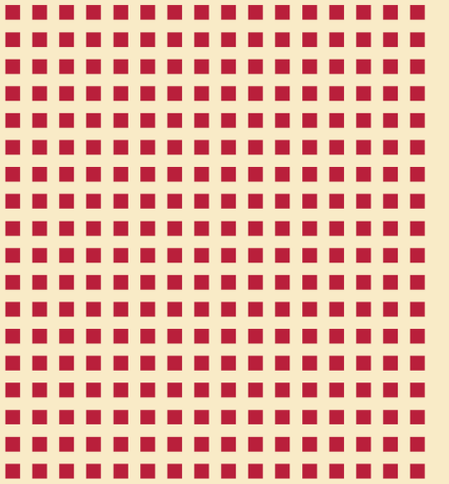
1. 288 squares
2. Learners compare their answers to question 1.

UNIT 5

WHOLE NUMBERS: MULTIPLICATION AND DIVISION

5.1 Multiplication

Counting takes a lot of time, especially when there are many objects to be counted. For instance, it may take a lot of time to count all the squares shown below.



A quicker way is to add 16 repeatedly: 16 + 16 + 16 + ...

But if you can calculate 18 × 16, it is even quicker!

To be able to calculate something like 18 × 16, you need to know some multiplication facts, such as 10 × 6 = 60, 8 × 6 = 48, 10 × 10 = 100 and 8 × 10 = 80.

In this unit you will learn many facts like these.

1. How many squares are shown on this page?
2. Compare your answer with that of a classmate.

5.2 Count objects

Mathematical notes

There is more to mental mathematics than the rapid recall of number facts such as multiplication tables and addition and subtraction bonds. Mental mathematics also involves building number concept and calculation strategies. Mental mathematics can involve jotting down numbers and making sketches.

The focus of this section is the 3 times table. Grade 3 learners multiply by 3; this section revises it but also develops a conceptual understanding of multiplication that includes the commutative property of multiplication and the order of operations.

The commutative property of multiplication states that when two numbers are multiplied the answer remains the same even if the numbers are swapped around. This is sometimes called the order property of multiplication. Learners are *not* expected to know the name of this property. They are just expected to know that they can swap the order of two numbers that are multiplied. They are expected to use this to make calculations easier.

Teaching guidelines

Try to move quickly through Section 5.2: aim to cover this section in 35 minutes. One possibility is to use

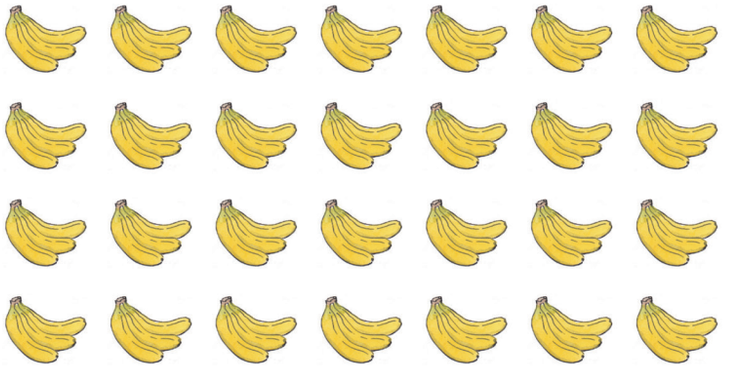
- questions 1 and 10 for mental mathematics, keeping in mind that this is also concept development
- question 1, the summary bar and tinted passage on page 61, questions 8(a) and (d) and the tinted passages on page 62, questions 9(a), 10, 11, 12 and 13, and the summary bar on page 63 for concept development,
- questions 2, 3, 4, 5, 7 and 8(c) and (e) for classwork, and
- questions 6, 8(b) and (f), and 9(b) to (d) for additional practice.

You can introduce this lesson by asking learners to write down the answers to question 1 while they look at the picture on page 61. Explain to them that this is called multiplication. Demonstrate how to write questions 1(a), (b) and (c) as number sentences using the multiplication sign.

Answers

1. (a) 3 (b) 15 (c) 30 (d) 27 (e) 21 (f) 18
2. $3 \times 9 = 27$ or $9 \times 3 = 27$
3. 3 6 9 12 15 18 21 24 27 30
   33 36 39 42 45 48 51 54 57 60

5.2 Count objects



1. (a) How many bananas are there in one bunch?
   (b) How many bananas are there in five bunches?
   (c) How many bananas are there in ten bunches?
   (d) How many bananas are there in nine bunches?
   (e) How many bananas are there in seven bunches?
   (f) How many bananas are there in six bunches?

What you did to answer questions 1(b) to (f) is called **multiplication**.

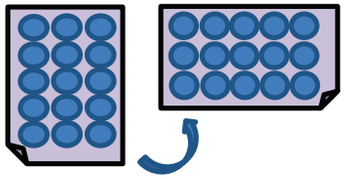
In question 1(b) you multiplied 3 by 5. This can be written **in symbols**: 5×3. You may write $5 \times 3 = 15$. You can also write $3 \times 5 = 15$, because $5 \times 3 = 3 \times 5$.

2. Write your answer to question 1(d) in symbols.
3. Copy the numbers below and count on in threes to write the first 20 numbers in this pattern:
   3 6 9 12 . . .

GRADE 4: MATHEMATICS [TERM 1] 61

Teaching guidelines

Ask learners to look at the pictures on page 63. You can read questions 10(a) and (b) while learners write down the answers in their exercise books. They could discuss question 11 in pairs before reporting back to the whole class. You can draw an array of 3 × 5 circles on a piece of paper. Show this to learners and then make a quarter turn with the paper. This shows that 3 columns of 5 circles become 5 columns of 3 circles, or vice versa. This demonstrates that $3 \times 5 = 5 \times 3$.



In questions 12 and 13 learners should try out several pairs of numbers to see whether it makes a difference if they swap them around. Take feedback on these questions and emphasise that changing the order of two numbers in a subtraction statement changes the calculation completely.

Use the second tinted passage on page 62 and questions 8(a) and (d) to explain to learners that when a number sentence contains multiplication and addition or multiplication and subtraction, you always do multiplication first, for example: $3 \times 10 - 3 \times 4 = 30 - 12 = 18$.

A template for the table in question 7 is provided in the Addendum (page 438).

Possible misconceptions

Learners sometimes overgeneralise in Mathematics. This is the reason for questions 12 and 13. You can change the order of two numbers that are multiplied or added without affecting the answer, for example $7 \times 3 = 3 \times 7$ and $70 + 30 = 30 + 70$. This is not true for subtraction or division, for example $6 - 4 \neq 4 - 6$ and $6 \div 3 \neq 3 \div 6$. Learners sometimes assume that they can swap the order of the numbers when subtracting without changing the answer. It is important that they see and experience that this cannot be done.

Answers

4. (a) 18 (b) 21 (c) 24 (d) 12

5. Learners check and correct their answers.

6. (a) 30 (b) 27 (c) 21 (d) 15
   (e) 6 (f) 9 (g) 18 (h) 12

7.

×	1	2	3	4	5	6	7	8	9	10
3	3	6	9	12	15	18	21	24	27	30

8. (a) $15 + 9 = 24$ (b) 36
   (c) $18 + 18 = 36$ (d) $30 - 12 = 18$
   (e) $27 - 12 = 15$ (f) $12 + 15 = 27$

4. Use the numbers in the counting pattern that you wrote for question 3 to say how much each of the following is.
   (a) 6×3 (b) 7×3 (c) 8×3 (d) 4×3

5. Check your answers to question 4 by counting bananas on page 61.

6. How much is each of the following?
   (a) 10×3 (b) 9×3 (c) 7×3 (d) 5×3
   (e) 2×3 (f) 3×3 (g) 6×3 (h) 4×3

You have now learnt some important multiplication facts that you should try to remember. This set of facts is sometimes called the "three times table". Knowing these facts will help you to solve many problems easily and quickly.

7. Copy and complete the three times table.

×	1	2	3	4	5	6	7	8	9	10
3	3	6	9	12						

8. Use your knowledge of the three times table to quickly find out how much each of the following is, just by doing addition or subtraction:
   (a) $5 \times 3 + 3 \times 3$ (b) 12×3
   (c) $6 \times 3 + 6 \times 3$ (d) $10 \times 3 - 4 \times 3$
   (e) $9 \times 3 - 4 \times 3$ (f) $4 \times 3 + 5 \times 3$

Each item in question 8 is a **plan** to do a calculation.
We can say each item is a **calculation plan** or **expression**.

People all over the world have agreed that when multiplication, addition and subtraction appear in the same plan, multiplication is done first unless otherwise indicated.

So, the plan $5 \times 3 + 3 \times 3$ tells you to multiply 5 by 3, then multiply 3 by 3 and add the two totals together.

Mathematical notes

Question 9 begins to show learners the relationship between multiplication and division. We nearly always use multiplication when we do division.

Teaching guidelines

Learners can use skip counting and the pictures of bananas on page 63 to answer question 9(a). They can use skip counting and the pictures of the bananas on page 61 to answer questions 9(b), (c) and (d).

Answers

9. (a) 6
   (b) 8
   (c) 7
   (d) 10

10. (a) 15
    (b) 15

11. $3 \times 5 = 5 \times 3$

12. Investigate, e.g. $3 + 5 = 8$ and $5 + 3 = 8$.
    Yes, it is also a property of addition.
    Note that learners can use any examples to test whether the answer stays the same when they change the order of two numbers added.

13. Investigate, e.g. $5 - 3 = 2$, but $3 - 5 \neq 2$.
    No, subtraction does not have this property.
    Note that learners can use any examples to test whether the answer stays the same when they change the order of two numbers subtracted.

9. (a) How many bunches of 3 make 18 bananas?
   (b) How many bunches of 3 make 24 bananas?
   (c) How many bunches of 3 make 21 bananas?
   (d) How many bunches of 3 make 30 bananas?

10. (a) How many bananas are there in 3 bunches of 5 bananas each?



(b) How many bananas are there in 5 bunches of 3 bananas each?



11. What do you notice about 3×5 and 5×3?

When two numbers are multiplied, they can be swopped around: the answer remains the same. This is a **property** of multiplication.

12. Do you think addition also has this property? Investigate whether two numbers can be swopped around when they are added.

13. Do you think subtraction also has this property? Investigate whether two numbers can be swopped around when the one number is subtracted from the other number.

GRADE 4: MATHEMATICS [TERM 1] **63**

5.3 Learn more multiplication facts

Possible misconceptions

Many learners do calculations by skip counting, or drawing dots or lines and counting in ones. This not only slows learners down, but reduces their chances of learning any mathematics. It is imperative that learners develop a good sense of both numbers and operations, and that they learn basic addition, subtraction and multiplication facts. Grade 4 learners should move beyond skip counting to knowing multiplication facts off by heart and being able to reconstruct some multiplication facts from others at a high speed.

Mathematical notes

Learners move from counting pictures of groups (bunches of 4 bananas) to counting using symbols to multiplying by 4 in a function table. The aim is to encourage learners to count in groups (skip count), first with the help of pictures if necessary (question 1), then skip count without the aid of pictures (question 2), then moving to multiplying (questions 3 and 4), and finally to doing multiplication in reverse, which is division (question 5).

Teaching guidelines

Aim to complete this section in 1 hour. Templates for all the tables are provided in the Addendum (pages 438 to 440). You can copy these to save time.

You can ask learners how many bananas there are in each bunch in the picture above question 1. Learners can then skip count in 4s as they point to bunches of bananas. Learners can also skip count in 4s backwards. This should be revision of Grade 3 work. In order to keep up the pace of work you could read questions 1, 2, 3 and 4 and let learners write the answers in their exercise books.

In question 5 learners work with multiplication in reverse, which is division. Learners can use the table in question 4 to answer questions 5(a) and (b).

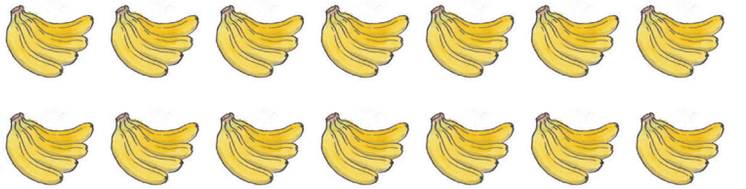
Answers

1. (a) 40 (b) 36 (c) 32 (d) 28
2. 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80
3. (a) 24 (b) 28 (c) 32 (d) 16 (e) 36 (f) 40
4.

×	1	2	3	4	5	6	7	8	9	10
4	4	8	12	16	20	24	28	32	36	40

5. (a) 5 (b) 7 (c) 14 (d) 12 (e) 3

5.3 Learn more multiplication facts

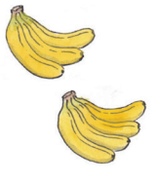


1. (a) How many bananas are there in ten bunches?
   (b) How many bananas are there in nine bunches?
   (c) How many bananas are there in eight bunches?
   (d) How many bananas are there in seven bunches?
2. Copy the numbers below and count on in fours to write the first 20 numbers in this pattern:

   4 8 12 16 . . .
3. Use the numbers in the counting pattern you wrote for question 2 to say how much each of the following is.
   (a) 6 × 4 (b) 7 × 4
   (c) 8 × 4 (d) 4 × 4
   (e) 9 × 4 (f) 10 × 4
4. Copy and complete the four times table.

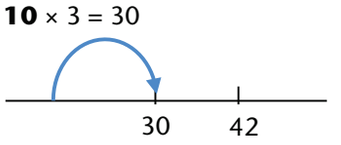
×	1	2	3	4	5	6	7	8	9	10
4	4	8	12	16						

5. (a) How many bunches of 4 make 20 bananas?
   (b) How many bunches of 4 make 28 bananas?
   (c) How many bunches of 3 make 42 bananas?
   (d) How many bunches of 3 make 36 bananas?
   (e) How many bananas should be in each bunch so that 5 bunches are 15 bananas?

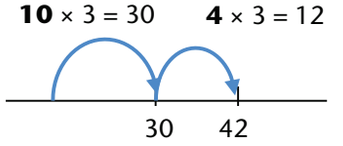


Notes on questions

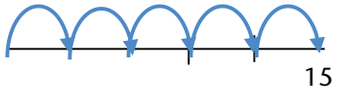
In questions 5(c) and (d) learners can draw on what they learnt about multiplying by 3 in Section 5.2. However, learners cannot simply copy the answers from the table on page 62, as the answers are higher than 10 × 3. Show learners how they can use what they know (e.g. 10 × 3 = 30) and an empty number line to find the answer.



After showing that 10 × 3 = 30 on the number line, some learners may know that 4 × 3 = 12 will take you to 42; other learners may skip count on: 33, 36, 39, 42.



In question 5(e) some learners may know that □ × 5 = 15 means the answer is 3. Other learners could draw a number line with 5 jumps (hops) and try skip counting in different numbers until they get the answer.



In question 6 remind learners that in Section 5.2 they learnt that when a number sentence contains multiplication and addition and/or subtraction, they should do the multiplication first. For example, question 6(a)
5 × 4 + 4 × 4 means: do the two multiplication sections first and then add them, i.e.
5 × 4 + 4 × 4 = 20 + 16 = 36.

Learners can use the table in question 7 to answer questions 9 and 10.

Answers

5. (f) 4

6. (a) 20 + 16 = 36 (b) 40 – 16 = 24 (c) 24 + 24 = 48
   (d) 48 (e) 36 – 16 = 20 (f) 16 + 20 = 36

7.

×	1	2	3	4	5	6	7	8	9	10
5	5	10	15	20	25	30	35	40	45	50

8. Learners describe their methods. Accept all reasonable answers. There are many possible methods that learners could use, including but not limited to: skip counting in 5s; adding on in 5s with or without the use of a number line; filling in known answers and using these to find the rest.

9. (a) 50 (b) 45 (c) 35
   (d) 25 (e) 10 (f) 30
   (g) 40 (h) 15 (i) 20

10. (a) 30 bananas (b) 50 bananas (c) 45 bananas
    (d) 40 bananas (e) 35 bananas

(f) How many bananas should be in each bunch so that 7 bunches is 28 bananas?

6. Use your knowledge of multiplication facts to quickly find out how much each of the following is, just by doing addition or subtraction:
   (a) 5 × 4 + 4 × 4 (b) 10 × 4 – 4 × 4
   (c) 6 × 4 + 6 × 4 (d) 12 × 4
   (e) 4 × 9 – 4 × 4 (f) 4 × 4 + 4 × 5

7. Copy and complete the five times table.
   You will have to make your own plan to find the facts.

×	1	2	3	4	5	6	7	8	9	10
5	5	10	15	20						

8. Describe the plan that you made to find the facts to one of your classmates.

9. How much is each of the following?
   (a) 10 × 5 (b) 9 × 5 (c) 7 × 5
   (d) 5 × 5 (e) 2 × 5 (f) 6 × 5
   (g) 8 × 5 (h) 3 × 5 (i) 4 × 5

10. (a) How many bananas are there in six bunches of five bananas each?
    (b) How many bananas are there in ten bunches of five bananas each?
    (c) How many bananas are there in nine bunches of five bananas each?
    (d) How many bananas are there in eight bunches of five bananas each?
    (e) How many bananas are there in seven bunches of five bananas each?

Notes on questions

In question 12 first ask learners to fill in any multiplication facts for 6 that they know, for example 1 × 6; 2 × 6 (which is double 1 × 6); 10 × 6; 3 × 6 = 6 × 3; 5 × 6 = 6 × 5; 4 × 6 = 6 × 4. They can then compare the 6 times table with the 3 times table. Ask whether they see any patterns. If learners do not see that ×6 = double ×3, then let them work with questions 1, 2 and 3 on pages 71 and 72.

In questions 14 and 15 learners can use a range of strategies to work out the multiplication facts that they do not know. These include but are not limited to:

- doubling to get ×2; doubling and doubling again or double ×2 to get ×4; doubling, doubling and doubling again to get ×8 or double ×4 to get ×8,
- halving ×10 to get ×5,
- doubling ×3 to get ×6, and
- using the fact that the order of multiplication does not matter, e.g. 5 × 9 = 9 × 5.

Answers

11. (a)

×	1	2	3	4	5	6	7	8	9	10
10	10	20	30	40	50	60	70	80	90	100

(b) 30 + 50 = 80 (c) 80 – 40 = 40

12.

×	1	2	3	4	5	6	7	8	9	10
6	6	12	18	24	30	36	42	48	54	60

13. (a) 48 (b) 54

14.

×	2	4	8	3	6	7	5	10	9
10	20	40	80	30	60	70	50	100	90
5	10	20	40	15	30	35	25	50	45
3	6	12	24	9	18	21	15	30	27
6	12	24	48	18	36	42	30	60	54
4	8	16	32	12	24	28	20	40	36

15. Learners describe their methods (plans).

16. (a) 48 (b) 36 (c) 48 (d) 36

11. (a) Copy and complete the ten times table.

×	1	2	3	4	5	6	7	8	9	10
10	10	20	30	40						

(b) How much is 3 × 10 + 5 × 10?

(c) How much is 8 × 10 – 4 × 10?

12. Copy and complete the six times table. If you wish, you may use the three times table that you made earlier to help you.

×	1	2	3	4	5	6	7	8	9	10
6										

13. There are six bottles of juice in one pack.

(a) How many bottles are there in 8 packs?

(b) How many bottles are there in 9 packs?

14. Copy and complete the table, to show the multiplication facts you have learnt so far. Try to do it without looking at the 3 times, 4 times, 5 times, 10 times and 6 times tables that you completed earlier.

×	2	4	8	3	6	7	5	10	9
10	20		80						
5									
3									
6		24							
4			32			28			

15. Did you use some of the facts that you knew well to make it easier to complete this table? Describe your plans to one of your classmates, and explain them if you can.

16. How much is each of the following?

(a) 6 × 8 (b) 4 × 9 (c) 8 × 6 (d) 9 × 4

5.4 Multiply by 7, 8 and 9

Mathematical notes

In Grade 3 learners solve problems using multiplication and division (grouping and sharing). But when they work with decontextualised calculations they are only expected to multiply and divide by 1, 2, 3, 4, 5 and 10. Skip counting, multiplying and dividing by 6, 7, 8 and 9 is new in Grade 4.

Teaching guidelines

Aim to complete this section in 40 minutes.

Remind learners that the order in which you multiply two numbers does not matter, for example $2 \times 8 = 8 \times 2 = 16$.

Ask learners to fill in all the answers they know in question 1 as quickly as they can. Then let them work out the answers to question 2. This will help them to complete the ×9 column. In question 4 you can show learners that by blocking off 1 column of dots at a time you can show 7×9; 7×8; 7×7. Templates for the tables are provided in the Addendum (page 441).

Answers

1.

×	2	3	4	8	1	6	7	5	10	9
8	16	24	32	64	8	48	56	40	80	72
7	14	21	28	56	7	42	49	35	70	63
9	18	27	36	72	9	54	63	45	90	81

2. (a) 63 (b) 72 (c) 81 (d) 90

3. Learners should at least know the ×1, ×2, ×3 and ×10 columns off by heart. Ask them to explain how they derived some answers from other answers. Learners' answers will differ. Accept all reasonable answers. Learners' answers might include, but not be limited to:

   I can complete the ×2 column by doubling the ×1 column, the ×4 column by doubling the ×2 column, the ×8 column by doubling the ×4 column.

   I can complete the ×6 column by doubling the ×3 column.

   I can complete the ×5 column by halving the ×10 column.

4. (d) $7 \times 10 = 70$

5.4 Multiply by 7, 8 and 9

1. Make a copy of the table and fill in the facts you already know.

×	2	3	4	8	1	6	7	5	10	9
8										
7										
9										

2. How much is each of the following? To find the answers, you may count in nines if you wish, or you can use different methods.

   (a) $9 + 9 + 9 + 9 + 9 + 9 + 9$

   (b) $9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9$

   (c) $9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9$

   (d) $9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9$

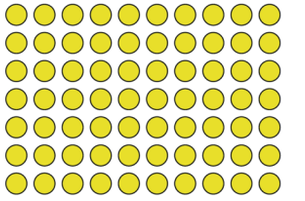
   Use your answers to answer question 3.

   Also use your answers to fill in more cells of your table.

3. (a) Make your own plans to find the facts that you need to fill in cells that are still open in your table for question 1.

   (b) Explain the plans that you made to one of your classmates.

4. There are 7 rows with 10 dots each in this diagram.



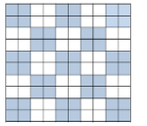
   Which of the following multiplication facts can you use to know how many dots there are in the diagram, altogether?

   (a) $7 \times 7 = 49$ (b) $7 \times 8 = 56$

   (c) $7 \times 9 = 63$ (d) $7 \times 10 = 70$

Notes on questions

In question 5 learners are only expected to state the number of small squares. Some learners may count squares within squares and get a much bigger number than what is stated in the answers below. For example, apart from the 100 1×1 squares there are 2×2 squares, 3×3 squares, 4×4 squares, 5×5 squares, 6×6 squares, 7×7 squares, 8×8 squares and 9×9 squares. These can overlap. If learners get very big answers for questions 5(a), (b), (c) and (d), ask them to explain how they counted. Discourage learners from counting in ones. They should rather treat each grid as an array, and skip count rows or columns, or multiply rows with columns.



In question 6, ask learners to first fill in the answers they know.

They should at least know the ×1, ×2, ×3, ×10 columns off by heart, as well as the ×3 row. Then ask them to use known multiplication facts to derive the others.

They can complete the ×2 column by doubling the ×1 column, the ×4 column by doubling the ×2 column, and the ×8 column by doubling the ×4 column.

They can complete the ×6 column by doubling the ×3 column.

They can complete the ×5 column by halving the ×10 column.

They can complete the ×9 column by subtracting one multiple from the ×10 column; for example $10 \times 8 = 80$, so $9 \times 8 = 80 - 8$. Similarly, $9 \times 6 = 60 - 6$; $9 \times 7 = 70 - 7$; $9 \times 9 = 90 - 9$.

They can complete the ×7 column by swapping the order of the numbers multiplied and filling in the answers they have calculated already, for example $8 \times 7 = 7 \times 8$; $9 \times 7 = 7 \times 9$, etc.

Question 7 contains rate problems.

Answers

5. (a) $10 \times 10 = 100$ (b) $7 \times 8 = 56$ (c) $9 \times 8 = 72$ (d) $8 \times 8 = 64$

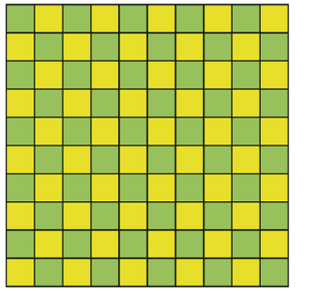
6.

×	1	2	4	8	3	6	7	5	10	9
8	8	16	32	64	24	48	56	40	80	72
3	3	6	12	24	9	18	21	15	30	27
6	6	12	24	48	18	36	42	30	60	54
7	7	14	28	56	21	42	49	35	70	63
9	9	18	36	72	27	54	63	45	90	81

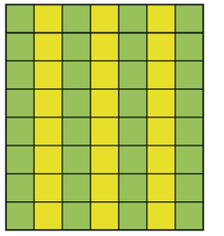
7. (a) R63 (b) R72 (c) R56 (d) R54 (e) R48 (f) R64

5. Use the multiplication facts that you know to state how many squares there are in each of the diagrams below.

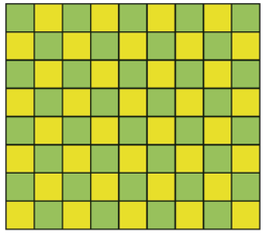
(a)



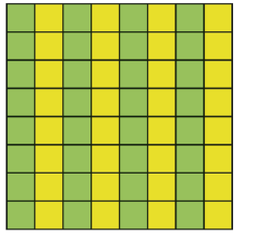
(b)



(c)



(d)



6. Copy and complete the table.

×	1	2	4	8	3	6	7	5	10	9
8				64						
3										
6										
7										
9										

7. What will each of the following cost in total?

(a) 9 apples at R7 each
(b) 8 cans of juice at R9 each
(c) 7 pencils at R8 each
(d) 6 pens at R9 each
(e) 6 oranges at R8 each
(f) 8 pears at R8 each

5.5 Multiplication in reverse

Mathematical notes

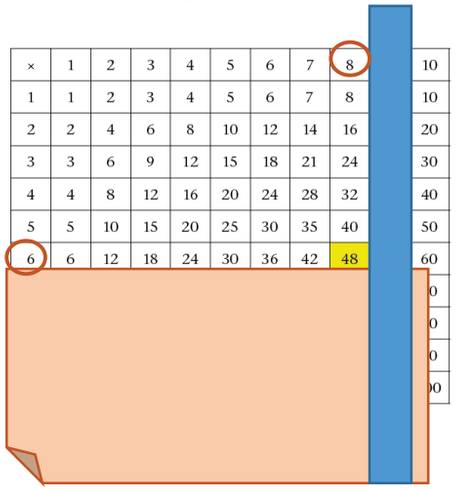
In this section learners use multiplication facts that they know to solve division problems (sharing and grouping) and division calculations. In Grade 4 learners always use multiplication to do division.

Teaching guidelines

Aim to complete this section in 50 minutes. One possibility is to use

- page 69 and the summary bar and tinted passage on page 70 for concept development,
- questions 2, 4(a), (b), (f) and (g), and 5(a) for classwork, and
- questions 3, 4(c) to (e) and (h) to (l), and 5(b) for additional practice.

You can explain to learners how to use the multiplication grid on page 69 to solve division calculations or problems. Problem B in the tinted passage on page 69: "How many loaves of bread at R6 each can Musi buy with R48?" can be thought of as: "How many sixes make up 48?" To find the answer, learners find the ×6 row, place their ruler/exercise book/a piece of paper under it and read across the ×6 row until they find 48. They can then place another straight edge on the right of the column with 48 in it, and read the top number in this column: 8. This means that 6 × 8 = 48. So Musi can buy 8 loaves of bread. Repeat this process to explain to learners how to use the multiplication grid to solve "7 × *which number* = 63" as a way of solving Problem A: "David paid R63 for 7 packets of sweets. How much did each packet cost?"



×	1	2	3	4	5	6	7	8	10
1	1	2	3	4	5	6	7	8	10
2	2	4	6	8	10	12	14	16	20
3	3	6	9	12	15	18	21	24	30
4	4	8	12	16	20	24	28	32	40
5	5	10	15	20	25	30	35	40	50
6	6	12	18	24	30	36	42	48	60

Regular use of the multiplication grid will help learners to learn their multiplication tables.

Explain to learners that 4 × ☐ = 24 means the same as 24 ÷ 4 = ☐. "24 divided by 4" can be thought of as "how many fours are there in 24?" or "4 × what is 24?" Explain that this means that they can always rewrite a division calculation as a multiplication calculation.

Answers

1. (a) 8 (b) 9

5.5 Multiplication in reverse

You will now learn to use your knowledge of multiplication facts to solve problems like the following.

A. David paid R63 for 7 packets of sweets. How much did each packet cost?

B. How many loaves of bread at R6 each can Musi buy with R48?

The following table can help you to solve problems like problems A and B above.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

1. Look at the table. Note that the numbers 48 and 63 are coloured in. Now use the table to find the answers to the following questions.

   (a) How many sixes make up 48?

   (b) 7 × *which number*? = 63

Teaching guidelines

Ask learners to complete question 2. They should see which questions they can answer automatically. They can use the multiplication grid on page 69 to answer the rest of question 2.

Suggest that one learner keeps the textbook open to page 69 and the learner next to him or her keeps it open to page 70. This way they will be able to read the questions and use the grid.

Encourage learners always to think: "*What do I already know that can help me here?*" Learners should multiply to solve division calculations. Learners should also see where they can use answers they have already worked out. For example, in question 3(e) they can use the answer to 3(d); similarly they can use 3(a) for 3(g); 2(i) for 4(a); 2(l) for 4(b); 2(h) for 4(c); 2(g) for 4(d); 2(f) for 4(k); and 4(f) for 4(g).

Critical knowledge

Multiplication and division are related.

You can use multiplication to check division.

The following are equivalent statements: $7 \times \square = 63$ and $\square \times 7 = 63$ and $63 \div 7 = \square$.

This means that learners can always change a division calculation, problem or number sentence into an equivalent multiplication calculation, problem or number sentence.

Learners should use what they know about multiplication to solve division calculations.

Answers

2. (a) 6 (b) 4 (c) 9
   (d) 4 (e) 6 (f) 7
   (g) 7 (h) 8 (i) 9
   (j) 10 (k) 9 (l) 7

3. (a) 5; $5 \times 2 = 10$ (b) 3; $3 \times 3 = 9$ (c) 6; $6 \times 2 = 12$ (d) 4; $4 \times 3 = 12$
   (e) 3; $3 \times 4 = 12$ (f) 5; $5 \times 4 = 20$ (g) 2; $2 \times 5 = 10$ (h) 5; $5 \times 5 = 25$

4. (a) 9 (b) 7 (c) 8
   (d) 7 (e) 10 (f) 10
   (g) 9 (h) 9 (i) 9
   (j) 4 (k) 7 (l) 8

5. (a) 4 mangoes (b) R9 for one bar of soap

What you did to find the answers to question 1 is called **division**.

2. Find the missing number in each of the following number sentences.
   Try to do this by using your knowledge of multiplication facts. The table on the previous page may also be helpful.
   (a) $4 \times \ldots = 24$ (b) $\ldots \times 8 = 32$ (c) $27 = 3 \times \ldots$
   (d) $\ldots \times 9 = 36$ (e) $3 \times \ldots = 18$ (f) $49 = \ldots \times 7$
   (g) $8 \times \ldots = 56$ (h) $\ldots \times 8 = 64$ (i) $63 = 7 \times \ldots$
   (j) $\ldots \times 6 = 60$ (k) $6 \times \ldots = 54$ (l) $35 = \ldots \times 5$

The correct answer for question 2(a) is 6, because $4 \times 6 = 24$.

There is a different way in which question 2(a) can be asked.

Instead of asking $4 \times ? = 24$, we can ask the question

"How much is 24 divided by 4?"

We can write "24 divided by 4" in symbols, like this: $24 \div 4$.

3. Calculate, and check your answers by doing multiplication.
   (a) $10 \div 2$ (b) $9 \div 3$ (c) $12 \div 2$ (d) $12 \div 3$
   (e) $12 \div 4$ (f) $20 \div 4$ (g) $10 \div 5$ (h) $25 \div 5$

4. How much is each of the following? You can get the first four answers from your work in question 2.
   (a) $63 \div 7$ (b) $35 \div 5$ (c) $64 \div 8$
   (d) $56 \div 8$ (e) $100 \div 10$ (f) $90 \div 9$
   (g) $90 \div 10$ (h) $81 \div 9$ (i) $45 \div 5$
   (j) $28 \div 7$ (k) $49 \div 7$ (l) $72 \div 9$

5. (a) One mango costs R8. How many mangoes can Christelle buy with R32?
   (b) Emma paid R45 for five bars of soap. What was the price of one bar of soap?

5.6 Multiplying in steps

Mathematical notes

In this section learners are shown that when they multiply by a number, they can split that number into factors and multiply by each factor in succession. For example, □ × 25 = □ × 5 × 5; □ × 27 = □ × 3 × 3 × 3, etc. In Term 2 Unit 5 learners will see how to use this to multiply by multiples of 10 (see Sections 5.1 and 5.2). The aim is to use this to make it easier to multiply mentally, by reconstructing multiplication facts that you don't know off by heart.

Teaching guidelines

Aim to complete this section in 30 minutes.

The aim of this section is to learn ways to quickly reconstruct multiplication facts from multiplication facts you already know.

You can use questions 1 to 4 for concept development. You can use this for whole-class teaching, but make sure that learners have their textbooks open because it is important that they see the flow diagrams as they work through the calculations. Learners can answer question 1 – it is easier to start with 2 × 6, then 3 × 6, then 4 × 6. Learners can then answer questions 2 and 3. Focus learners' attention on the fact that the following are all equal to 24: 4 × 6; 4 × 2 × 3; 4 × 3 × 2. These are equivalent statements. Learners already know that 2 × 3 = 6, so what they are learning here is that they can split a number into factors and multiply by each factor in stages; this will not change the answer. Let learners check whether 3 × 6 = 3 × 2 × 3 = 3 × 3 × 2 and whether 2 × 6 = 2 × 2 × 3 = 2 × 3 × 2. Let them take any other number and check whether ×6 is always equal to × 2 × 3 and × 3 × 2.

Explain to learners that when they do not know a multiplication fact off by heart, they can use this strategy to quickly reconstruct the multiplication fact by multiplying by factors they know off by heart. Explain that this can be represented by two flow diagrams, each with one operator, or one flow diagram with two operators.

Answers

1. (a) 24 (b) 18 (c) 12
2. (a) 6 (b) 18 (c) 4 (d) 12
3. (a) 12 (b) 24 (c) 9 (d) 18 (e) 6 (f) 12

Possible misconceptions

Check that learners understand that they have been splitting numbers into factors here and that if they split numbers into bonds (using + or –) they need to multiply both parts, for example 8 = 5 + 3, so 3 × 8 = 3 × 5 + 3 × 3.

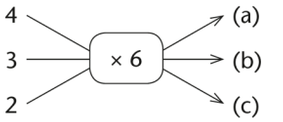
5.6 Multiplying in steps

A flow diagram consists of three parts:

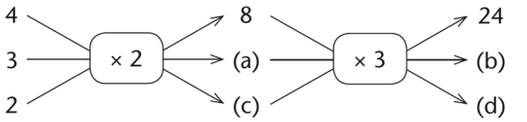
- the **input numbers**, for example 4, 3 and 2 in the flow diagrams below
- the **operator(s)**, for example × 6 in question 1, and × 2 and × 3 in question 2
- the **output numbers**, for example the number 24 in question 2.

In question 2, the number 8 is an output number for the first flow diagram, and it is also an input number for the second flow diagram.

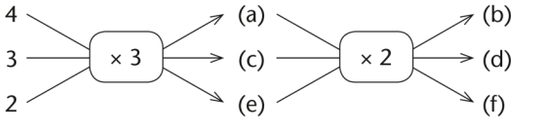
1. What are the output numbers at (a), (b) and (c)?



2. What are the numbers at (a), (b), (c) and (d)?



3. Write the numbers that are missing below.



Critical knowledge

When two numbers are multiplied, the answer remains the same even if the order of the two numbers is swapped around, for example 2 × 3 = 3 × 2.

When three or more numbers are multiplied, you can change the way in which the operations are grouped and the answer will remain the same, for example 6 × 7 = (2 × 3) × 7 = 2 × (3 × 7) = 2 × 21 = 42.

You can use known number facts to create new number facts. One way to do this is to split numbers into factors and multiply by the factors in stages. If you don't know the six times table or a particular number fact, for example 6 × 7, you can split 6 into 2 × 3 and multiply by each factor separately.

So 3 × 7 = 21; 21 × 2 is simply double 21 = 42. So 6 × 7 = 2 × 3 × 7 = 3 × 7 × 2 = 21 × 2 = 42.

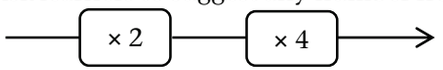
Stated differently: to multiply by 6, you can multiply by 2 and then by 3, or first multiply by 3 and then by 2. Similarly, to multiply by 8, you can

- multiply by 2 then by 4, or multiply by 4 then by 2, or
- multiply by 2, by 2 again, and by 2 again, which is the same as double, double, double.

To multiply by 5, you can first multiply by 10, then halve, for example 7 × 5 = 70, half of 70 is 35.

Teaching guidelines

Ask learners to suggest any number from 2 to 9. Use this example to multiply



Repeat this with other examples.

Learners can check with each number chosen whether they get the same results when they multiply it by × 4 × 4; × 2 × 2 × 2; × 4 × 2 and × 8. Learners will see that × 8 = × 2 × 4 and × 4 × 2 and × 2 × 2 × 2, but not × 4 × 4. Ask learners to multiply out the various factors, i.e. what does 2 × 4 =; what does 4 × 4 =; what does 4 × 2 =; they will see that except for 4 × 4 all the other answers are 8. Remind learners that multiplying by 2 is the same as doubling. Explain that a quick way to multiply by 8 is to double, double again, and double a third time. Let learners test this out with some big number, for example 31 × 8:

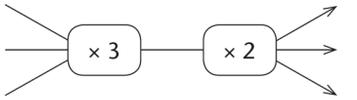
Double 31 = 62
Double 62 = 124
Double 124 = 248, so 31 × 8 = 248.

Once learners have seen that × 5 gives the same results as × 10 ÷ 2, they can use × 10 and then halve to multiply by 5 mentally, for example 645 × 5 = half of 6 450 = 3 225.

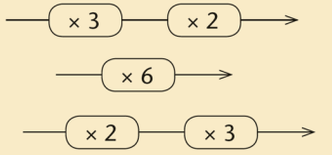
Answers

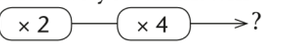
4. (a) Flow diagrams B, C and D (b) Learners investigate
5. × 5

Each flow diagram in question 3 has one operator. Two flow diagrams with one operator each can be combined into one flow diagram with two operators:

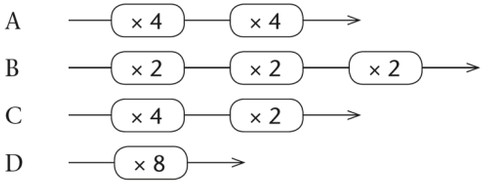


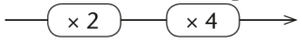
The three flow diagrams below are different. However, they produce the same results, so we say they are **equivalent**.



4. (a) Which of Flow diagrams A to D below do you think will produce the same results as 

?



(b) Use 2, 3 and 5 as input numbers and calculate the output numbers to investigate which of A, B, C and D produce the same output numbers as 

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Flow diagram D above has only one operator, but it produces the same results as Flow diagram C which has two operators.

5. Which flow diagram with one operator only will produce the same results as the flow diagram below?



Grade 4 Term 1 Unit 6 Time

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
6.1 Days, weeks, months and years	Focusing on longer lengths of time (years, months, weeks and days) and reading calendars	73 to 76
6.2 Measuring time	Making a water clock and experiencing some of the difficulties with measuring time	77 to 79
6.3 Understanding the 12-hour clock	Reading clocks and calculating time intervals using hours and minutes in 12-hour time	79 to 82
6.4 Understanding the 24-hour clock	Reading analogue and digital clocks using 12-hour and 24-hour time	83 to 84
6.5 Hours, minutes and seconds	Reading time in hours, minutes and seconds on analogue and digital clocks	84 to 85

CAPS time allocation	6 hours
CAPS page references	27 and 55

Mathematical background

Some Grade 4 learners can read clocks and watches, but many learners find it difficult. There are three issues that make the concept of time difficult:

- Time cannot be seen, touched or physically experienced like length, capacity or volume, area and mass. We measure time by looking at environmental changes, changes in the position of the hands of an analogue clock, or the numbers on a digital clock face.
- Unlike the number system and other forms of measurement, the numbers do not get bigger forever. We measure time in modular units: when we reach certain numbers, for example 60 seconds, 60 minutes, 24 hours, 365 days, the numbers "wrap around" and go back to the beginning. This is different to the way primary school learners work with numbers in other aspects of mathematics.
- In primary school mathematics, numbers are organised in groups and powers of ten. In the topic of time, numbers are also organised in groups of 60, 24, 7, 28, 29, 30, 31, 365 and 366.

Much time is spent on reading clocks when learners study time. However, the topic of time involves more than just that. There are three aspects of time that need to be developed:

- the duration of time
- the passing and sequencing of time
- identifying a point in time by, for example, reading a clock.

All three aspects of time are developed in this unit.

Resources

This topic involves a number of practical activities for which various resources are needed. These resources are listed in the various sections.

6.1 Days, weeks, months and years

Mathematical notes

In this section learners work with days, weeks, months and years. They also read calendars and talk about public holidays.

Resources

You could use two calendars from the current year (one with the weeks starting on Mondays and one with the weeks starting on Sundays) to replace the two calendars on pages 73 and 75. You will, however, then need to check all the questions against the current year's calendar.

Teaching guidelines

You can check whether learners know the number of days in a year, the number of months in a year, the number of days in a week, and the number of weeks in a year. You can also check whether they know how leap years differ from other years. The extra day in February every four years is the culmination of four quarter days.

You can use the tinted passage to fill in any gaps that learners may have. Learners can also use it to revise this information for tests or exams.

Notes on questions

Explain to learners that when a public holiday falls on a Sunday, the Monday also becomes a public holiday. In question 4, some learners may count the yellow blocks in the calendar on page 73 and give an answer of 13, but other learners may count from the list of public holidays on page 74 and give an answer of 12.

Possible misconceptions

The school week starts on Mondays. The calendar shown on page 75 has each week beginning on a Monday. However, the calendar shown on page 73 has each week beginning on a Sunday. In the Jewish and Christian traditions Sunday is the first day of the week. However, Monday is the first day of the school and working week.

Both types of calendars are provided here with the aim to teach learners to always check whether the calendar starts the week with a Sunday or a Monday.

UNIT 6 TIME

6.1 Days, weeks, months and years

There are **365 days** in a normal calendar year.

There are **12 months** in a year. Eleven of the months have either 30 or 31 days.

But February, the second month of the year, has 28 days for three consecutive years. In the fourth year, called a **leap year**, February has 29 days. (Why?)

There are **52 complete weeks** in a year. Each week has **7 days**.

We use **calendars** to measure time in years, months, weeks, and days.

Below is the calendar for 2016. The public holidays are marked in yellow.

JANUARY 2016

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

FEBRUARY 2016

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

MARCH 2016

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

APRIL 2016

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

MAY 2016

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

JUNE 2016

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

JULY 2016

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

AUGUST 2016

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

SEPTEMBER 2016

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

OCTOBER 2016

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

NOVEMBER 2016

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

DECEMBER 2016

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Teaching guidelines

You might want to supply learners with a calendar and a list of the public holidays for the current year. You could adapt questions 2, 3, 4 and 5(a) and (b) to focus on the current year.

Learners will find it easier to complete question 5 if they can see the questions (on page 74) and the calendar (on page 73) at the same time. You could suggest that pairs of learners organise their textbooks so that one learner has page 74 open and the other learner has page 73 open.

Answers

1. (a) April, June, September, November
   (b) January, March, May, July, August, October, December
   (c) February
2. Accept any legitimate ways that learners suggest.

   Learners can recite the rhyme:

   30 days has September,
   April, June and November,
   all the rest have 31
   except for February,
   which has twenty-eight
   – rain or shine –
   but in leap years, twenty-nine.

   Learners can count on their knuckles:

   Starting with January *on* a knuckle, February *off* a knuckle, March on a knuckle, etc. All the months that "land on knuckles" have 31 days, as long as you count both July and August on a knuckle. The other months have 30 days, except for February.
3. Yes. We can see on the calendar that there are 29 days in February 2016.
4. 12
5. (a) Sunday
   (b) In South Africa, when a public holiday falls on a Sunday the following day becomes a public holiday as well.
   (c) On this day we celebrate the first time that all South Africans could vote. Most South Africans were not able to vote before 27 April 1994.
   (d) September 24 is Heritage Day.
6. Answers will differ from class to class and year to year.
7. Learners' answers to this question will differ, because they were born on different days.

1. (a) Name the months with 30 days.
   (b) Name the months with more than 30 days.
   (c) Name the months with fewer than 30 days.
2. Is there a pattern that you can use to quickly say how many days there are in any month?
3. Is 2016 a leap year? How do you know that?
4. How many public holidays are there in 2016?

Here is a list of the public holidays:

January 1	New Year's Day	June 16	Youth Day
March 21	Human Rights Day	August 9	Women's Day
March 25	Good Friday	September 24	Heritage Day
March 28	Family Day	December 16	Day of Reconciliation
April 27	Freedom Day	December 25	Christmas Day
May 1	Workers' Day	December 26	Day of Goodwill

5. (a) On what day of the week is Workers' Day in 2016?
   (b) 2 May is also indicated as a public holiday on the 2016 calendar but it is not in the list of public holidays above. Why is that? Discuss with a classmate.
   (c) April 27 is known as Freedom Day in South Africa. Why is this specific day important for our nation? Find out from someone if you do not know.
   (d) Does September have a public holiday? If so, what is it called and when is it?
6. How many months, weeks and days have passed since the beginning of the year until today?
7. How old are you today in years, months, weeks and days?

Teaching guidelines

Learners will find it easier to answer the questions if they can see the questions (on page 76) and the calendar (on page 75) at the same time. You could suggest that pairs of learners organise their textbooks so that one learner has page 75 open and the other learner has page 76 open.

You might like to adapt this activity to the current year by providing learners with a calendar and a list of school and public holidays for the current year. You could adapt questions 8 and 9(a) accordingly.

Study the calendar below and answer the questions on page 76.

My school calendar

JANUARY							FEBRUARY							MARCH							APRIL						
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
			1	2	3	4							1							1			1	2	3	4	5
5	6	7	8	9	10	11	2	3	4	5	6	7	8	2	3	4	5	6	7	8	6	7	8	9	10	11	12
12	13	14	15	16	17	18	9	10	11	12	13	14	15	9	10	11	12	13	14	15	13	14	15	16	17	18	19
19	20	21	22	23	24	25	16	17	18	19	20	21	22	16	17	18	19	20	21	22	20	21	22	23	24	25	26
26	27	28	29	30	31		23	24	25	26	27	28		23	24	25	26	27	28	29	27	28	29	30	31		
														30	31												

MAY							JUNE							JULY							AUGUST						
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
				1	2	3	1	2	3	4	5	6	7			1	2	3	4	5						1	2
4	5	6	7	8	9	10	8	9	10	11	12	13	14	6	7	8	9	10	11	12	3	4	5	6	7	8	9
11	12	13	14	15	16	17	15	16	17	18	19	20	21	13	14	15	16	17	18	19	10	11	12	13	14	15	16
18	19	20	21	22	23	24	22	23	24	25	26	27	28	20	21	22	23	24	25	26	17	18	19	20	21	22	23
25	26	27	28	29	30	31	29	30						27	28	29	30	31			24	25	26	27	28	29	30
																					31						

SEPTEMBER							OCTOBER							NOVEMBER							DECEMBER						
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
	1	2	3	4	5	6				1	2	3	4							1		1	2	3	4	5	6
7	8	9	10	11	12	13	5	6	7	8	9	10	11	2	3	4	5	6	7	8	7	8	9	10	11	12	13
14	15	16	17	18	19	20	12	13	14	15	16	17	18	9	10	11	12	13	14	15	14	15	16	17	18	19	20
21	22	23	24	25	26	27	19	20	21	22	23	24	25	16	17	18	19	20	21	22	21	22	23	24	25	26	27
28	29	30					26	27	28	29	30	31		23	24	25	26	27	28	29	28	29	30	31			
														30													

SCHOOL TERMS	First Term	Second Term	Third Term	Fourth Term
Coastal provinces Eastern Cape • KwaZulu-Natal • Northern Cape • Western Cape	14.01 – 25.03	13.04 – 26.06	20.07 – 02.10	12.10 – 09.12
Inland provinces Free State • Gauteng • Mpumalanga • North West • Limpopo	21.01 – 01.04	13.04 – 26.06	20.07 – 02.10	12.10 – 09.12

PUBLIC HOLIDAYS					
New Year's Day	1 January	Thursday	Youth Day	16 June	Tuesday
Human Rights Day	21 March	Saturday	National Women's Day	9 August	Sunday
Good Friday	3 April	Friday	Public holiday	10 August	Monday
Family Day	6 April	Monday	Heritage Day	24 September	Thursday
Freedom Day	27 April	Monday	Day of Reconciliation	16 December	Wednesday
Workers' Day	1 May	Friday	Christmas Day	25 December	Friday
			Day of Goodwill	26 December	Saturday

Answers

8. No, different calendars are organised differently. Many calendars have Sunday as the beginning of the week, while others have Monday as the beginning of the week.

9. (a) 1, 15, 22
   (b) 6, 13, 27
   (c) There are 7 days in a week, so you add or subtract in 7s.

10. (a) Learners' answers will differ.
    (b) Term 2 starts on 13 April; Term 2 ends on 26 June.
    (c) 3 weeks and 2 days
    (d) When the school closes on 9 December.
    (e) Term 2

11. The Term 1 dates for schools in coastal provinces are given in brackets.

Term no. and dates		No. of week days	No. of public holidays	Actual no. of school days
1	Starts: 14 (21) January Ends: 25 Mar (1 April)	51	0	51
2	Starts: 13 April Ends: 26 June	56	3	53
3	Starts: 20 July Ends: 02 October	55	2	53
4	Starts: 12 October Ends: 9 December	43	0	43
	TOTAL	205	5	200

8. Study the school calendar. The first column in each month is a Monday. In the calendar of 2016, the first column is a Sunday. Is this a mistake in one of the calendars? Why or why not? Discuss with a classmate.

9. (a) On the school calendar, 8 February is on a Sunday. What other dates in February will also be on a Sunday in this year? Find the answer without looking at the calendar.
   (b) If 20 June is on a Monday in a certain year, what other dates in June will also be on a Monday?
   (c) How did you think to get the answer in (b)?

10. (a) Is your school in a coastal or inland province?
    (b) When does your Second Term start and when does it end?
    (c) How long are your winter school holidays? Give your answer in weeks and days.
    (d) When do the summer holidays start?
    (e) In which school term does Workers' Day fall?

11. Copy and complete the table for your school.

Term no. and dates		No. of days	No. of public holidays	Actual no. of school days
1	Starts: ..... Ends:			
2	Starts: ..... Ends:			
3	Starts: 20 July Ends: 2 October			
4	Starts: 12 October Ends: 9 December			
	TOTAL			

6.2 Measuring time

Mathematical notes

This section helps learners to discover one way in which time was measured in the past, i.e. using water clocks. Learners will make their own water clocks.

Resources

Ask learners to bring empty 2 litre plastic bottles to school. You will need one bottle for every 4 to 6 learners. You will also need cutting knives or other knives, and other sharp implements, for example nails for making a hole in the bottle top. You could also ask learners to make the water clocks at home, following steps 1 to 3.

Possible misconceptions

Most informal measurement depends on counting: we count the number of strides, we count the number of teaspoons or cups, etc. This water clock is divided into time units based on counting. This works if you compare times measured by the same water clock. However, it is not helpful when comparing times measured by two different clocks. This is because people count at different speeds.

In general, most people are not good at estimating and judging time lengths. This is the basis of the English idiom "time flies when you are having fun".

Teaching guidelines

Learners can follow steps 1 to 6 to make a water clock. You might want to ask unemployed parents to assist during this lesson to prevent learners from hurting themselves or others when using the knives and other sharp implements. Other options are to make the models yourself or to ask learners to make the models at home and to bring them to class.

Answers

1. These are instructions, there are no answers.

6.2 Measuring time

Many years ago water clocks were used to keep track of short periods of time. These clocks measured time based on how much water flowed from one container into the next. You are going to make a simple water clock.

1. Form groups of 4 to 6 learners. Each group will need the following: *a plastic 2-litre bottle, a sharp knife or scissors, a tool with a sharp narrow point, a bottle or jug of water and a permanent marker.*

Step 1  *Cut the body of the bottle in half.*	**Step 2**  *Puncture the bottle top. The hole should not be smaller than this circle:* ●
Step 3  *Screw the bottle top back on. Slip the top part of the bottle into the bottom part.*	**Step 4**  *Pour water into the top half. As soon as the water starts to flow into the bottom half, one of the group members must start to count from 1 to 240.*

Answers

2. The time taken will differ if different water clocks are used. This is because people count at different speeds. These water clocks were based on counting speeds.

3. The marks on the different clocks will be at different heights.

4. Learners' answers will differ.
   Instruction time is legislated at $27\frac{1}{2}$ hours a week. This is 1 650 minutes.
   Different schools will break up the time differently.

Step 5	Step 6
Use the permanent marker to mark how much water has flowed into the bottom half when 60, 120, 180 and 240 counts are reached. 	*Empty the top and bottom half of the bottle.* *Repeat steps 3 to 5.* [Source: http://www.wikihow.com/Make-a-Water-Clock-(Clepsydra)]

You can now use the water clock to measure the time it takes to do something in units of 60 counts.

2. Each group uses their water clock to measure how long one of the learners in the class can hold her or his breath. Compare the time taken according to the different water clocks. What do you notice? Explain why this is so.

3. Place all the water clocks next to each other. What do you notice about the marks you made earlier at 60, 120, 180 and 240?

Counting is not a good way to measure time, because some people count faster than others. People decided long ago to use the changing of days to measure time.

The sun appears each day, it disappears later, and it appears again the next day. The period of time that starts when the sun appears on one day, and ends when the sun appears on the next day, is called **one day**. The day is divided into 24 equal periods which are called **hours**.

1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

4. For approximately how many minutes are you in school each day of the week?

Answers

5. They take the same time. 65 seconds = 1 minute and 5 seconds
6. They take the same time. 120 minutes = 2 hours

6.3 Understanding the 12-hour clock

Mathematical notes

In the Intermediate Phase, learners work with both 12-hour time (using a.m. and p.m.) and 24-hour time. This section focuses on 12-hour time; in the next section learners will work with 24-hour time. Both Sections 6.3 and 6.4 focus on analogue time. Section 6.5 focuses on digital time.

Teaching guidelines

You can check that learners understand that watches are worn on wrists and clocks are freestanding or hang on a wall.

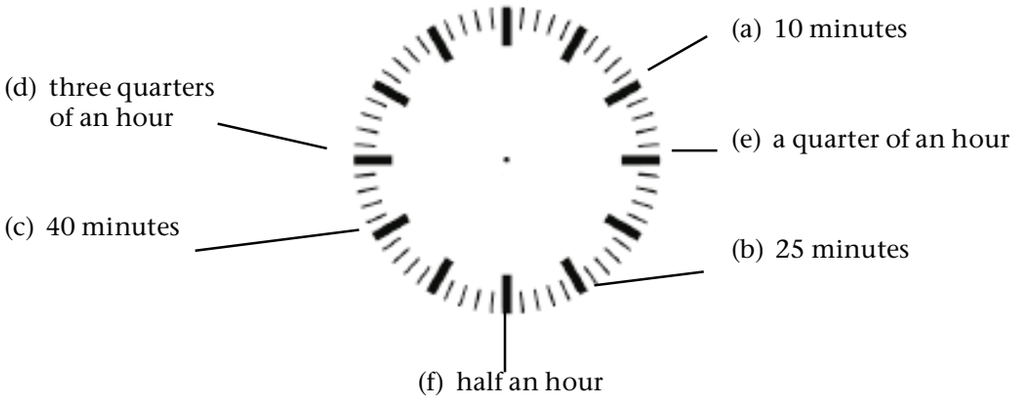
You can check whether learners know which is the hour hand and which is the minute hand. Also check whether learners remember how many minutes there are in an hour. Check that they can deduce how many minutes are in $\frac{1}{2}$ hour, $\frac{1}{4}$ hour and $\frac{3}{4}$ hour. You can use the tinted passage to fill in any gaps that learners may have. Learners can also use it to revise this information for tests or exams.

Notes on questions

In question 1, make sure that learners understand that they need to start counting from the 12th hour position.

Answers

1.



5. It takes Learner A 65 seconds to write ten words. It takes Learner B 1 minute and 5 seconds to write ten words. Who takes the longest to write ten words?
6. It takes Grandma 120 minutes to walk to the shop. It takes Grandpa 2 hours to walk to the shop. Who takes the longest?

6.3 Understanding the 12-hour clock



This clock shows 12 hours. It measures the time from midnight to midday and from midday to midnight. The starting point (or the zero point) is at the top of the clock where the 12 is.

The clock face is divided into 60 sections to show 60 minutes. The short hand is the **hour hand**. The long hand is the **minute hand**.

The minute hand moves around the clock face once every 60 minutes. While the minute hand moves around the clock once, the hour hand moves to the next hour.

One hour is the same length of time as **60 minutes**. So:

15 minutes = $\frac{1}{4}$ hour

30 minutes = $\frac{1}{2}$ hour

45 minutes = $\frac{3}{4}$ hour



1. Copy this circle. It has the same markings as the 12-hour clock above. Start counting the minutes at 12. Show where the markings are for the following number of minutes:
   (a) 10 minutes (b) 25 minutes
   (c) 40 minutes (d) three quarters of an hour
   (e) a quarter of an hour (f) half an hour

Possible misconceptions

Learners sometimes struggle to understand minutes *past* the hour, and minutes *to* the hour. Sometimes it helps to act this out.

Resources

12 sheets of paper with a number from 1 to 12 on each sheet.

Two long sticks or pieces of string.

Teaching guidelines

Let learners sit in a large circle. Place stones to mark every minute, and use the numbers on the pieces of paper to show the hours (1 to 12). One learner sits in the middle of the circle holding a piece of string or a stick while another learner holds the other end of the string or stick. This is the minute hand.

Use another piece of string or a stick to represent the hour hand. Start with the hands positioned at any hour, for example 12 o'clock. Ask learners to say what time the clock shows. Then ask the learner with the minute hand to walk around the inside of the circle while the rest of the class says: "It is ... minutes *past* 12 o'clock." Stop the learner when he/she gets to 12:15 and ask the class what fraction of the circle the learner has walked around. Do this again at 12:30. Then explain that for the rest of the circle, the learner will be walking back towards the 12. So we no longer count minutes *past* the hour, we count minutes *to* the next hour. Continue as before but let the class say: "It is ... minutes *to* 1 o'clock," etc.

Answers

2. (a) 1:00 1 o'clock
   (b) 1:15 quarter past 1
   (c) 1:30 half past 1
   (d) 1:45 quarter to 2

3. (a) 8:05 5 past 8
   (b) 8:10 10 past 8
   (c) 8:20 20 past 8

2. What times do these clocks show? Write the time in numbers and in words, for example: *6:15 quarter past 6*.

(a) 

(b) 

(c) 

(d) 

Minutes past the hour

As the minute hand moves from 12, at the top of the clock, we talk about minutes *past* the hour. We say: 1 minute past ..., 2 minutes past ..., 3 minutes past ... until we get to 30 minutes past, or half past the hour.

3. What times do these clocks show? Write the time in numbers and in words.

(a) 

(b) 

(c) 

Minutes to the hour

As the minute hand moves on past the 30 minute mark, we talk about minutes *to* the hour. We say 29 minutes to ..., 28 minutes to ..., 27 minutes to ... and so on, until we get to 1 minute to ... and then reach the next hour.

Notes on questions

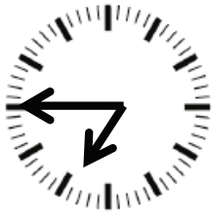
This is quite a long section. You could aim to do questions 1 to 6 in class, and let learners do questions 7 to 12 for additional practice.

Note that there are blank clock faces in the Addendum (page 442) that you could photocopy for learners for question 5, or for further practice.

Answers

4. (a) 8:35 25 to 9
   (b) 8:40 20 to 9
   (c) 8:45 quarter to 9

5. (a)



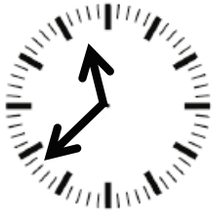
(b)



(c)



(d)



6. (a) 6: 00 a.m. (b) 7:15 a.m.
   (c) 8: 00 a.m. (d) 2:10 p.m.

4. What times do these clocks show? Write the time in numbers and in words.

(a) 

(b) 

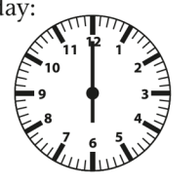
(c) 

5. Copy the circle on page 79. Draw hands to show the time:
   (a) quarter to 7 (b) 25 minutes past 3 o'clock
   (c) 4 o'clock (d) 22 minutes to 12 o'clock

Before midday and after midday

You already know that a 12-hour clock shows only 12 hours. The short hand goes around the clock once in 12 hours and twice around the clock in 24 hours. So, when we talk about time on a 12-hour clock we must say whether the time is **before midday (a.m.)** or **after midday (p.m.)**.

6. Give the times on the clocks. Remember to write a.m. or p.m.
   (a) Nathi wakes up to start his day:



   (b) Nathi starts walking to school:



   (c) School starts:



   (d) Soccer practice starts after school:



Notes on questions

As suggested earlier, questions 7 to 12 could be used for additional practice.

Answers

7. (a) 10 minutes (b) 20 minutes
   (c) 7 minutes (d) 15 minutes

8. (a) 30 minutes (b) 15 minutes
   (c) 120 minutes (d) 150 minutes
   (e) 10 minutes (f) (12 × 60) + 7 = 727 minutes
   (g) (12 × 60) + 20 = 740 minutes (h) 12 × 60 = 720 minutes

9. 5 hours
   Here learners can count up: 7:30; 8:30; 9:30; 10:30; 11:30; 12:30. They could also subtract: 12 – 7 = 5.

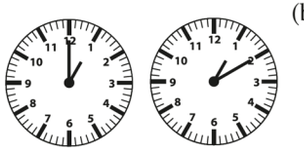
10. 3:10 p.m.
    Here learners can count up 2 to 3 and add the minutes.

11. 9:27 a.m.
    Quarter past 9 = 9:15 15 + 12 = 27

12. Two and a quarter hours = (60 minutes × 2) + 15 minutes = 135 minutes

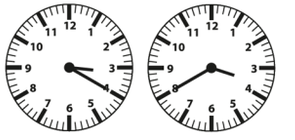
7. The clocks below show when an activity started and when it ended. Say how long each activity took.

(a)



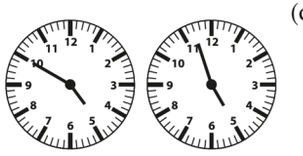
Start End

(b)



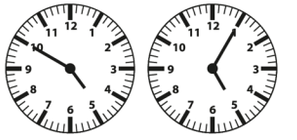
Start End

(c)



Start End

(d)



Start End

8. How many minutes are there in each of these time intervals?
   (a) half an hour (b) a quarter of an hour
   (c) 2 hours (d) two and a half hours
   (e) 6:15 a.m. to 6:25 a.m. (f) 12:18 p.m. to 12:25 a.m.
   (g) 12:50 p.m. to 1:10 a.m. (h) 7:34 a.m. to 7:34 p.m.

9. A school starts at 7:30 a.m. and finishes at 12:30 p.m. How many hours are the learners at school?

10. Soccer practice starts at ten past two in the afternoon and it is one hour long. What time does soccer practice end?

11. Margaret started sweeping her garden at a quarter past 9 in the morning and finished 12 minutes later. What time did she finish?

12. Which is longer: two and a quarter hours or 124 minutes? Explain your answer.

6.4 Understanding the 24-hour clock

Mathematical notes

Both 24-hour time and digital clocks are new in Grade 4.

Resources

Sheets of paper or card for each learner.

Sets of cards numbered as shown in the "Teaching guidelines" below.

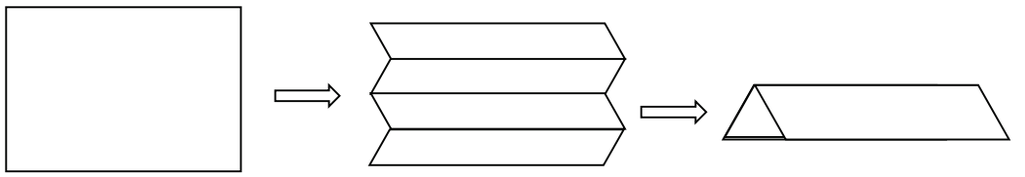
String or wire.

Teaching guidelines

It is useful for learners to see both digital and analogue clocks showing the same time. You can show working examples of both in your classroom. You might also like to make models of both analogue and digital clocks.

To make a model of a digital clock you can do the following:

- Fold a sheet of thick paper or card into quarters lengthwise.



- Fold it again to form a triangular prism, to make the stand.
- Make four sets of cards to show hours and minutes.

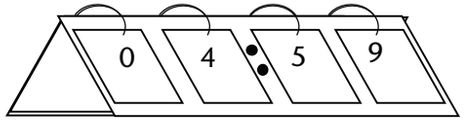
The left-hand set needs cards: 0 1 2

The set second from the left needs cards: 0 1 2 3 4 5 to 9

The set second from the right needs cards: 0 1 2 3 4 5

The set on the far right needs cards: 0 1 2 3 4 5 to 9

- Attach these cards with string to the stand.

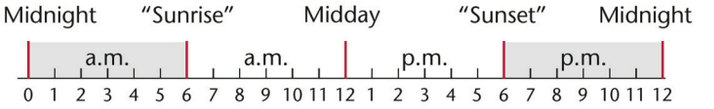


6.4 Understanding the 24-hour clock

The 24-hour clock



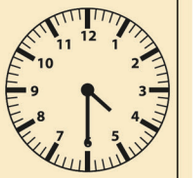
The 12-hour clock



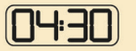
Two kinds of clocks

An **analogue clock** is usually a 12-hour clock. The 12-hour clock on the right shows half past four.

The hour hand is halfway between 4 and 5. The minute hand is on 30 minutes.



Analogue 12-hour clock



Digital 24-hour clock

A **digital clock** is usually a 24-hour clock. The time on the 24-hour clock on the right is **04:30**; this is 4 hours and 30 minutes after midnight or 4:30 a.m.

If you wake up at six o'clock in the morning, the time on a 24-hour clock will show **06:00**. This is the same as 6 a.m. If we have supper at six o'clock in the evening, the time on the 24-hour clock will show **18:00**. This is the same as 6 p.m.

On the 24-hour clock, midnight is 24:00. But because the new day begins at the same time as the old day ends, midnight is usually indicated as 00:00.

Possible misconceptions

Learners may be confused about how to write midday and midnight in 24-hour time. You may need to clarify for them that midday is written 12:00 p.m. or 12:00 (in digital format) and midnight is written 12 a.m. or 00:00 (see the tinted passage on page 85). This is simply a convention that has been adopted for the sake of clarity.

Answers

1. (a) 07:00 (b) 19:00 (c) 00:00
   (d) 12:00 (e) 11:00 (f) 23:00
2. (a) 23:00 (b) 11:00 (c) 02:00
   (d) 14:00 (e) 18:00 (f) 18:00
3. (a) 13:20 (b) 12:40 (c) 15:15 (d) 04:25
4. (a) 10:00 a.m. 10 o'clock in the morning
   (b) 10:15 a.m. quarter past 10 in the morning
   (c) 10:30 a.m. half past 10 in the morning
   (d) 10:45 a.m. quarter to 11 in the morning
   (e) 12:00 a.m. midday or 12 noon (f) 12:00 p.m. midnight
   (g) 8:35 a.m. 25 to 9 in the morning (h) 4:35 p.m. 25 to 5 in the afternoon

6.5 Hours, minutes and seconds

Resources

For question 1 you will need a stopwatch. Digital stopwatches are usually easier to read than analogue stopwatches. Many cellphones have a stopwatch function, or the possibility of downloading a free stopwatch app. If you have access to more stopwatches, learners can use these in question 4.

Learners will need tennis balls to bounce for question 2.

Teaching guidelines

If you do not have enough tennis balls for the whole class to do question 2 simultaneously, let learners work in groups. Let most of the class complete the other questions while one group is busy with the tennis balls.

Answers

1. Practical activity
2. (a) Learners' estimates will differ.
   (b) The number of times learners bounce the ball will differ.

1. Write the 24-hour time for the following:
   (a) 7 o'clock in the morning (b) 7 p.m.
   (c) 12 midnight (d) noon or midday
   (e) 11 a.m. (f) 11 p.m.

2. Write the 24-hour time for the following:
   (a) an hour before midnight (b) an hour before midday
   (c) two hours after midnight (d) two hours after midday
   (e) six hours before midnight (f) six hours after midday

3. Write the 24-hour time for the following:
   (a) 20 minutes after 1 p.m.
   (b) 20 minutes to 1 p.m.
   (c) quarter past 3 in the afternoon
   (d) 25 minutes past 4 a.m.

4. Write these 24-hour times as 12-hour times. Write the times in numbers and in words.
   (a) 10:00 (b) 10:15 (c) 10:30 (d) 10:45
   (e) 12:00 (f) 24:00 (g) 08:35 (h) 16:35

6.5 Hours, minutes and seconds

1. Ask your teacher for a stopwatch. A stopwatch counts seconds and minutes. Let a classmate time you. Practise until you can count to 10 in 10 seconds.
2. (a) Estimate how many times you can bounce a tennis ball in one minute.
   (b) Ask your teacher for a tennis ball. Let the teacher time you on the stopwatch. Bounce the tennis ball and count the number of bounces in one minute.

Teaching guidelines

If you only have one instrument for measuring seconds (stopwatch, cellphone, watch or clock with seconds hand), let most of the class continue with the other questions while you help groups of learners to complete question 4.

Answers

3. Learners' answers will differ.
4. Learners' activities will differ; their estimates will also differ.
5. (a) B (b) C (c) A

3. How many activities can you think of that will take about one second to complete? Make a list.
   Here are three examples to get you going: switching on a light; snapping your fingers once; sneezing.
4. As a class, think of three activities that will take less than a minute to complete. Estimate how many seconds it will take a classmate to do each of the three activities. Let a classmate do each activity while the teacher times him or her with a watch or stopwatch. How close were your estimates?

Many 12-hour and 24-hour clocks show seconds too.

These clocks both show 40 seconds past 7 o'clock in the evening.





5. Match the times on the 24-hour clocks with the times given in words below:

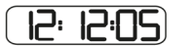
A



B



C



(a) 12 minutes and 5 seconds after midnight

(b) 12 minutes and 5 seconds after midday

(c) 20 minutes and 30 seconds before 6 p.m.

Grade 4 Term 1 Unit 7 Data handling

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 Collect and organise data	The concept of data as measurements that vary	86 to 89
7.2 Representing data in tables	Data that are represented in tables can be compared more easily	90
7.3 Pictographs and bar graphs	Frequency comparisons of data at a glance	91 to 92
7.4 Information in pie charts	Proportional comparison of data at a glance	92 to 94
7.5 Information in bar graphs	Length of bars are used to represent frequencies	95

CAPS time allocation	10 hours
CAPS page references	30 and 57 to 58

Mathematical background

Some ideas that differentiate data handling from mathematics:

- **The answer to data questions is in the information from lots of data gathered.**
  Data handling is necessary where measurements and frequencies vary, and therefore one measurement cannot provide accurate information about a situation. Lots of different data can be confusing, so we organise the data that we collect in different ways to get a "picture" of the situation.
- **The numbers we use in data handling always have some unit of measurement, or some description of a category they belong to.**
  In Mathematics, learners mostly work with abstract numbers. In data handling the numbers must be interpreted in a context. The number 2 can be 2 cm or 2 fish, depending on the question.
- **Data questions are always answered with a story about the context.**
  Data handling starts when we need to answer a question about a situation where the property we look at varies. The numerical answers we get through data handling must be interpreted to answer the question about the situation.

Resources

Rulers; cut-out paper circles for folding into quarters.

7.1 Collect and organise data

Mathematical notes

This unit provides a controlled context to work through the complete cycle of data handling. You can use similar problem types to work through the cycle in learners' own contexts, as the CAPS requires that this be done a few times in a year.

Resources

Rulers.

Teaching guidelines

Aim to spend about 2 hours on this section. Read the story to the class. You must pose the question that will guide the work in this unit: Jabu wants to know the size of the fish so that he can plan how much food he needs, and when to sell his fish. The questions learners will answer are all about the different kinds of fish he has, and their sizes.

Learners will need rulers to take the measurements of 20 fish. Learners work in pairs to answer question 1 on page 87. They will find it easier if one learner keeps his/her book open on page 87 while the other learner keeps his/her book open on pages 88 to 89.

Learners may share the measuring work to save time. Discuss with them how to measure as accurately as possible.

Critical knowledge

Data are about trends. Information is grouped to make it easier to "see" the trends.

Possible misconceptions

Learners tend to answer statistical questions with reference to only one of the data points. For example, when you ask how big the Huna goldfish are, they may give the length of the longest or the shortest fish. Learners are used to giving a single number as an answer when they solve word problems in Mathematics. In Statistics we also want to find a single number to answer a question, but the number must be representative of all the numbers that we gathered as data.

In Grade 4 we prepare learners' intuitions about representativeness, but they do not yet use a single measurement to represent all measurements. In this unit we develop ways to compare groups of data by using language like "between", "less than" and "more than".

UNIT 7 DATA HANDLING

In this unit you are going to need your ruler to measure the length of fish. The measurements and other information about the fish will be our data to tell a story about a fish farmer.

7.1 Collect and organise data

Jabu farms with goldfish. He sells the fish to people who keep them as pets. He has three kinds of goldfish: Huna (blue-grey), Hibuna (gold) and Syubunkin (spotty).

Goldfish live up to 20 years and they require good care.

Fish of different sizes have different prices. We are going to imagine we use data to help Jabu find the sizes of his goldfish.



Huna goldfish



Hibuna goldfish



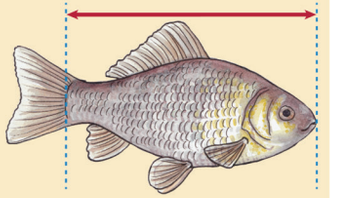
Syubunkin goldfish

Jabu has 20 goldfish in Tank A. Imagine that they are the goldfish that you see on pages 88 and 89. You have to measure these goldfish and make a list of their measurements.

Plan ahead

Make a plan so that you will know which measurements are of Huna fish and which are of Hibuna fish.

Plan how to measure accurately. Measure from the nose to the end of the body. Fish don't keep their tails still, so you can't measure their tails accurately!

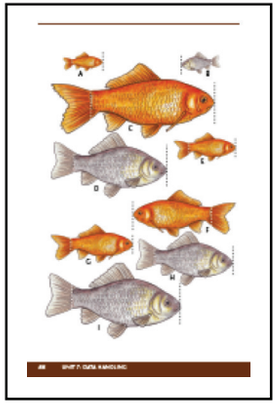


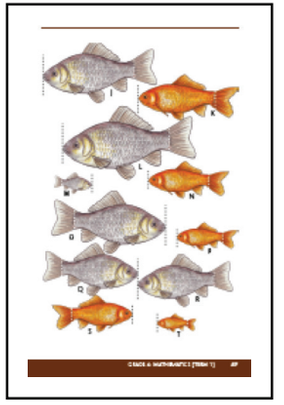
86 UNIT 7: DATA HANDLING

Notes on questions

Observe the ways different pairs of learners use to organise the data informally. Let them discuss which ways are more helpful when they have to answer questions 2 and 3.

Question 2 draws attention to language. For example, there is a difference between saying "the fish are longer than 2 cm" and "the fish are 2 cm and longer". Sentence B is therefore *not true*, because some Hibunas are 2 cm long. Sentences A and C are true, but Sentence D *best describes* the size of the Hibunas.





Please note: Learner Book pages 88 and 89 shown alongside only contain sketches of fish. No questions or instructions appear on these pages, so there are no further teaching notes specifically related to these pages.

Answers

1. (a) Answers will differ. Learners may decide to each measure one type of fish, or each measure all the fish on one page, or split the work in some other way. This is their data gathering plan that they must write down.
   (b) Some learners may simply write down the numbers in a jumbled way, others may make separate lists for the measurements of the Hibuna and the Huna fish. Others may start to organise their measurements into simple tables.
   Hibunas (gold): 2 cm, 8 cm, 3 cm, 5 cm, 4 cm, 5 cm, 4 cm, 3 cm, 4 cm, 2 cm
   Hunas (blue-grey): 2 cm, 6 cm, 5 cm, 7 cm, 6 cm, 7 cm, 2 cm, 6 cm, 5 cm, 5 cm
   (c) Learners compare measurements of fish. Where they differ, they decide on the source of the difference. They have to show each other how they measured.
   (d) Learners re-do question (b) if necessary.
2. Sentence D: The Hibuna fish are between 2 cm and 8 cm long, but most are between 3 cm and 5 cm long.
3. There are many possible answers. Learners' answers will differ. Let learners compare and discuss their descriptions. One possible description is:
   The Huna fish are between 2 cm and 7 cm long, but most are 5 cm and longer.

1. Work with a classmate.
   (a) Decide how you will share the work and make sure that you do not leave out fish or measure them twice. Write down what you decided.
   (b) Now measure the fish and write down your measurements in your book.
   (c) Compare your measurements with another pair of classmates. If your measurements are not the same, discuss the reasons and write them down.
   (d) Measure again where you differed. Measure accurately. If you have new measurements, write them down.

The measurements you made are called **data**. Data are measurements of a property that **varies**. The lengths of goldfish vary because the fish are not all the same age, and because they do not all eat exactly the same amount of food, and just because some fish grow faster and get bigger than others.

The measurements you made may also be a little different from the measurements your classmates made, because you may not have been completely accurate. We must plan to be completely accurate when we gather data.

2. Jabu asks how long the fish in Tank A are. Which of sentences A to D below best describes the lengths of the Hibuna fish?
   A The Hibuna fish are not more than 8 cm long.
   B The Hibuna fish are all longer than 2 cm.
   C The Hibuna fish are between 2 cm and 8 cm long.
   D The Hibuna fish are between 2 cm and 8 cm long, but most are between 3 cm and 5 cm long.
3. Look at your measurements. Write a sentence to tell Jabu how big his Huna fish are.

7.2 Representing data in tables

Mathematical notes

A well-constructed tally table shows how the data are distributed. If it is done neatly, as on page 90, it looks like a graph.

Teaching guidelines

Aim to spend about 2 hours on this section.

Discuss the structure of the tally table with the class. Draw learners' attention to the clever plan to group the tallies into fives so that they can be counted quickly.

Notes on questions

The questions aim to make learners aware of the use of words or phrases such as "most", "same as", "more than", "less than", etc. Other terminology that is used to describe intervals of measurements informally, such as "between", "up to", or "longer than" is also important. In question 2, let learners decide which statements are true, and correct those that are not true.

Possible misconceptions

You can expect learners to answer question 1 by giving the exact number of fish for each measurement, for example: "There are 3 fish that are 2 cm long and 7 fish that are 3 cm long ..." Discuss with them that such answers do not help us to understand the situation because we cannot remember all the information.

Learners sometimes use the expression "most of" to describe the category with the most data, or the longest bar in a bar graph. However, a category can have more data than any of the others without this category representing most of the data in the data set. For example, in the tally table we can see that the biggest size category is 4 cm fish. But only 15 fish out of 50 are 4 cm long. Most of the fish in the tank, the other 35 fish, are not 4 cm long. It is important to help learners to understand this distinction.

Answers

1. There are many possible descriptions. Two examples are:
   "The Syubunkin fish are between 2 cm and 8 cm long, but most are between 3 cm and 6 cm long."
   "The Syubunkin fish are between 2 cm and 8 cm long, and many are 4 cm or 5 cm long."

7.2 Representing data in tables

In Tank B Jabu has 50 Syubunkin fish. He measured them and wrote his measurements in this **tally table**. It helps him to organise his measurements. He makes a mark (tally) in the appropriate place in the table. Have a look at what he did.

Syubunkin in Tank B

Length	Tallies	Total number of fish
2 cm	𝍸	3
3 cm	𝍸 //	7
4 cm	𝍸 𝍸 𝍸	15
5 cm	𝍸 𝍸 /	11
6 cm	𝍸 //	7
7 cm	///	3
8 cm	////	4

1. What can you tell about the length of the fish from Jabu's tally table? Write down your story and compare it with that of a classmate.
2. Do you agree or disagree with the following statements? Explain why.
   (a) Most Syubunkins are between 5 cm and 8 cm long.
   (b) Most Syubunkins are 4 cm long.
   (c) The number of Syubunkins that are 3 cm long is the same as the number of Syubunkins that are 6 cm long.
   (d) Half of the Syubunkins are 5 cm up to 8 cm long.
3. Jabu asks you to put together the information about the fish in Tank A (that is, the fish on pages 88 and 89). Make a tally table like the one above to show the lengths of the Hibuna and Huna fish that you measured.

Answers *(continued)*

2. (a) Disagree, because the same number of fish (25) are between 5 cm and 8 cm long as between 2 cm and 4 cm long.
   Or, disagree, and give another interval that contains more than half of the fish.
   (b) Disagree, because only 15 of the 50 Syubunkins are 4 cm long; the majority (35 out of 50) Syubunkins have different sizes.
   (c) Agree, because there are 7 Syubunkins that are 3 cm long and 7 Syubunkins that are 6 cm long.
   (d) Agree

3. Learners copy and complete one table for the Hibunas and one for the Hunas in Tank A (the fish on pages 88 and 89).

Hunas (blue-grey) in Tank A

Length	Tallies	Total
2 cm	//	2
3 cm		0
4 cm		0
5 cm	///	3
6 cm	///	3
7 cm	//	2
8 cm		0

Hibunas (gold) in Tank A

Length	Tallies	Total
2 cm	//	2
3 cm	//	2
4 cm	///	3
5 cm	//	2
6 cm		0
7 cm		0
8 cm	/	1

7.2 Representing data in tables

In Tank B Jabu has 50 Syubunkin fish. He measured them and wrote his measurements in this **tally table**. It helps him to organise his measurements. He makes a mark (tally) in the appropriate place in the table. Have a look at what he did.

Syubunkin in Tank B

Length	Tallies	Total number of fish
2 cm	///	3
3 cm	𝍸 //	7
4 cm	𝍸 𝍸 𝍸	15
5 cm	𝍸 𝍸 /	11
6 cm	𝍸 //	7
7 cm	///	3
8 cm	////	4

1. What can you tell about the length of the fish from Jabu's tally table? Write down your story and compare it with that of a classmate.
2. Do you agree or disagree with the following statements? Explain why.
   (a) Most Syubunkins are between 5 cm and 8 cm long.
   (b) Most Syubunkins are 4 cm long.
   (c) The number of Syubunkins that are 3 cm long is the same as the number of Syubunkins that are 6 cm long.
   (d) Half of the Syubunkins are 5 cm up to 8 cm long.
3. Jabu asks you to put together the information about the fish in Tank A (that is, the fish on pages 88 and 89). Make a tally table like the one above to show the lengths of the Hibuna and Huna fish that you measured.

7.3 Pictographs and bar graphs

Teaching guidelines

Aim to spend about 2 hours on this section.

Prepare the frame of the pictograph on the board or on a poster so that you can use it in the class discussion. Alert learners to the heading and the key. These are important aspects of a pictograph. Stress that all the fish must be drawn the same size and must be evenly spaced.

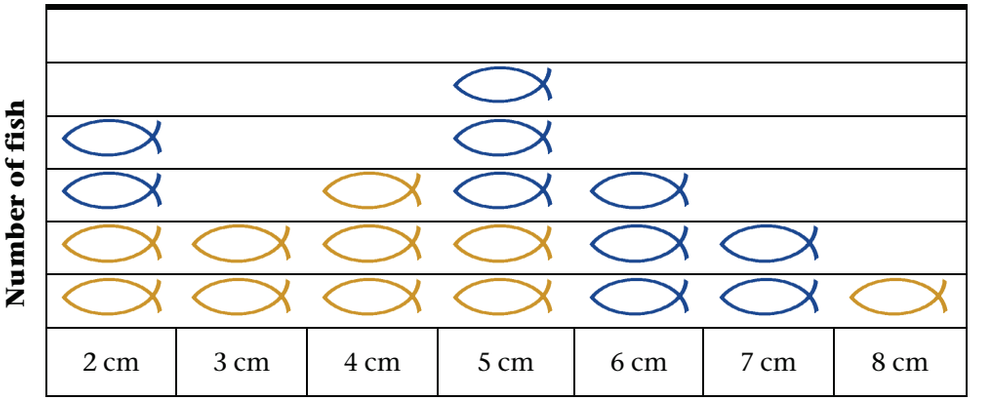
Learners must redraw the frame of the pictograph in their exercise books.

Critical knowledge

The pictures must all be the same size and evenly spaced. The heading and key are crucial parts of the graph.

Answers

1. **Number of Hunas and Hibunas of different lengths in Tank A**



Key: One Huna One Hibuna

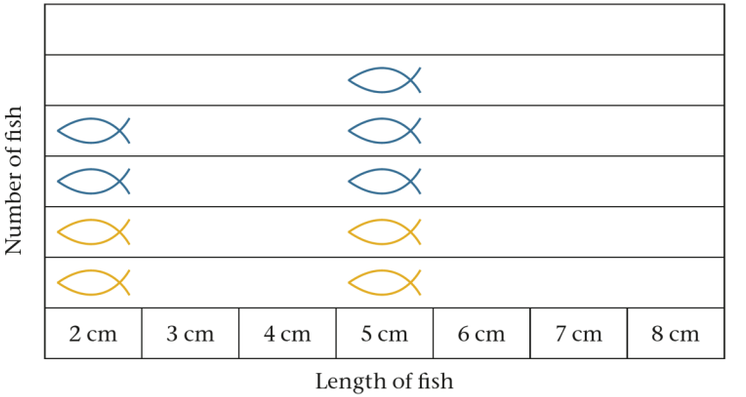
2. (a) **Small:** 6 fish: 2 Hunas and 4 Hibunas
   (b) **Medium:** 8 fish: 3 Hunas and 5 Hibunas
   (c) **Large:** 6 fish: 5 Hunas and 1 Hibuna

7.3 Pictographs and bar graphs

1. The graph below is called a **pictograph**. Complete a pictograph of your own to show the Huna and Hibuna fish of different lengths.

A pictograph has a **heading** and a **key** to tell the meaning of the pictograph.

Number of Huna and Hibuna of different lengths in Tank A



Key: One Huna One Hibuna

2. Jabu wants a bar graph of the small, medium and large fish in Tank A.
   (a) If a fish is shorter than 4 cm it is *small*. How many small fish are there in Tank A?
   (b) A fish that is 4 cm or 5 cm long is *medium*. How many medium fish are there in Tank A?
   (c) A fish that is 6 cm or longer is *large*. How many large fish are there in Tank A?

Teaching guidelines

Once learners have completed question 2 they should copy the frame for the bar graph given on page 92 and fill in the data. Emphasise once again that the graph needs a heading, and that each axis needs a label. Once learners have completed the graph for Hunas in Tank A, they should draw a second graph (adapt the graph heading) for Hibunas in Tank A.

Answers

2. (d)

Number of Huna fish of different lengths in Tank A

Number of fish			
8			
7			
6			
5			
4			
3			
2			
1			
	Small	Medium	Large

Groups

Number of Hibuna fish of different lengths in Tank A

Number of fish			
8			
7			
6			
5			
4			
3			
2			
1			
	Small	Medium	Large

Groups

(d) Complete the bar graph below. Draw bars to show the number of fish. Use one block for each fish you count. Draw another bar graph for the Hibuna.

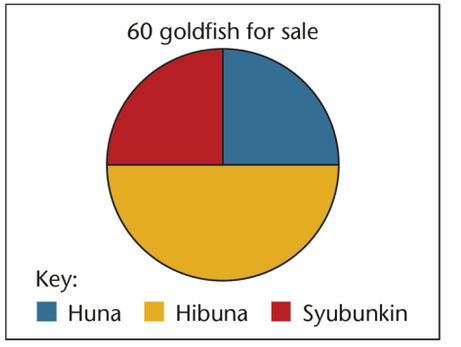
Number of Huna of different lengths in Tank A

Number of fish			
8			
7			
6			
5			
4			
3			
2			
1			
	Small	Medium	Large

Groups

7.4 Information in pie charts

Jabu sees an advertisement on the internet of a pet shop owner who wants to sell her tank of fish. She wants R480 for 60 goldfish. The advertisement has only this pie chart.



A pie chart has a **heading** and a **key** to tell the meaning of the pie chart.

In a pie chart the parts are **fractions** of the pie (circle). For example, we work out *what fraction* of the fish are Huna and then we colour *the same fraction* of the pie.

1. (a) What is the heading of the chart?
   (b) What does the key tell you?
   (c) What does the graph tell you about the fish in this tank?

7.4 Information in pie charts

Mathematical notes

Pie charts are useful to compare proportions of data.

The proportion of each category of data is represented as part of the area of a circle. These areas are called sectors of a circle.

In Grade 4 learners will compare these proportions by using fractions.

Resources

If possible, cut-out paper circles for folding into quarters (see the note about question 3 in "Notes on questions" on the next page).

Large copies of the graphs on posters or drawn on the board.

Teaching guidelines

Aim to spend about 2 hours on this section.

Prepare a pie chart on the board or on a poster to use for class discussions.

Discuss with learners that these pie charts do not show exact numbers, but show the relative amounts of each grouping as fractions of circles.

Use class discussions to develop comparisons like "twice as many" and "half as many".

Critical knowledge

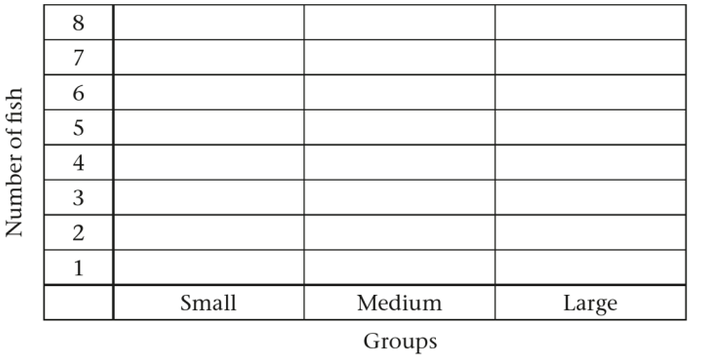
Pie charts help us to make comparisons about categories of data. The size of the sector represents the relative proportion of the data. Pie charts do not show the exact numbers. Learners should use their knowledge of fractions to estimate the relative size of the sectors.

Answers

1. (a) 60 goldfish for sale
   (b) It tells us what kind of fish each colour represents.
   (c) Half of the fish (30 fish) are Hibunas.
   One quarter of the fish (15 fish) are Hunas.
   One quarter of the fish (15 fish) are Syubunkins.

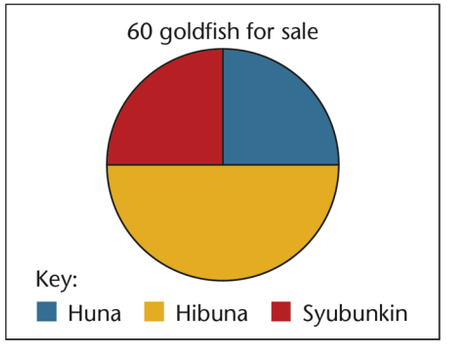
(d) Complete the bar graph below. Draw bars to show the number of fish. Use one block for each fish you count. Draw another bar graph for the Hibuna.

Number of Huna of different lengths in Tank A



7.4 Information in pie charts

Jabu sees an advertisement on the internet of a pet shop owner who wants to sell her tank of fish. She wants R480 for 60 goldfish. The advertisement has only this pie chart.



A pie chart has a **heading** and a **key** to tell the meaning of the pie chart.

In a pie chart the parts are **fractions** of the pie (circle). For example, we work out *what fraction* of the fish are Huna and then we colour *the same fraction* of the pie.

1. (a) What is the heading of the chart?
   (b) What does the key tell you?
   (c) What does the graph tell you about the fish in this tank?

Notes on questions

For question 3, let learners cut out a circle and fold it in quarters, then cut out the quarter segments to help them estimate the sizes of the sectors of the pie chart.

In question 4 it is important to stress the word "*about*" – learners are not asked exact numbers. Learners' estimates may differ and lead them to interpret the statements in questions 4(b), (c) and (d) differently. Whether learners agree or disagree depends on how much difference they are willing to see as "almost the same".

Answers

2. R480 – R240 = R240
   R240 ÷ 60 = R4
   Yes, Jabu is correct.

3. Learners' answers will differ. A sample answer is provided below. Check all learners' answers to see which make sense.

   Fish between 7 cm and 9 cm long make up the smallest group in the tank.

   Fish between 4 cm and 6 cm long make up about $\frac{1}{3}$ of the fish in the tank.

   Fish that are 10 cm to 12 cm long make up the biggest group.

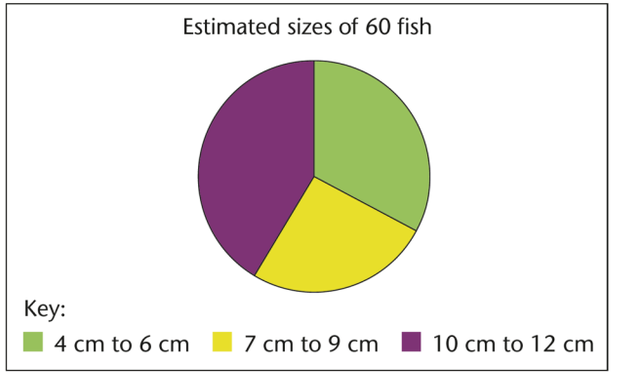
4. (a) Disagree, because the yellow sector is less than half of the circle.
   (b) Agree, the green sector is a little less than one third of the circle.
   (c) Agree, the purple sector is a little more than one third of the circle.
   (d) Disagree, because the yellow sector cannot fit into the purple sector twice.

   As mentioned under "Notes on questions", whether learners agree or disagree depends on how much difference they are willing to see as "almost the same".

Jabu says: "A fish tank costs about R240. If the fish tank is part of the sale, then the pet shop owner asks about R4 per fish. That's a bargain!"

2. Check if Jabu is correct about the price.

Jabu asked the pet shop owner to let him know what size the fish are. She sent this pie chart:



3. What story does the pie chart tell about the sizes of the 60 goldfish? Write the information in your book.

4. Read each statement and say whether you agree or disagree. Explain why.

   (a) More than half of the fish are between 7 cm and 9 cm long.

   (b) About a third of the fish are between 4 cm and 6 cm long.

   (c) About a third of the fish are between 10 cm and 12 cm long.

   (d) The big fish (10 cm to 12 cm) are almost twice as many as the medium fish (7 cm to 9 cm).

Notes on questions

Question 7 is a challenge and an extension question.

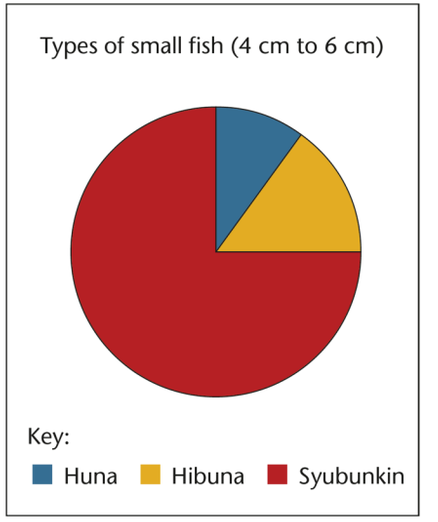
Answers

5. Learners' answers may differ. A sample answer is provided below.
   Most of the small fish, about $\frac{3}{4}$, are Syubunkins. Less than $\frac{1}{8}$ are Hunas, and slightly more than $\frac{1}{8}$ are Hibunas.

6. (a) Agree, draw lines or fold another circle to demonstrate.
   (b) Disagree, because the blue and yellow sectors, which show fish other than Syubunkins, make up about one quarter of the circle.
   (c) Agree, draw lines or fold another circle to demonstrate.
   (d) Disagree, the blue sector is smaller than the yellow sector.

7. 15 of the fish are small Syubunkins between 4 cm and 6 cm.
   Learners might use different ways to find the answer. Here is one way:
   From question 4 (see 4(c)), learners know that about 1 third of all the fish are small. One third of 60 is 20. So 20 of all the fish are small.
   Three quarters of the small fish are Syubunkins.
   One quarter of 20 is 5, so 3 quarters of 20 is 15.

Jabu does not want to buy big fish. Big fish need more food and that is expensive. He asked the pet shop owner which fish are between 4 cm and 6 cm long. She sent this pie chart.



5. What story does the pie chart tell about the kinds of small fish? Write the information in your book.
6. Read each statement and say whether you agree or disagree. Explain why.
   (a) About three quarters of the small fish are Syubunkins.
   (b) About one third of the small fish are not Syubunkins.
   (c) About one eighth of the small fish are Hunas.
   (d) The number of small Hunas is the same as the number of small Hibunas.
7. Use the information in all the previous pie charts to work out how many of the fish are small Syubunkins.

7.5 Information in bar graphs

Mathematical notes

The scales on the frequency axes differ between the two bar graphs.

Teaching guidelines

Aim to spend about 2 hours on this section.

Prepare the bar graphs on the board or on posters for use during class discussions.

Possible misconceptions

Learners may simply look at the visual heights of the bars and ignore the scale. Learners may also struggle to understand that while the scale for the graph of the large fish is counted in twos, they have to infer that the numbers in between can be used to give frequencies. They may also say there are six and a half large Syubunkins, instead of seven.

Critical knowledge

Bar graphs also help us to make comparisons about categories of data. However, you cannot only look at the relative height of the bars, you also need to consider the scale of the graph.

The heading of the graph, the labels of the axes and the names of the categories are crucial parts of the graph.

Notes on questions

Question 5 is a challenge and an extension question.

Answers

1. 6
2. 12
3. There are the same number of medium and large Hibunas: 6 of each.
4. There are 8 medium Syubunkins and 7 large Syubunkins.
5. There are 6 medium and 6 large Hibunas. So there are 12 Hibunas that are medium or large. There are 30 Hibunas altogether. This means that the number of small Hibunas is 30 – 12 = 18. There are 18 small Hibunas.

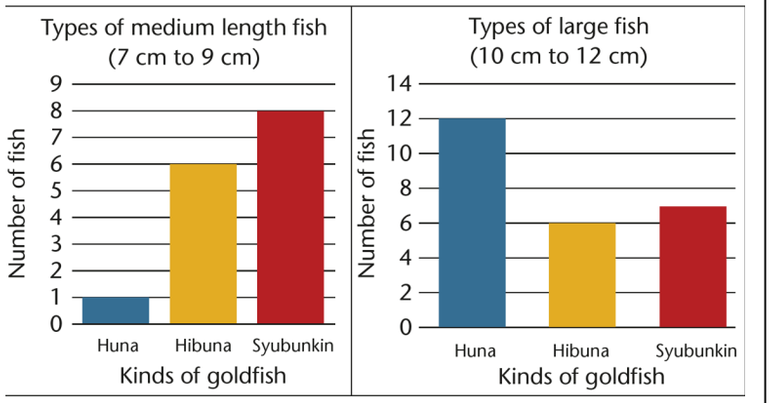
   There are 1 medium Huna and 12 large Hunas. So there are 13 Hunas that are medium or large. There are 15 Hunas altogether. This means that the number of small Hunas is 15 – 13 = 2. There are 2 small Hunas.

7.5 Information in bar graphs

Jabu wants even more information.

He says: "I know now that three quarters of the small fish she wants to sell are Syubunkins. And I know the Syubunkins are about one quarter of all the fish she wants to sell. I also want to know how many of the Hunas and Hibunas are small."

Jabu asked the seller for bar graphs. This is what she sent:



Read the graphs to answer the questions. Look carefully at the number lines.

1. How many Hibunas are medium length fish?
2. How many Hunas are large fish?
3. Compare the number of Hibunas that are medium and large.
4. Compare the number of Syubunkins that are medium and large.
5. Work out how many Hunas and Hibunas are small. Use the information in all the previous bar graphs and pie charts.

Grade 4 Term 1 Unit 8 Properties of two-dimensional shapes

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
8.1 Surfaces with different shapes	Rectangular shapes, including square shapes	96 to 98
8.2 Surfaces with other shapes	Other shapes that are polygons, and shapes that are not polygons	98 to 101
8.3 Make drawings on grid paper	Introducing a useful tool for drawing neat, accurate shapes	102 to 103

CAPS time allocation	5 hours
CAPS page references	21 to 22 and 59 to 61

Mathematical background

Two-dimensional shapes do not exist on their own in the real world. They occur as faces of many three-dimensional objects. They are also found by looking carefully at objects around us, for example the silhouette of a mountain may be triangular; three or more stars in the sky may form the corners of triangles, quadrilaterals, pentagons, etc.; the shadow of a football may be circular or oval. We can draw (i.e. represent) two-dimensional shapes on paper and other flat surfaces. This allows us to focus our mathematical attention on them more easily.

Two-dimensional shapes have properties. Different kinds of shapes may share some properties. For example, squares and rectangles share the property "all four corners look the same". Different kinds of shapes will also have properties that differ. For example, squares have the property "all four sides look the same, i.e. they are the same length," but rectangles have the property "opposite sides are the same length". Identifying properties allows us to relate different shapes to each other. For example, a square is a special kind of rectangle. All the properties of rectangles also apply to squares, but squares have some extra properties. Being able to identify these properties also improves spatial sense.

In the Intermediate Phase learners begin to focus on the properties of shapes and how different shapes are related to each other. Drawing shapes and talking about them is a very important first step. Begin by letting learners draw shapes freehand (using only a pen or pencil). Ask them how they could improve on their freehand drawings. For polygon shapes an extra tool, the straight edge, is very useful. It allows one to make sure that the sides of a drawn polygon are straight, which is a great improvement over freehand drawing.

Ask learners how they could further improve on their drawings. How can one make sure the straight sides of one's drawings are properly placed? Grid paper is an excellent tool for this. Grid paper is also a very helpful tool for freehand drawing of curved shapes such as circles and ovals ("egg shapes").

Note: The underlying idea in this unit is the properties of two-dimensional shapes. The straight edge and grid paper are *tools* to help (scaffold) learners to begin identifying and reasoning about these properties. Do not force them into this, but allow them the space to begin asking and answering questions such as: "*What makes this figure a rectangle?*", "*Why is that a better drawing of a square than this drawing?*", "*Have I drawn a circle or is it an egg shape?*", etc. Each learner should begin to develop a few criteria about how well a drawing represents a certain shape. This is the beginning of geometric reasoning.

Resources

Scrap paper; boxes with rectangular faces; boxes with square faces; square grid paper; pencils; rulers or straight edges.

8.1 Surfaces with different shapes

Mathematical notes

The focus in this section is on squares and other rectangles.

A square is a special kind of rectangle. Squares have all the properties of rectangles:

- They are closed shapes.
- All four sides are straight.
- All angles are the same size so they look the same.
- Opposite sides are the same length.

Squares have one additional property:

- All sides are the same length.

Teaching guidelines

Aim to cover this section in about 1 to $1\frac{1}{2}$ hours.

In Grade 3 learners should have learnt to recognise triangles, squares, rectangles and circles. However, learners were not expected to group figures according to the number or length of their sides, or to think about the corners (angles) of the figures.

You can introduce the lesson by drawing around the rectangular faces of a box. It is best if you show examples of differently shaped rectangles. If possible, find a cube and draw around the faces of the cube to show squares.

Allow learners to draw freehand squares and rectangles. Let them compare and talk about their drawings in suitable groups. Move around and engage each group in conversation to guide the talking about the properties of the shapes they draw, for example:

- How straight are the sides?
- Are the four sides the same length or different lengths?
- Are the four corners the same size or different sizes? Do they all look the same?

Resources

A piece of paper for each learner: it can be previously used paper.

A box in which all the faces are rectangles (like a cereal box or tea box).

A box in which all the faces are squares (like a hand or facial cream box or a square tissue box).

Answers

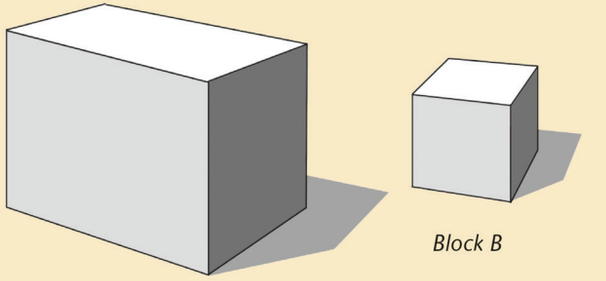
1. Learners' own freehand drawings of squares

UNIT 8 PROPERTIES OF TWO-DIMENSIONAL SHAPES

8.1 Surfaces with different shapes

These boxes or blocks have flat surfaces.
The surfaces are called **faces**.

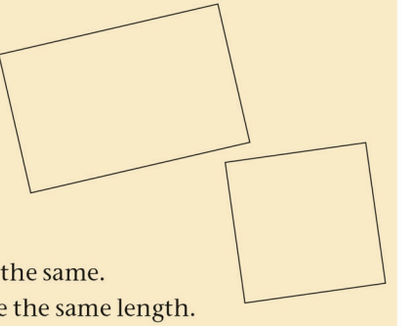




Block B

Block A

Each face of Block A is a **rectangle**.
The four corners look exactly the same.



Each face of Block B is a **square**.
The four corners look exactly the same.
The four sides of a square have the same length.

1. Use your pencil only and try to draw a square. Do not use a ruler or some other straight object.

   Your drawing must be bigger than the square shown here.

   Use about half of a page.

Notes on questions

The aim of questions 2 and 3 is to help learners to focus on the features or properties of squares.

In question 4, when the straight edge is introduced, explain that its purpose is to allow us to draw the sides of our shapes as straight as possible. Ask learners what else we could use as a straight edge.

The folded paper straight edge has additional teaching value. If it is folded carefully it forms a rectangle. You may open a rewarding class discussion about this. Why is it a rectangle? How can we fold a square? How can we fold it so it is not a rectangle or a square?

Teaching guidelines

Once learners have completed question 3 you can write a list of the properties of a square on the board.

You can demonstrate to learners how to fold a straight edge from a piece of paper. It is important that you make a sharp crease each time. You can show learners how to rub their fingernail or the side of a pencil over the fold to make a sharp crease.

Answers

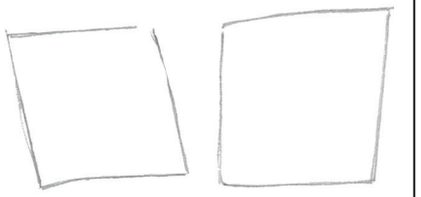
2. Learners' choices of the best square may differ.

3. Any of the following statements are valid. Learners are not expected to provide all of these answers.
   (a) The corners are not the same size and/or the bottom line is not straight and/or it is not closed.
   (b) All sides are not equal in length and/or the corners are not the same size.
   (c) The sides are not straight.
   (d) The sides are not straight and/or not equal in length.

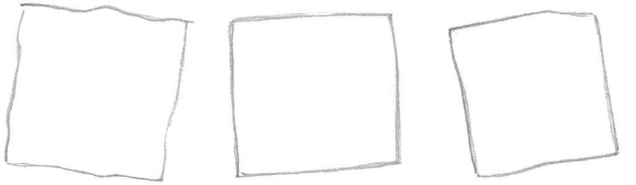
4. Learners fold their own straight edge.

2. Other learners also tried to draw a square, without using a ruler or other straight object.

   Which of these drawings is the best attempt to draw a square?



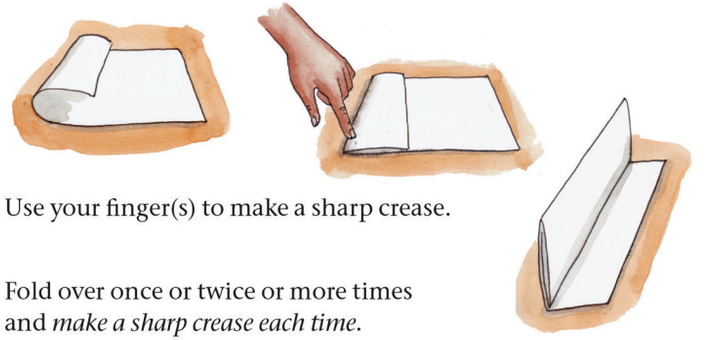
Drawing A *Drawing B*



Drawing C *Drawing D* *Drawing E*

3. In each case, state why it is not a good drawing of a square. Describe what is wrong.
   (a) Drawing A (b) Drawing B
   (c) Drawing C (d) Drawing E

4. To get a straight line, like the side of a rectangle or a square, you can fold a sheet of paper as shown below.



Use your finger(s) to make a sharp crease.

Fold over once or twice or more times and *make a sharp crease each time.*

GRADE 4: MATHEMATICS [TERM 1] **97**

Teaching guidelines

In the Foundation Phase, two-dimensional shapes are treated as completely distinct from each other. Some learners will find it strange to think of a square as a special type of rectangle. Using non-mathematical examples can help learners to think about groupings as sub-sets. For example: All boys are people and all girls are people; boys and girls are special kinds of people. But not all people are boys, nor are all people girls. Or: Cattle, sheep, goats and dogs are all animals, but not all animals are cattle or sheep or goats or dogs, etc.

Once learners have completed question 6, talk to them about the difference between squares and other rectangles: When is a rectangle a square? When is a rectangle not a square?

Possible misconceptions

Some learners may struggle to distinguish squares and rectangles clearly from more general types of quadrilaterals, or to distinguish squares from rectangles. Such learners will have to be supported individually. We suggest letting them draw freehand shapes under your personal guidance to achieve this.

Answers

5. Learners' own drawings of squares
6. Learners' own drawings of rectangles

8.2 Surfaces with other shapes

Mathematical notes

When we group figures according to their properties/features, we call this classifying. The focus in this section is on classifying more figures with straight sides according to their shapes.

Polygons are closed figures with straight sides only. Non-polygons are figures with one or more curved side. Learners must know the shapes and names of polygons up to and including hexagons: see the text at the summary bar on page 100. Learners identify, name and describe polygons by counting and referring to their number of sides.

This section also helps learners to distinguish between shape and size – when do two figures (e.g. two triangles) have the same shape and size, when do they have the same shape but different sizes, and when do they have different shapes and sizes.

You have made yourself a tool that you can use to draw straight lines. A tool like this is called a **straight edge**.

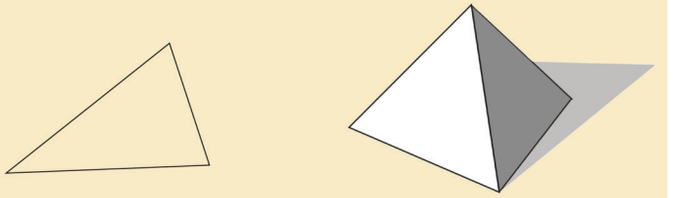
Keep your straight edge! You will use it again later!



5. Use your straight edge to draw a square as well as you can.
6. Also draw a rectangle that is not a square, as well as you can.

8.2 Surfaces with other shapes

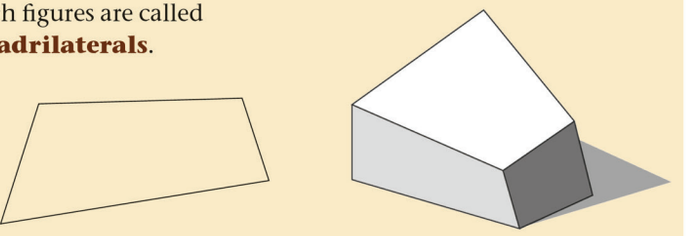
Some of the faces of Block C are **triangles**.



Block C

The faces of Block D have four sides each.

Such figures are called **quadrilaterals**.



Block D

A quadrilateral has four straight sides.

Teaching guidelines

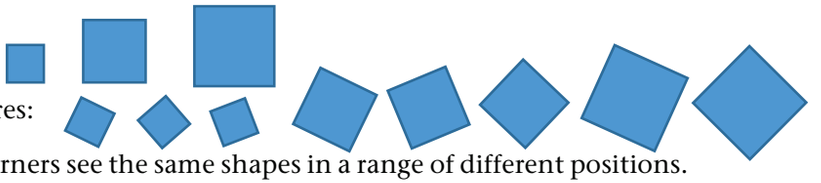
This is a long section: aim to complete it in about 2 hours.

You can start by drawing a range of quadrilaterals of different shapes and sizes on the board. Explain to learners that a **quadrilateral** is a closed figure with four straight sides. This term will be new to them. Ask them if they can think of any figures that they know that are quadrilaterals. Draw examples of figures with four straight sides that are not closed. Explain that because not all the sides meet, we say that the figure is open. Such figures are not quadrilaterals.

Possible misconceptions

Some letters of the alphabet have the same shape, but are different because they face different directions, for example p, q, d and b. Sometimes learners think that if figures face a different direction they are different shapes. This misconception often occurs with squares:

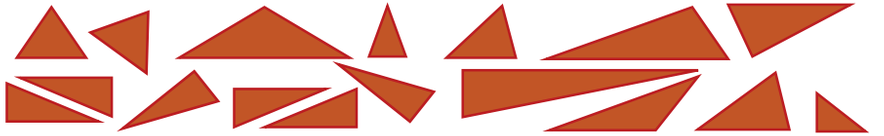
These are all squares:



These are also all squares:

It is important that learners see the same shapes in a range of different positions.

It is also important that learners see a range of shapes that look different. For example, these are all triangles:



Page 60 of the CAPS shows some different examples of quadrilaterals, pentagons and hexagons.

Notes on questions

Questions 1, 2, 3, 11 and 12 are about quadrilaterals, including squares and other rectangles.

Answers

1. Figures that are not quadrilaterals: A, C, E, F, H, I, J, K, L
2. Figures with four corners looking the same: B, N, O
3. (a) Rectangles: B, N, O
   (b) Rectangles but not squares: O (B and N are squares)

Rectangles and squares are also quadrilaterals. The four corners of a rectangle or a square look the same.

1. Which of these figures are *not* quadrilaterals?

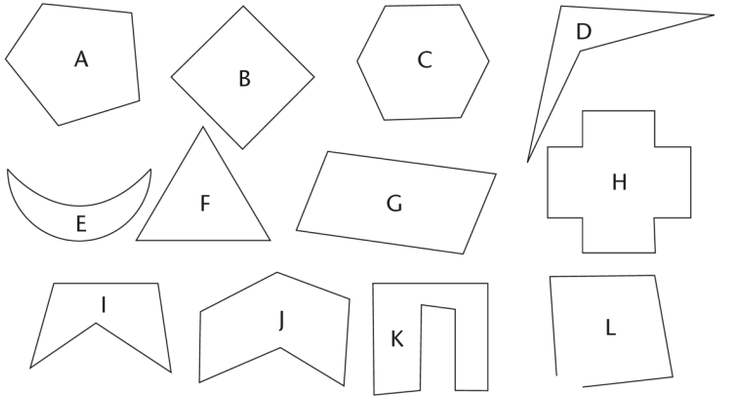
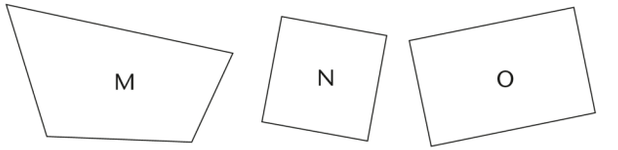


Figure L is not called a quadrilateral because it is **open** at one place. Only **closed** figures, such as those below, are called quadrilaterals.

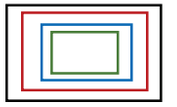


2. The corners of Figure M do not all look the same.
   In which of all the figures on this page do the four corners look the same?
3. (a) Which of the figures on this page are rectangles?
   (b) Which figures on this page are rectangles but not squares?

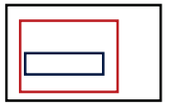
Teaching guidelines

Pentagons and hexagons are new in Grade 4. You can draw a variety of pentagons on the board (page 60 of the CAPS shows some examples). Ask learners what is the same about each of the figures. Use the summary bar to explain what a pentagon is. You can repeat this with hexagons.

Once learners have completed question 4, you can explain, with illustrations, how figures with the same name can have the same shape but different *sizes*. You can use the tinted passage on page 100 to guide you. Consider illustrating this with rectangles too, as shown here, so that learners see two different examples.



Then continue to show how figures with the same name, for example rectangles, can have different *shapes*, as shown alongside.



Notes on questions

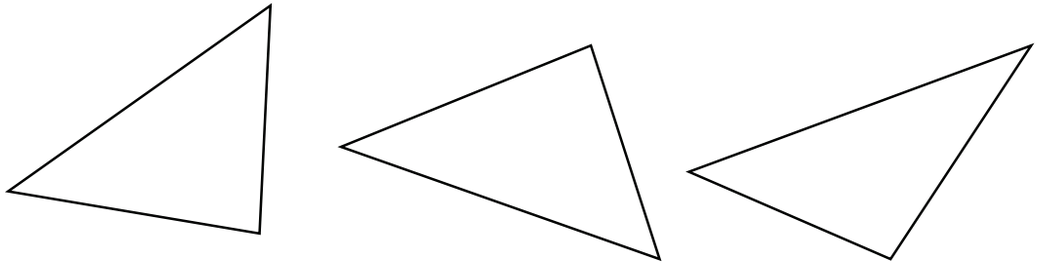
Questions 5 to 12 are about identifying sameness and difference for a range of figures. Learners will need to identify what is the same and what is different using shape and size. This is an important aspect of developing reasoning in geometry.

If time permits, encourage discussion around these issues. Encourage learners to use drawings during these discussions to resolve differences of opinion and to make themselves better understood. Let learners make freehand drawings as well as drawings using a straight edge. Move around the classroom to check that each group is on track and focused on the important issues.

Answers

4. (a) Not a polygon: E, L (E does not have straight sides, L is not closed)
   (b) Pentagons: A, I
   (c) Hexagons: C, J
   (d) Not triangles, quadrilaterals, pentagons or hexagons: H, K, E, L

5. Learners' own drawings of triangles, for example:



A closed figure with *four straight sides* is called a **quadrilateral**.

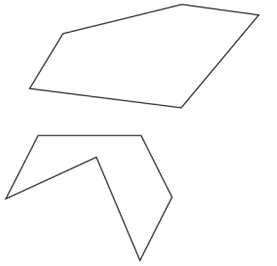
A quadrilateral of which all four corners look the same is called a **rectangle**.

A closed figure with *three straight sides* is called a **triangle**.

A closed figure with *five straight sides* is called a **pentagon**.

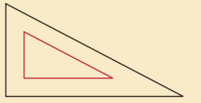
A closed figure with *six straight sides* is called a **hexagon**.

Any closed figure with *straight sides* is called a **polygon**.

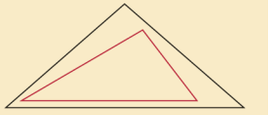


4. (a) Which figures in question 1 are not polygons?
   (b) Name all the pentagons in question 1.
   (c) Name all the hexagons in question 1.
   (d) Name the polygons in question 1 that are not triangles, quadrilaterals, pentagons or hexagons.

The red and black triangles on the right have the same shape. The one is just bigger than the other.

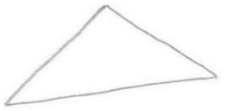


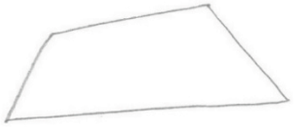
These two triangles do not have the same shape.

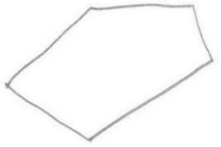


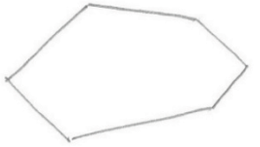
5. Use your straight edge and a sharp pencil to draw three triangles of about the same size, but with different shapes.

Answers

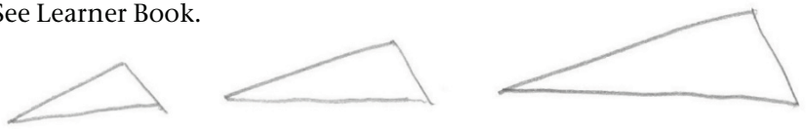
6. (a) 

(b) 

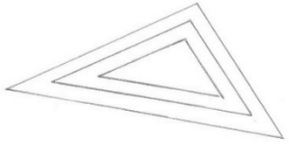
(c) 

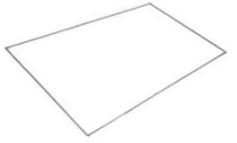
(d) 

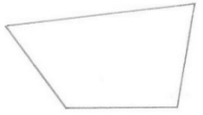
7. See Learner Book.

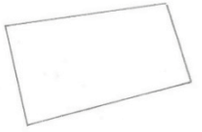
8. 

9. The purpose of the question is to make learners think about what they did, not to get a neat, "correct" description, which they are not capable of at this time. Learners may mention that they try to make the angles or corners "the same" or "equal".

10. 

11. 

12. 



13. 

When you make a drawing without using a straight edge or other tool, it is called a **freehand drawing**.

Artists mainly make freehand drawings.

6. Make freehand drawings of the following:
   (a) a triangle (b) a quadrilateral that is not a rectangle
   (c) a pentagon (d) a hexagon

7. Make a freehand drawing of three triangles with the same shape, one inside the other.

   Make your drawings large so that they fill a whole page.



8. Draw three triangles, not inside each other, with the same shape but different sizes. Do not use a straight edge.

9. How did you try to make sure that your three triangles have the same shape? Write a short paragraph about it. You may make drawings to help you explain.

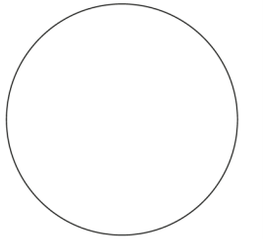
10. Now use your straight edge to again draw three triangles with the same shape. Draw your triangles inside each other.

11. Use your straight edge to draw a quadrilateral that is not a square or a rectangle.

12. Draw two more quadrilaterals that are different from each other, and are not squares or rectangles.

13. A figure like this is called a **circle**.

    Make a good freehand drawing of a circle. You can make a nice circle by going round and round and making it better all the time.



GRADE 4: MATHEMATICS [TERM 1] **101**

8.3 Make drawings on grid paper

Mathematical notes

This section introduces another important and useful tool, namely grid paper. Carefully drawn grid paper is regular, i.e. it is made up of identical squares. This allows one person (or two or more people) to draw exactly the same shape, exactly the same size, as many times as they wish.

Teaching guidelines

Aim to spend about 1 hour to $1\frac{1}{2}$ hours on this section.

Be explicit about the role of grid paper as a **tool**, and not as a separate, new topic in this unit. Allow learners to draw their own grid paper while you draw a larger version on the board. They will watch you and will hopefully try to draw as well as you do. Also make sure that each learner is drawing a regular grid. If the lines are unevenly spaced it will not be a very useful tool. For the best results the grid must have squares.

Try to make time for a discussion around how the grid makes it easier to draw shapes correctly. Ask questions that aim to justify opinions, for example: "*Why do we get a rectangle when we join these four points?*" or "*Why must the circle pass through those points?*" Encourage learners to do extra, self-directed exploration of the grid with some feedback, time permitting. If at all possible, provide learners with copies of a regular grid. You can copy square grid paper from the Addendum (page 433), or ask learners to prepare three grids at home, following the method shown in question 1.

Possible misconceptions

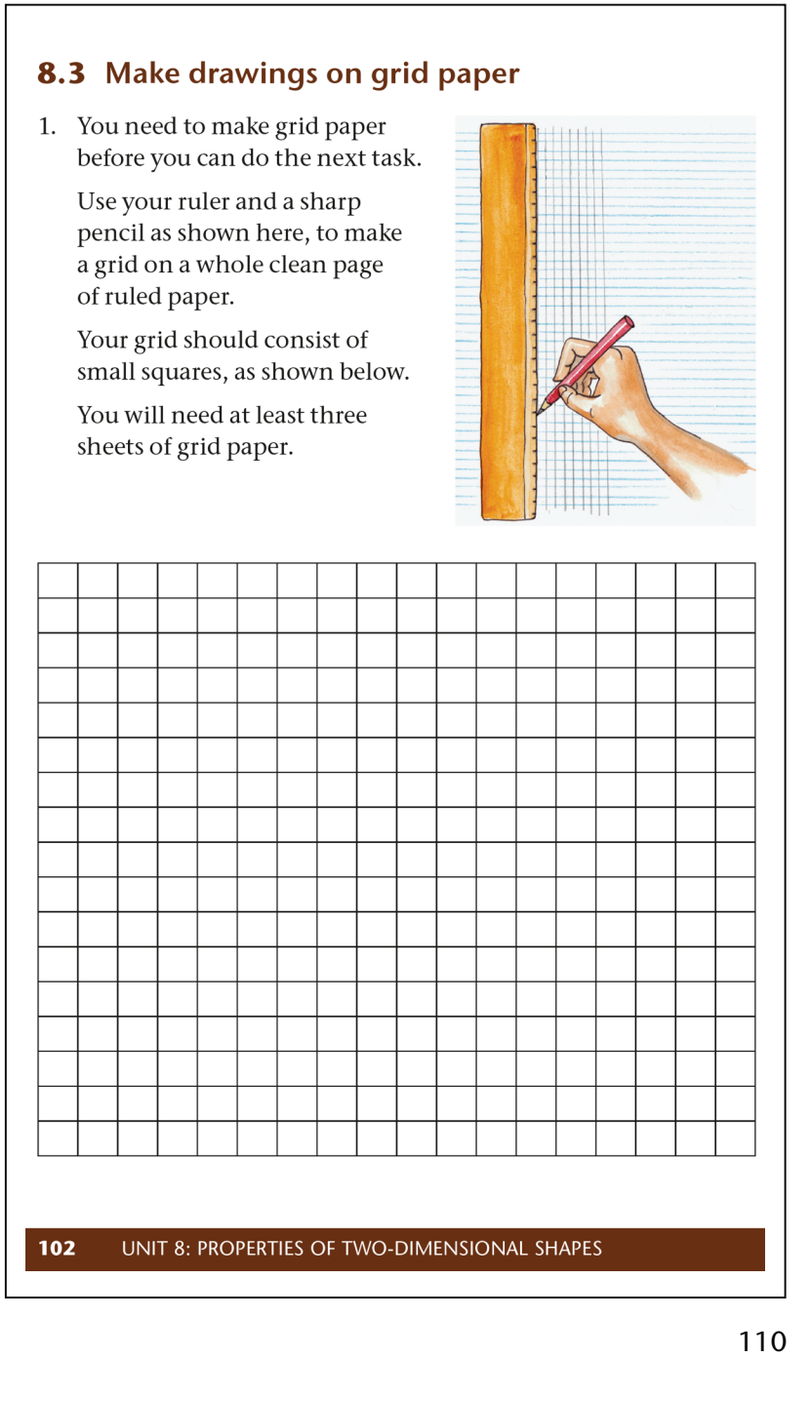
If a grid is not regular or at least pretty close to regular, its value as a tool for drawing diminishes. It will be difficult to see the properties of the different shapes on such a grid. Learners who really struggle to draw a regular grid may be provided with a partially drawn grid which they must complete.

Resources

Three sheets of square grid paper: one sheet for question 2; one sheet for question 3; one sheet for questions 4, 5 and 6.

Answers

1. Learners draw their own grid lines: check as they work that they are forming squares.



8.3 Make drawings on grid paper

1. You need to make grid paper before you can do the next task.

   Use your ruler and a sharp pencil as shown here, to make a grid on a whole clean page of ruled paper.

   Your grid should consist of small squares, as shown below.

   You will need at least three sheets of grid paper.

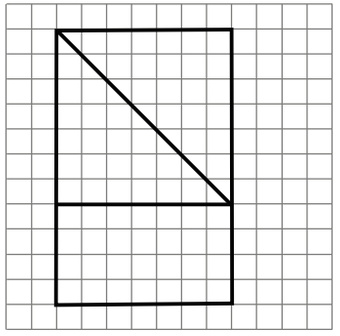
Notes on questions

In questions 2(a) and (b) learners will draw the figures by drawing along the grid lines. In question 2(c) one line will not be drawn along the grid lines. When learners draw the seven-sided polygon and the triangles in question 4, some of the sides of the figure will not coincide with grid lines. In questions 5 and 6 the circle and oval will touch the grid lines but not be drawn along them. Check that learners understand that drawing figures on grid paper does not mean that all the sides need to be on grid lines.

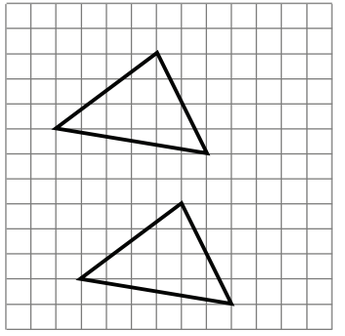
Questions 5 and 6 are challenging because learners have to draw freehand. Their circles and ovals (egg shapes) may not be very neat. Engage them in a discussion about how they tried to draw the shapes, even though their drawing skills may not be up to executing their ideas. Ideas in mathematics are often clearer than our representations of them.

Answers

2. (a), (b), (c) Example below.



3. (a), (b) Example below.

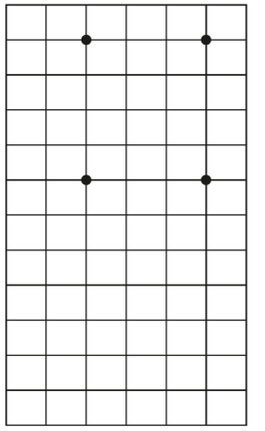


When learners have completed question 3(b), you may ask them to individually describe, in writing and with sketches if they prefer, what they did to ensure that the second triangle is the same shape and size as the first one. It would be best if they do this as homework. The purpose of the question is not to obtain any "correct" answer, but to provide each learner with a valuable opportunity to form mathematical ideas based on his/her own actions.

Some learners may explain their work in terms of the relative positions of the vertices of the triangle, for example: "The left corner is four squares left of the top corner, and three squares down."

Answers for questions 4 to 6 are given on the next page.

2. The places where the grid lines meet are called **grid points**.



   (a) Draw a large rectangle on a grid by joining four grid points. Use your straight edge. Your rectangle must not be a square.

   (b) Draw a square as big as possible inside the rectangle.

   (c) Draw a line inside your square so that it forms two triangles with the same shape.

3. (a) Draw a large triangle by joining three points on one half of a new grid sheet.

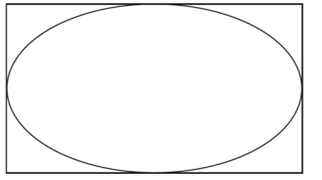
   (b) Draw another triangle, of exactly the same shape and size, on the other half of the grid sheet.

4. (a) Draw a polygon with seven sides on grid paper.

   (b) Divide your polygon into triangles.

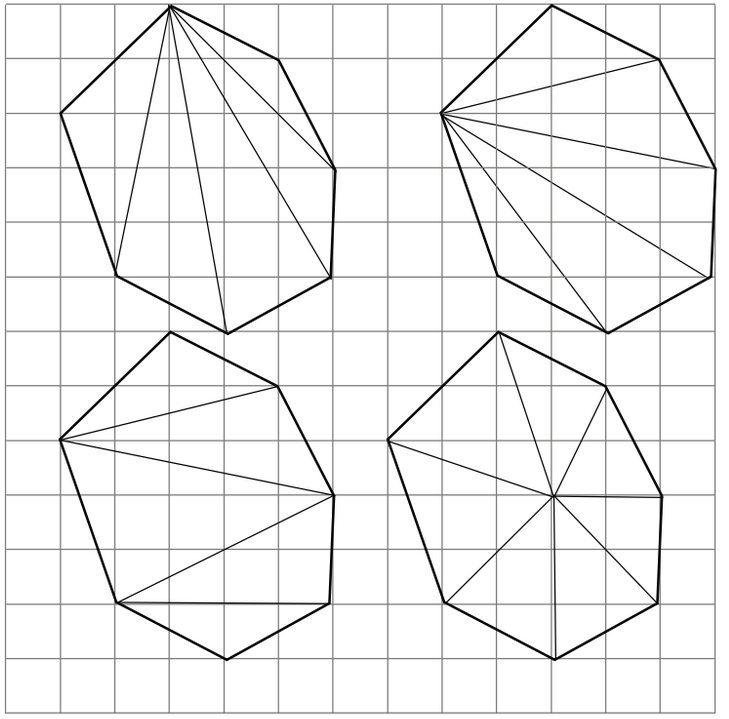
5. Draw a square on less than half of a grid sheet.
   Then draw a circle as neatly as you can tightly inside the square.

6. Draw a rectangle that is not a square on less than half of a grid sheet.
   Then draw a figure that looks like an egg tightly inside the rectangle.



Answers (*continued*)

4. (a) and (b). Some of the many ways in which a polygon with 7 sides can be divided into triangles are shown below.



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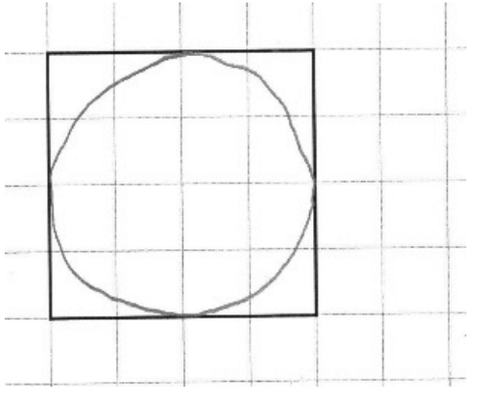
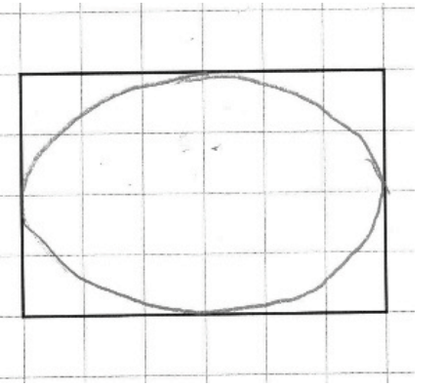


6.



Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
9.1 Learn to multiply with bigger numbers	Breaking numbers down into place value parts and multiplying each part	104 to 106
9.2 Build knowledge for multiplication	Multiplying by units and by 20, 30, 40 and 50	106 to 107
9.3 Use your knowledge of multiplication facts	Multiplying by multiples of ten and units	107 to 108
9.4 Build more knowledge of multiplication facts	Multiplying by units and by 60, 70, 80 and 90	108 to 110
9.5 Multiplying three or more numbers	When multiplying two or more numbers, the answer remains the same whether or not you change the order in which you multiply	110 to 111
9.6 Use your knowledge in a different way	Using multiplication to divide	111 to 113
9.7 Practise and use your skills	Solving problems with multiplication and division	113 to 114
9.8 Dividing into equal parts	Solving more problems with multiplication and division Introducing the concept of remainders	115 to 116

CAPS time allocation	5 hours
CAPS page references	13 to 15 and 62 to 65

The first unit of multiplication and division in Term 1 focused on multiplying 1-digit numbers and dividing 1-digit numbers into 2-digit numbers. It built learners' awareness of multiplication facts such as the multiplication tables. The main focus of this unit is on multiplying by 2-digit numbers (by breaking the numbers down into place value parts) and dividing 2-digit and 3-digit numbers by 2-digit and 1-digit numbers. Learners will also be reminded that the order in which we multiply numbers does not impact on the answer we get (the commutative property of multiplication). They will learn that changing the order of the numbers that we multiply can make the multiplication task easier, for example $25 \times 7 \times 4 = 25 \times 4 \times 7 = 100 \times 7 = 700$. This is an easier calculation than $25 \times 7 \times 4 = 175 \times 4$.

Mathematical background

When we multiply multi-digit numbers, we break down the numbers. Usually we break them down into place value parts and multiply each of the parts separately, for example $476 \times 7 = 400 \times 7 + 70 \times 7 + 6 \times 7 = 2\,800 + 490 + 42 = 3\,332$.

We can also break numbers up into factors, as shown in Unit 5 Section 5.6 (pages 71 and 72). For example: $13 \times 8 = 13 \times 2 \times 2 \times 2 = 26 \times 2 \times 2 = 52 \times 2 = 104$. Learners can use this to make calculations easier. It is important for them to know that if they are multiplying two or more numbers, they can change the order of the numbers multiplied without changing the answer. The reason for doing this is to make calculations simpler.

In this unit the main focus is on multiplying 2-digit numbers by breaking them down into place value parts, for example $6 \times 54 = 6 \times 50 + 6 \times 4$. In order to do this successfully, learners need to be able to multiply quickly by multiples of 10, for example by 50. This skill is developed and practised in this unit.

9.1 Learn to multiply with bigger numbers

Mathematical notes

This section uses arrays to show that you can break numbers down into place value parts to multiply them. Two-digit numbers are multiplied after they have been broken down into a tens part and a units part. In question 1, for example, 7 × 46 is broken down into 7 × 40 + 7 × 6.

Critical knowledge



An array can represent multiplication, for example the array alongside (from question 1) represents 4 × 10. Learners should not count each ring individually, but rather count 4 rings in a row and 10 rings in a column and know that this means "4 times a group of 10". They could also skip count to get 10 times a row of 4: 10 × 4.

You can break down a number into place value parts, multiply each part separately and add the products.

You can split numbers into multiples to make multiplying easier, for example: 60 × 4 = 10 × 6 × 4 = 10 × 24 = 240.

The order in which you multiply numbers does not impact on the answer, for example: 80 × 7 = 8 × 10 × 7 = 8 × 7 × 10 = 56 × 10 = 560.

Teaching guidelines

Try to move quickly through Section 9.1: aim to cover this section in 45 minutes. One possibility is to use

- questions 1 and 2, and the tinted passage on page 104 for concept development,
- questions 3, 4, 5 and 6 for classwork, and
- questions 7, 8 and 9 for additional practice.

You can start the lesson by focusing on multiplying by 10. You can ask learners to page back to the multiplication grid on page 69. Ask them to focus on either the ×10 row or the ×10 column. Ask them: "*What is the same about all the numbers in that row or column?*", "*What happens to a number when you multiply it by ten?*" They can then practise this with a range of 2-digit numbers, for example 13 × 10; 27 × 10; 39 × 10, etc.

When you multiply by 10, the number becomes 10 times bigger, for example 10 × 4 = 40; 40 is 10 times bigger than 4. Teachers sometimes tell learners that when you multiply by 10 you add a zero onto the number. This is not true, for example 4 × 10 ≠ 4 + 0.

Answers

1. Yes

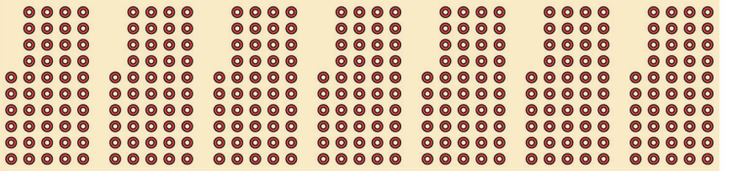
UNIT 9

WHOLE NUMBERS: MULTIPLICATION AND DIVISION

9.1 Learn to multiply with bigger numbers

In this unit you will learn to multiply with bigger numbers, for example how to calculate 6 × 54 or 73 × 5.

If you can do such calculations, you can easily and quickly find out how many rings there are in the diagram below, without counting all the rings.

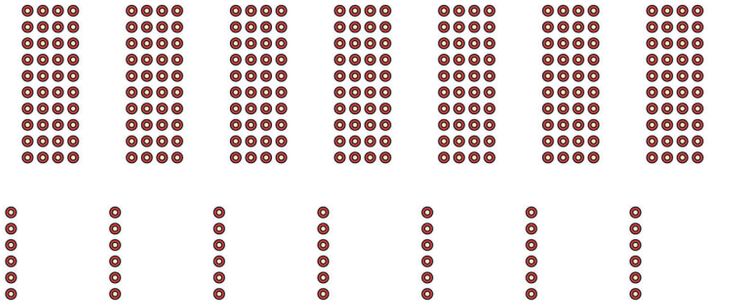


Zweli knows that 7 × 40 = 280.

He also knows that 7 × 6 = 42.

Zweli thinks he can use this knowledge to quickly find out how many rings there are in the above diagram.

1. Do you think the diagram below has the same number of rings as the diagram above?



Teaching guidelines

You can help learners to generalise about multiplying by multiples of 10, for example multiplying by 30. Let learners work through questions 4(a), 5(a), 6, 8 and 9 on page 163. They must understand that multiplying by a multiple of 10 can be done by breaking the number up into factors, for example $11 \times 80 = 11 \times 8 \times 10 = 88 \times 10 = 880$.



Ask learners how many rings there are in each of the repeated larger arrays at the bottom of page 104. Learners can skip count to get to 4 times a column of 10 rings (4×10) or $10 \times$ a row of 4 rings (10×4). Then ask learners how many of these arrays there are. Ask them to write a number sentence that represents this row of arrays. Repeat these questions but refer to the single columns of rings at the bottom of page 104. Learners can then work though question 2 on page 105.

Then refer learners back to the tinted passage on page 104. Ask them what is the same about the arrays in the tinted passage and those in question 1. Use this to motivate that when we multiply by a 2-digit number we can break the number down into place value parts (tens and units), multiply each part separately and then add the answers. For example, $46 \times 7 = 40 \times 7 + 6 \times 7 = 280 + 42 = 322$. Remind learners that when they see multiplication, addition and/or subtraction in the same expression they should multiply first. Learners can use a similar approach for questions 4, 5 and 6. A template of the table in question 7 is provided in the Addendum (page 443). You can photocopy it to save learners time.

Answers

2. (a) Yes
   (b) $280 + 42 = 322$

3. 30 60 90 120 150 180 210 240 270 300

4. $6 \times 30 + 6 \times 7 = 180 + 42 = 222$

5. 222, because it is also 6×37

6. 402. Learners' methods may differ. One option modelled on questions 1 and 2 is: $6 \times 60 + 6 \times 7 = 360 + 42 = 402$

7.

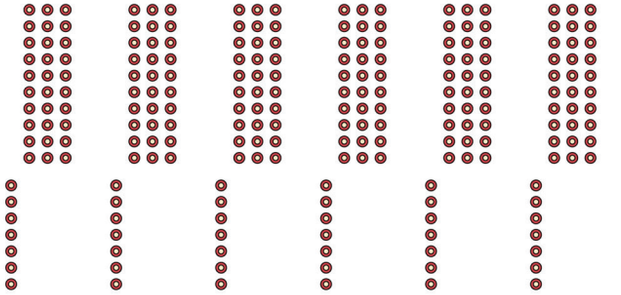
×	2	3	4	5	6	7	8	9	10
30	60	90	120	150	180	210	240	270	300
50	100	150	200	250	300	350	400	450	500

2. (a) Do you agree that in the diagram in question 1, there are 7×40 rings and another 7×6 rings?
   (b) If $7 \times 40 = 280$ and $7 \times 6 = 42$, how many rings are there altogether?

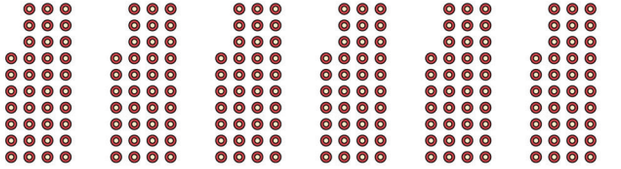
3. Copy the four numbers below. Then add 30 each time to write the next six numbers in the pattern.

   30 60 90 120 . . .

4. Work out how many rings there are in this diagram.



5. How many rings do you think there are in this diagram?



6. What is 6×67? How did you work that out?

7. Complete the table.

×	2	3	4	5	6	7	8	9	10
30	60	90	120						
50				250					

Answers

8. $60 \times 5 + 4 \times 5 = 300 + 20 = 320$
9. $54 \times 6 = 50 \times 6 + 4 \times 6 = 300 + 24 = 324$

9.2 Build knowledge for multiplication

Critical knowledge

Multiplying by multiples of ten, namely 20, 30, 40, 50.

Teaching guidelines

Try to move quickly through Section 9.2: aim to cover this section in 30 minutes. One possibility is to use

- questions 1(c), (k) and (l) for concept development,
- questions 1(a), (b) and (d) to (j), 2, 3 and 4 for classwork, and
- questions 5, 6 and 7 for additional practice.

Learners can begin by skip counting in 20s and completing the 20 times row in question 2. (In order to save time, you can photocopy the template of the table in question 2 – see the Addendum on page 443.) You can work though questions 1(c), (k) and (l) with the whole class. Remind learners that when they see multiplication, addition and/or subtraction in the same expression they should multiply first, for example $4 \times 20 + 4 \times 6 = 80 + 24 = 104$.

Answers

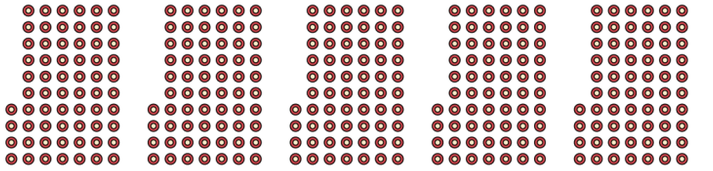
1. (a) 100 (b) 120 (c) 80
   (d) 180 (e) 140 (f) 100
   (g) 60 (h) 160 (i) 200
   (j) 160 (k) 104 (l) 104

2.

×	2	3	4	5	6	7	8	9	10
20	40	60	80	100	120	140	160	180	200
30	60	90	120	150	180	210	240	270	300
40	80	120	160	200	240	280	320	360	400
50	100	150	200	250	300	350	400	450	500

3. (a) 150 (b) 180 (c) 120
   (d) 270 (e) 240 (f) 210
   (g) 90 (h) 190 (i) 190

8. Work out how many rings there are in this diagram.



9. Work out how many apples there are in 6 boxes with 54 apples in each box.

9.2 Build knowledge for multiplication

In this section you will learn more multiplication facts so that you can calculate quickly with larger numbers.

1. How much is each of the following?
   (a) 5×20 (b) 6×20 (c) 4×20
   (d) 20×9 (e) 20×7 (f) 5×20
   (g) 3×20 (h) 20×8 (i) 10×20
   (j) 8×20 (k) $4 \times 20 + 4 \times 6$ (l) 4×26

2. Complete the 20 times row and the 30 times row.

×	2	3	4	5	6	7	8	9	10
20	40	60	80						
30				150					
40									
50									

3. How much is each of the following?
   (a) 5×30 (b) 6×30 (c) 4×30
   (d) 30×9 (e) 30×8 (f) 7×30
   (g) 30×3 (h) $5 \times 30 + 5 \times 8$ (i) 5×38

Answers

4. See the 40 times row in the table in question 2.
5. (a) 200 (b) 240 (c) 160 (d) 360
   (e) 280 (f) 200 (g) 120 (h) 320
   (i) 400 (j) 320 (k) 322 (l) 414
6. See the 50 times row in the table in question 2.
7. (a) 250 (b) 300 (c) 200 (d) 450
   (e) 350 (f) 250 (g) 150 (h) 400
   (i) 500 (j) 400 (k) 392 (l) 504

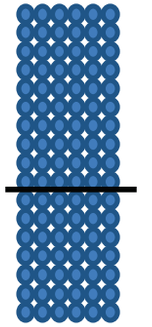
9.3 Use your knowledge of multiplication facts

Teaching guidelines

Try to move quickly through Section 9.3: aim to cover this section in 30 minutes. One possibility is to use

- questions 1(e), (f), (i) and (j), 2(a) and (e), and 3 for concept development,
- questions 1(a) to (d), (g), (h), (k) and (l), and 2(b), (c), (d) and (f) for classwork, and
- question 4 for additional practice.

You can demonstrate to learners how they can use the table from question 2 in Section 9.2 to find the answers for questions in this section.



Questions 2(e) and (f) begin to prepare learners for question 3. You can introduce learners to this by showing them how you can split an array to make it easier to multiply. For example, if you want to multiply 6 × 17 you can split 17 into 10 + 7 (as shown in the array) and multiply each part separately: 6 × 10 + 6 × 7. When learners calculate questions 2(e) and (f), ask them to circle the answers to 10 × 30 and 10 × 4 in their table. They can then circle the answer to 6 × 30 in their table. Ask them what they notice. Learners can check whether this works in the other examples in question 2.

Answers

1. (a) 9 (b) 9 (c) 6 (d) 3 (e) 5 (f) 40
   (g) 12 (h) 10 (i) 20 (j) 7 (k) 9 (l) 7
2. (a) 160 (b) 270 (c) 460 (d) 160 (e) 180 (f) 180

4. Now complete the 40 times row in your table.

5. How much is each of the following?
   (a) 5 × 40 (b) 6 × 40 (c) 4 × 40
   (d) 40 × 9 (e) 40 × 7 (f) 5 × 40
   (g) 3 × 40 (h) 40 × 8 (i) 10 × 40
   (j) 8 × 40 (k) 7 × 46 (l) 46 × 9

6. Complete the 50 times row in your table.

7. How much is each of the following?
   (a) 5 × 50 (b) 6 × 50 (c) 4 × 50
   (d) 50 × 9 (e) 50 × 7 (f) 5 × 50
   (g) 3 × 50 (h) 50 × 8 (i) 10 × 50
   (j) 8 × 50 (k) 7 × 56 (l) 56 × 9

9.3 Use your knowledge of multiplication facts

Use the multiplication facts for 20, 30, 40 and 50 that you have developed in Section 9.2 to do questions 1 and 2.

1. Write the missing number in each number sentence.
   (a) 30 × . . . = 270 (b) 450 = 50 × . . .
   (c) 120 = . . . × 20 (d) . . . × 40 = 120
   (e) 30 × . . . = 5 × 30 (f) 4 × . . . = 8 × 20
   (g) 50 × . . . = 600 (h) 400 = 40 × . . .
   (i) 800 = 40 × . . . (j) 30 × . . . = 210
   (k) 40 × . . . = 360 (l) 280 = 40 × . . .

2. Calculate.
   (a) 3 × 20 + 5 × 20 (b) 5 × 30 + 4 × 30
   (c) 40 × 4 + 50 × 6 (d) 9 × 40 – 5 × 40
   (e) 10 × 30 – 4 × 30 (f) 6 × 30

Answers

3. There is a quicker way. Subtract 5 from 9 and multiply the answer, i.e. 4, by 40:
   $(9 - 5) \times 40 = 4 \times 40 = 160$
4. (a) 120 (b) 250 (c) 138 (d) 138
   (e) 190 (f) 190 (g) 270 (h) 232

9.4 Build more knowledge of multiplication facts

Teaching guidelines

Try to move quickly through Section 9.4: aim to cover this section in 30 minutes. One possibility is to use

- the summary bar on page 110 for concept development,
- questions 1, 2, 5, 6(a), (b), (c), (g) and (h), and 7(a) and (b) for classwork, and
- questions 3, 4, 6(d), (e) and (f), and 7(c) and (d) for additional practice.

There are no new concepts in this section. Learners merely practise multiplying by different multiples of 10, namely 60, 70, 80, 90, and they also practise multiplying by 100.

Remind learners that they can break numbers down into their place value parts and multiply each part separately, for example $43 \times 5 = 40 \times 5 + 3 \times 5 = 200 + 15 = 215$.

Templates for the tables in questions 1, 4 and 5 are provided in the Addendum (pages 444 to 446).

Answers

1.

×	2	3	4	5	6	7	8	9	10
60	120	180	240	300	360	420	480	540	600
70	140	210	280	350	420	490	560	630	700
80	160	240	320	400	480	560	640	720	800
90	180	270	360	450	540	630	720	810	900
100	200	300	400	500	600	700	800	900	1 000

2. (a) 300 (b) 420 (c) 360 (d) 540
   (e) 560 (f) 200 (g) 270 (h) 480
   (i) 70 (j) 480 (k) 640 (l) 810

3. (a) 350 (b) 480 (c) 160 (d) 360
   (e) 490 (f) 400 (g) 540 (h) 560

3. When you calculate $9 \times 40 - 5 \times 40$, is it necessary to first calculate 9×40 and 5×40 separately, or is there a quicker way?
4. Calculate.
   (a) $9 \times 30 - 5 \times 30$ (b) $8 \times 50 - 3 \times 50$
   (c) $3 \times 40 + 3 \times 6$ (d) 3×46
   (e) $5 \times 30 + 5 \times 8$ (f) 5×38
   (g) 6×45 (h) 4×58

9.4 Build more knowledge of multiplication facts

1. Copy and complete this table.

×	2	3	4	5	6	7	8	9	10
60									
70									
80									
90									
100									

2. How much is each of the following?
   (a) 5×60 (b) 6×70
   (c) 90×4 (d) 60×9
   (e) 80×7 (f) 5×40
   (g) 3×90 (h) 60×8
   (i) 10×7 (j) 8×60
   (k) 8×80 (l) 90×9
3. How much is each of the following?
   (a) 5×70 (b) 8×60
   (c) 40×4 (d) 40×9
   (e) 70×7 (f) 5×80
   (g) 6×90 (h) 70×8

Notes on questions

Questions 6(a) and (b) prepare learners to answer question 6(c). Learners first copy from the table (in question 5) the answer to 8 × R7 (the units part), then they copy the answer to 20 × R7 (the tens part), and then they use these two answers to write down the answer to 28 × R7. This approach is repeated in questions 6(d), (e) and (f). This models for learners what they should do to get the answers to questions 6(g) and (h), i.e. break the numbers down into a tens part and a units part, then copy the answers from the table in question 5, and then combine the products.

Answers

4.

×	10	50	40	80	30	60	70	100	90
10	100	500	400	800	300	600	700	1 000	900
5	50	250	200	400	150	300	350	500	450
2	20	100	80	160	60	120	140	200	180
4	40	200	160	320	120	240	280	400	360
8	80	400	320	640	240	480	560	800	720
3	30	150	120	240	90	180	210	300	270
6	60	300	240	480	180	360	420	600	540
7	70	350	280	560	210	420	490	700	630
9	90	450	360	720	270	540	630	900	810

5.

Number of pancakes	1	2	3	4	5	6	7	8	9
Cost (in rands)	7	14	21	28	35	42	49	56	63

Number of pancakes	10	20	30	40	50	60	70	80	90
Cost (in rands)	70	140	210	280	350	420	490	560	630

6. (a) R56 (b) R140 (c) R196 (d) R42
   (e) R350 (f) R392 (g) R602 (h) R301

4. Complete the table.

×	10	50	40	80	30	60	70	100	90
10				800					
5									
2									
4			160						
8				640					
3									
6									
7									
9									

5. Mia is selling pancakes. One pancake costs R7. Complete this table to help her calculate the cost of *any* number of pancakes up to 100.



Number of pancakes	1	2	3	4	5	6	7	8	9
Cost (in rands)	7	14							

Number of pancakes	10	20	30	40	50	60	70	80	90
Cost (in rands)	70	140							

6. What is the cost of each of the following, if you buy from Mia?
   (a) 8 pancakes
   (b) 20 pancakes
   (c) 28 pancakes
   (d) 6 pancakes
   (e) 50 pancakes
   (f) 56 pancakes
   (g) 86 pancakes
   (h) 43 pancakes

Answers

7. (a) 216 (b) 312 (c) 312 (d) 567

9.5 Multiplying three or more numbers

Mathematical notes

From questions 1, 2 and 3 it is expected that learners will see that they can swap the order of the numbers that they multiply without changing the answer. This is useful for learners to know because it can sometimes make a calculation easier if the order in which you multiply the numbers is swapped around. For example, in question 2(a) it is easier to multiply $2 \times 5 \times 7 = 10 \times 7$ than it is to multiply $2 \times 7 \times 5 = 14 \times 5$, and similarly $25 \times 7 \times 4$ is easier if you multiply $25 \times 4 \times 7 = 100 \times 7 = 700$.

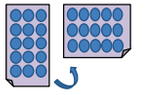
In Section 5.2 of Unit 5 learners were shown that they can swap the order of two numbers that are multiplied. Swapping the order of three or more numbers being multiplied is an extension of this.

Teaching guidelines

Try to move quickly through Section 9.5: aim to cover this section in 30 minutes. One possibility is to use

- the summary bar and tinted passage on page 111 and questions 4(a) and (b) for concept development,
- questions 1, 2 and 3 for classwork, and
- questions 4(c) to (f) for additional practice.

The numbers and calculations in questions 1, 2 and 3 are easy. Learners can do these as mental calculations. You can read the questions to them, so that they keep up their pace of work. Once learners have completed question 3 you can remind them that they have already seen in Section 5.2 that when you multiply two numbers you can swap the order in which they are multiplied without changing the answer. Remind learners that this can help to make calculations easier. You can use questions 4(a) and (b) to show learners how changing the order in which they multiply numbers can make the calculations easier.



Answers

1. (a) 30 (b) $2 \times 3 \times 5$ (c) Learners check and correct.
2. (a) 30 (b) $3 \times 5 \times 2$ (c) Learners check and correct.

Mia calculates the cost of 27 pancakes like this:
I look at the cost of 20 pancakes and add that to the cost of 7 pancakes.

To calculate 68×7, the 68 can be broken down into 60 and 8:
$60 \times 7 = 420$ and $8 \times 7 = 56$.
So, $68 \times 7 = 420 + 56$
which is 476.

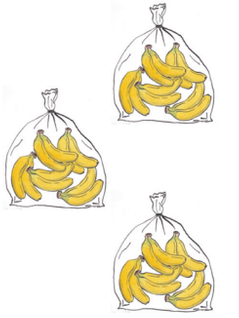
7. How much is each of the following?
   (a) 36×6 (b) 78×4 (c) 8×39 (d) 9×63

9.5 Multiplying three or more numbers

1. Mrs Mentoor bought 2 bags of bananas with 3 bunches of 5 bananas in each bag.
   (a) How many bananas did she buy?
   (b) Write down how you can calculate the number of bananas, without actually counting them, but by multiplication.
   (c) You can check your calculation by counting the bananas.



2. Mr Nene bought 3 bags of bananas with 5 bunches of 2 bananas in each bag.
   (a) How many bananas did he buy?
   (b) Write down how you can calculate the number of bananas, without actually counting them, but by multiplication.
   (c) You can check your calculation by counting the bananas.



Answers

3. (a) 30
   (b) The answer is still 30.
4. (a) 70 Probably easiest to calculate $2 \times 5 \times 7 = 10 \times 7$
   (b) 700 Probably easiest to calculate $25 \times 4 \times 7 = 100 \times 7$
   (c) 940 Probably easiest to calculate $4 \times 5 \times 47 = 20 \times 47$
   (d) 120 Probably easiest to calculate $2 \times 5 \times 12 = 10 \times 12$
   (e) 850 Probably easiest to calculate $10 \times 17 \times 5 = 170 \times 5$
   (f) 770 Probably easiest to calculate $2 \times 55 \times 7 = 110 \times 7$

9.6 Use your knowledge in a different way

Mathematical notes

This section links multiplication and division for learners. It also focuses their attention on using multiplication to solve division. Learners are only introduced to the word *division* and the symbol for division towards the end of this section. This is deliberate so that they focus on the connection between the two operations and using multiplication to solve the division problems.

Teaching guidelines

Try to move quickly through Section 9.6: aim to cover this section in 30 minutes. One possibility is to use

- questions 1(b), 2(a), 3(a), 4(a) and (b), and 5(a) for concept development,
- questions 1(a), (c) and (d), 2(b), (c) and (d), 5(b), 7 and 8 for classwork, and
- questions 3(b), (c) and (d), 6 and 9 for additional practice.

Learners can use question 1 to solve the calculations in questions 2 and 3. Encourage them to always ask themselves: "*What have I done before that can help me here?*" Model this by asking learners which calculation in question 1 can help them to answer questions 2(a), 3(a) and 5(a).

Answers

1. (a) 420 (b) 320
   (c) 420 (d) 450
2. (a) 4 (The answer can be seen in 1(b).) (b) 6 (The answer can be seen in 1(c).)
   (c) 9 (The answer can be seen in 1(d).) (d) 7 (The answer can be seen in 1(a).)

3. Daisy wrote $2 \times 3 \times 5$ for question 1.
   She wrote $3 \times 5 \times 2$ for question 2.
   Her answer for both calculation plans was 30.
   (a) How many bananas are there altogether in 5 bags with 2 bunches of 3 bananas in each bag?
   (b) What will happen if we change the order of the numbers in the calculation plan $5 \times 2 \times 3$?

When three or more numbers are multiplied, they can be swopped around: the answer remains the same. This is a **property** of multiplication.

If you have to multiply three or more numbers, you can rearrange the numbers so that the calculation is easier.

4. Do the following multiplications as quickly as you can. Try to do it in a way that will make it easy for you.
   (a) $2 \times 7 \times 5$
   (b) $25 \times 7 \times 4$
   (c) $4 \times 47 \times 5$
   (d) $12 \times 2 \times 5$
   (e) $10 \times 17 \times 5$
   (f) $2 \times 7 \times 55$

9.6 Use your knowledge in a different way

1. How much is each of the following?
   (a) 7×60 (b) 4×80
   (c) 6×70 (d) 9×50
2. Find the missing number in each number sentence. Your answers for question 1 may help you.
   (a) $\ldots \times 80 = 320$ (b) $\ldots \times 70 = 420$
   (c) $\ldots \times 50 = 450$ (d) $\ldots \times 60 = 420$

Answers

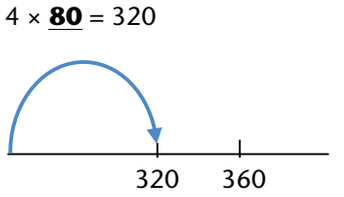
3. (a) 80 (The answer can be seen in 1(b).) (b) 70 (The answer can be seen in 1(c).)
   (c) 50 (The answer can be seen in 1(d).) (d) 60 (The answer can be seen in 1(a).)

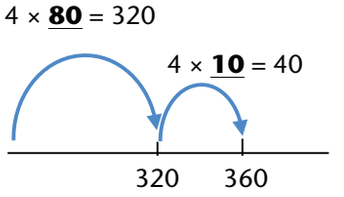
Notes on questions

Once learners have completed question 4(a), remind them that in Sections 9.3 and 9.4 they have often multiplied two numbers by the same number, for example 3 × 40 + 3 × 6, and that they have found this was the same as multiplying 3 × 46.

They can use this when they multiply to divide. For example, they know that 4 × 80 = 320 and this can help them to find 4 × ☐ = 360. This can be shown on a number line. From the number line alongside they can see that there is still 40 that needs to be shared amongst 4: 4 × ☐ = 40. 4 × 80 + 4 × 10 = 360. So, 4 × 90 = 360.

In question 5(b), help learners to understand that because R40 is half of R80, the answer to 5(b) must be twice that of 5(a).





Answers

4. (a) R90 (R320 = 4 × R80 and R40 = 4 × R10, so R360 = 4 × R90)
   (b) R95 (R360 = 4 × R90 and R20 = 4 × R5, so R380 = 4 × R95)
5. (a) 4 books
   (b) 8 books
6. (a) 10 pencils
   (b) 20 pencils
   (c) 5 pencils
   (d) 10 pencils
   (e) 12 pencils

3. Now find the missing number in each of these number sentences. Your answers for question 1 may again help you.
   (a) 4 × . . . = 320 (b) 6 × . . . = 420
   (c) 9 × . . . = 450 (d) 7 × . . . = 420

4. (a) How much money will each person get if R360 is shared equally between four people?
   (b) If R20 more is available to be shared equally between four people, there is R380 which can be shared. How much will each person get?

What you did when you found the missing numbers in questions 2, 3 and 4 is called **division**.
When you found the missing number 80 for the number sentence 4 × . . . = 320, you divided 320 by 4. You can write this as 320 ÷ 4.
What you will do in question 5 is also called division.

320 ÷ 4 = 80
4 × 80 = 320
Division is called the **inverse** of multiplication.

5. (a) How many books at R80 each can you buy with R320?
   (b) How many books at R40 each can you buy with R320?

6. (a) How many pencils at R10 each can you buy with R100?
   (b) How many pencils at R5 each can you buy with R100?
   (c) How many pencils at R5 each can you buy with R25?
   (d) How many pencils at R5 each can you buy with R50?
   (e) How many pencils at R5 each can you buy with R60?

Answers

7. 4 Learners do not have to show the answer on a number line.
8. (a) 4
   (b) 4
9. (a) 8
   (b) 90

9.7 Practise and use your skills

Teaching guidelines

Try to move quickly through Section 9.7: aim to cover this section in 1 hour. One possibility is to use

- questions 1, 4 and 10 for concept development,
- questions 2, 3, 7, 9 and 11 for classwork, and
- questions 5, 6 and 8 for additional practice.

Possible misconceptions

This unit is about multiplying and dividing. The assumption is that learners will use multiplication to divide. However, in question 2 learners will need to use adding on or subtracting to find the answers. Learners may wrongly assume that question 3 can be solved by multiplication because they have been working with multiplication in this unit.

Answers

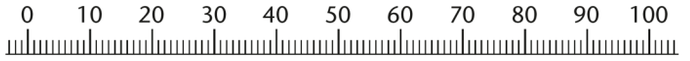
Note: Learners should estimate first and write down their estimate.

1. R438. Learners can either multiply the price of each item by 6, or add the prices of the three items and then multiply by 6.

2. R229. Learners could add on or subtract to get the answer.

3. R19. Learners need to find $\square \times 4 = R76$.

7. Use the number line below to find out how many 15s there are in 60.



8. Now find the missing numbers in these number sentences:
   (a) . . . × 15 = 60 (b) 60 ÷ 15 = . . .

9. (a) How many 15s are there in 120?
   (b) How much is 6 × 15?

9.7 Practise and use your skills

Answer the questions below.

Ask your teacher if there are words that you do not know.

Read each question carefully and make sure that you understand what is asked.

Each time first estimate the answer and write your estimated answer down.

When you have solved the problem, look at the answer and decide if that answer is possible in the real situation.

1. The Williams family of six people are having lunch at a café. They each order a hamburger for R38, chips for R19 and a cooldrink for R16. How much will the family's lunch cost in total?

2. Loyiso wants to buy a soccer ball that costs R387. He has saved R158. How much more money does he need?

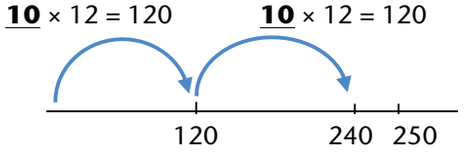
3. Mr Faku pays R76 for 4 kg tomatoes. What is the price of 1 kg tomatoes?



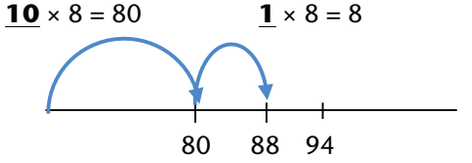
Notes on questions

In question 9 learners can use doubling to get the answers to (a) and (b). In question 9(a), 6 = double 3 or 2 × 3, so in 6 hours Werner earns double what he earns in 3 hours. In question 9(b), what Werner earns in 12 hours is double what he earns in 6 hours, or 4 times what he earns in 3 hours. Learners should try to use what they already know with each sub-question in question 9.

In question 10 learners can use a number line and multiply in stages.



In question 11 learners can also use a number line and multiply in stages.



Answers

4. 6 × 4 × 5 = 6 × 20 = 120 cookies
5. The Lucky Store: R6 per avocado
   The Big Grocer: R7 per avocado
   The Lucky Store gives the best deal.
6. 20 + 30 + 36 + 56 = 142 bananas
7. (a) 3 bottles. This is because 3 × R25 = R75.
   (b) R10
8. 5 T-shirts. This is because 5 × R16 = R80.
9. (a) R180
   Because 6 hours is double 3 hours. He earned R90 for 3 hours. 2 × R90 = R180.
   (b) R360
   Because 12 hours is 4 × 3 hours. He earned R90 for 3 hours. 4 × R90 = R360.
   (c) R30
   Because 3 × R30 = R90. He earned R90 in 3 hours, so in 1 hour he earned 1 third of R90.
   (d) R120
   Because 4 × R30 = R120. 4 hours is 4 × 1 hour.
10. (a) 20 crates Because 20 × 12 = 240. (b) 10 bottles
11. (a) 11 bags Because 8 × 11 = 88. (b) 6 gem squashes

4. Mrs Jacobs is baking cookies. She places 6 rows of cookies on a baking tray. There are four cookies in one row. If she fills five baking trays, how many cookies is she baking?
5. The Lucky Store charges R90 for a box of 15 avocado pears. The Big Grocer charges R63 for a bag of 9 avocado pears. Which is the best deal?
6. Mr Zweli bought one large box of bananas.
   He saw that there were
   5 bunches with 4 bananas in a bunch,
   6 bunches with 5 bananas in a bunch,
   12 bunches with 3 bananas in a bunch and
   7 bunches with 8 bananas in a bunch.
   How many bananas did Mr Zweli buy?
7. Kholeka has saved R85. She wants to buy bottles of juice for her birthday party. Each bottle costs R25.
   (a) How many bottles can she buy?
   (b) How much money will she have left?
8. Suzi buys T-shirts on a sale. Each T-shirt costs R16. Suzi has R90. How many T-shirts can she buy?
9. Werner is paid R90 for three hours' work. How much will he get paid for the following number of hours?
   (a) 6 hours (b) 12 hours
   (c) 1 hour (d) 4 hours
10. Dirkie packs bottles into crates. He packs 12 bottles in each crate. There are 250 bottles.
    (a) How many crates can he fill?
    (b) How many bottles will be left over?
11. Mr Daniels sells gem squashes in bags of 8. He has a large box with 94 gem squashes.
    (a) How many bags can he fill?
    (b) How many gem squashes are left over?



9.8 Dividing into equal parts

Mathematical notes

Question 1 provides examples of different sorts of situations that can be solved using multiplication.

In this section learners are also introduced to the concept of remainders. Remainders are parts that remain undivided, for example 60 ÷ 8 = 7 remainder 4 (the 4 has not been divided by 7). This implies that (8 × 7) + 4 = 60. Questions 1(e), (f) and (g) lay the basis for understanding remainders.

Teaching guidelines

Try to move quickly through Section 9.8: aim to cover this section in 45 minutes. One possibility is to use

- questions 1, 2(c) and (e), and 3(a) for concept development,
- questions 2(a), (b), (d) and (f) to (i), 3(b), 4, 5 and 8 for classwork, and
- questions 6, 7 and 9 for additional practice.

Once you have worked through question 1 with the class, you can show them how question 2(c) helps them to get the answer to question 2(e) and how question 2(e) provides the answer to question 3(a).

Answers

1. (a) R64
   (b) R16
   (c) 64 quarter-apples
   (d) 16 tables
   (e) 16 blue beads
2. (a) 77 (b) 63 (c) 56
   (d) 42 (e) 60 (f) 53
   (g) 34 (h) 84 (i) 35
3. (a) 7 loaves with R4 change
   (b) 9 rows

9.8 Dividing into equal parts

Knowledge of **multiplication facts** makes you powerful.

You can answer many different questions by using one multiplication fact.

For example, if you know the multiplication fact 4 × 16 = 64, you can answer all the different parts of question 1 easily. You will also be able to answer many other questions.

1. (a) Large bottles of juice cost R16 each.
   How much will four of these bottles cost altogether?
   (b) Janice paid R64 for four facecloths. They cost exactly the same. What did she pay for each facecloth?
   (c) Sixteen apples are cut into quarters.
   How many quarter-apples is this, altogether?
   (d) Four people will sit at each table at a wedding function.
   Sixty-four people are invited to the wedding.
   How many tables are needed?
   (e) For every blue bead in a string of beads, there are four red beads. How many blue beads are there, if there are 64 red beads in total?



2. How much is each of the following?
   (a) 7 × 11 (b) 7 × 9 (c) 7 × 8
   (d) 7 × 6 (e) (7 × 8) + 4 (f) (7 × 7) + 4
   (g) (7 × 4) + 6 (h) 7 × 12 (i) 7 × 5
3. (a) A loaf of bread costs R8 at a spaza shop. How many loaves of bread can you buy with R60, and how much change will you get?
   (b) There are 63 tiles on a floor. The tiles are laid in rows of 7 tiles each. How many rows are there?

Answers

4. (a) 7 remainder 4 (b) 8 remainder 2
   (c) 21 (d) 11
5. (a) 8 (b) 6 (c) 8 remainder 2
   (d) 9 (e) 10 (f) 6
   (g) 5 remainder 8 (h) 11 remainder 3 (i) 9
   (j) 7 (k) 20 (l) 4
   (m) 15 (n) 4 (o) 26 remainder 2
   (p) 3 (q) 30 (r) 15
6. 13 minutes
7. 7 rubber bands
8. 12 times
9. (a) 8 remainder 1 (b) 16 remainder 1 (c) 4
   (d) 6 (e) 9 (f) 7
   (g) 12 (h) 6 remainder 3 (i) 3 remainder 2
   (j) 8 (k) 14 (l) 19 remainder 1

Your answer for question 3(a) can be written like this:
60 ÷ 8 = 7 **remainder** 4.

4. How much is each of the following?
   (a) 53 ÷ 7 (b) 34 ÷ 4
   (c) 84 ÷ 4 (d) 77 ÷ 7
5. How much is each of the following?
   (a) 48 ÷ 6 (b) 48 ÷ 8 (c) 50 ÷ 6
   (d) 81 ÷ 9 (e) 100 ÷ 10 (f) 60 ÷ 10
   (g) 58 ÷ 10 (h) 58 ÷ 5 (i) 63 ÷ 7
   (j) 63 ÷ 9 (k) 80 ÷ 4 (l) 80 ÷ 20
   (m) 60 ÷ 4 (n) 60 ÷ 15 (o) 80 ÷ 3
   (p) 75 ÷ 25 (q) 90 ÷ 3 (r) 90 ÷ 6
6. Devin takes 65 minutes to wash 5 cars.
   How long does it take him to wash one car?
7. The teacher has 35 rubber bands. The learners sit and work in 5 equal groups. How many rubber bands will each group get if the teacher gives every group the same number of rubber bands?
8. Rico is 7 years old. His grandpa is 84 years old.
   How many times older is Rico's grandpa than Rico?
9. Calculate:
   (a) 65 ÷ 8 (b) 65 ÷ 4 (c) 36 ÷ 9
   (d) 54 ÷ 9 (e) 27 ÷ 3 (f) 42 ÷ 6
   (g) 60 ÷ 5 (h) 45 ÷ 7 (i) 20 ÷ 6
   (j) 24 ÷ 3 (k) 98 ÷ 7 (l) 58 ÷ 3

Term 2

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Grade 4 Term 2 Unit 1 Whole numbers

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
1.1 Compare bigger numbers	The concept of thousand and going beyond thousand	119 to 122
1.2 Counting thousands	Several thousands, and number names and symbols for numbers up to 10 000	123 to 124
1.3 Represent numbers in different ways	Representing numbers as symbols, in place value parts and expanded notation	125 to 126
1.4 Count beyond 1 000	Counting verbally and on a number line, comparing and ordering numbers	126 to 127

CAPS time allocation	1 hour
CAPS page references	14 and 67 to 68

In this unit learners develop and consolidate a sense of numbers into the thousands. Learners use place value cards to build up numbers and to break numbers down into place value parts. The former lays the basis for writing number symbols and the latter lays the basis for expanded notation. Once learners have a good sense of numbers into the thousands, it is easy for them to order and compare these numbers. This is done in the last section of this unit.

Mathematical background

Apart from knowing the number names and other ways to represent numbers, learners need to develop a sense of the quantities represented by bigger numbers. Learners do not often experience quantities (collections) beyond 1 000 in their everyday lives, hence learning activities need to provide them with experiences of such collections. This helps learners to develop, for numbers beyond one thousand, an intuitive sense of "how many" the numbers represent.

Since counting collections of 1 000 or more objects one by one is not practical, the structure of larger numbers – the way they are built up with parts of ten, hundred and thousand – needs to be used.

In most classrooms it is not feasible to provide each learner with structured apparatus to count beyond 1 000. Instead we provide images of objects grouped in tens and hundreds. In Term 1 Unit 1 of the Learner Book, learners have already encountered images of 100 as 10 groups of 10. This is reinforced in the images in this unit. Learners also see 10 groups of 100 making 1 000. Focus learners' attention on being able to recognise 10 groups of 10 as 100, and 10 groups of 100 as 1 000, rather than counting each object separately. This lays an important basis for understanding the value of large numbers.

Resources

Each learner should have their own set of place value cards. A master copy for place value cards is given in the Addendum (pages 416 to 418).

In addition, you should have a set of large place value cards for demonstration purposes as given in the Addendum (pages 419 to 432).

1.1 Compare bigger numbers

Mathematical notes

This section deals with number concept, number names and number symbols for numbers just beyond 1 000. The remaining sections in this unit deal with numbers up to 9 999. All the concepts raised here will form the basis of the rest of the number work in Grade 4. These concepts will be touched on repeatedly throughout the year. Try to keep the pace of work moving so that you complete this unit within the allocated time.

Critical knowledge

Learners need to move beyond an understanding of counting as *counting one by one* to a more sophisticated sense of counting: *counting in groups*. Counting forwards and backwards in twos, threes, fives and other small counting units is useful and promotes knowledge of basic addition and subtraction facts, and skills such as filling up to the nearest multiple of ten. However, counting in tens, hundreds and thousands is specifically important to develop a sense of larger numbers.

Teaching guidelines

To help learners to move away from the idea of counting one by one, you can ask questions like: *"How many cubes are there in each of the yellow groups?"* and *"Do all the yellow groups have the same number of cubes?"* Repeat these types of questions for the heaps of sticks (i.e. the groups of 10 bundles) on page 120: *"How many bundles of sticks are there in each heap?"*, *"How many sticks are there in each bundle inside each heap?"*, *"How many sticks are there altogether in a heap?"*, *"Do all the heaps have the same number of sticks?"* The aim is for learners to count the groupings in hundreds.

To allow learners to compare the number of cubes and sticks (as asked in question 2) more easily, you can ask one learner to keep page 119 open and the learner next to him or her page 120.

Notes on questions

The summary bar tells learners that 10 hundreds = 1 000. Question 1 then asks them to recognise that this also means that 100 tens = 1 000. This is an application of the commutative property of multiplication: $10 \times 100 = 100 \times 10 = 1\,000$.

Question 2 may prompt learners to consider the group sizes and numbers of groups.

Answers

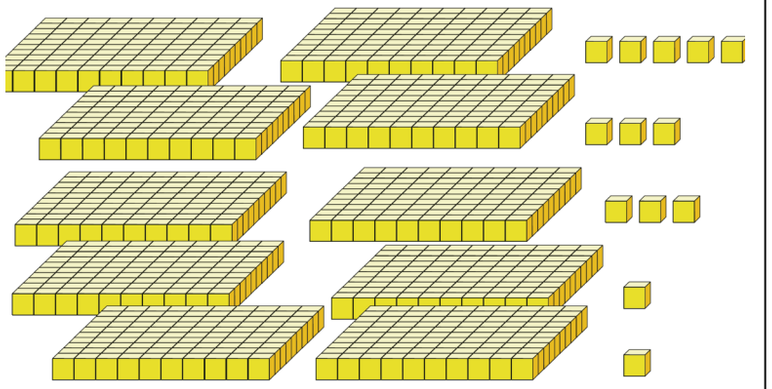
1. 100 tens
2. The number of cubes and sticks is the same.
3. one thousand and thirteen
4. one thousand and thirteen

UNIT 1 WHOLE NUMBERS

1.1 Compare bigger numbers

Ten hundreds is **one thousand**.
The number symbol for one thousand is **1 000**.

1. How many tens make up 1 000?
2. Which are more, the cubes below or the sticks on the next page?



3. How many cubes are shown here,
   one thousand and five or
   one thousand and twelve or
   one thousand and thirteen or
   one thousand and fourteen?
4. How many sticks are shown on the next page?

GRADE 4: MATHEMATICS [TERM 2] 119

Possible misconceptions

Remember to ask learners to always state the full number name, for example “one thousand and thirteen” rather than saying “one-oh-one-three”.

Some numbers that we deal with in everyday life, such as phone numbers, car registration numbers or ID numbers, comprise a set of single unit digits. These kinds of numbers are really labels and their digits do not have place value. For these kinds of numbers it is appropriate to say zero-seven-two-four-nine-nine-seven-four-seven-three.

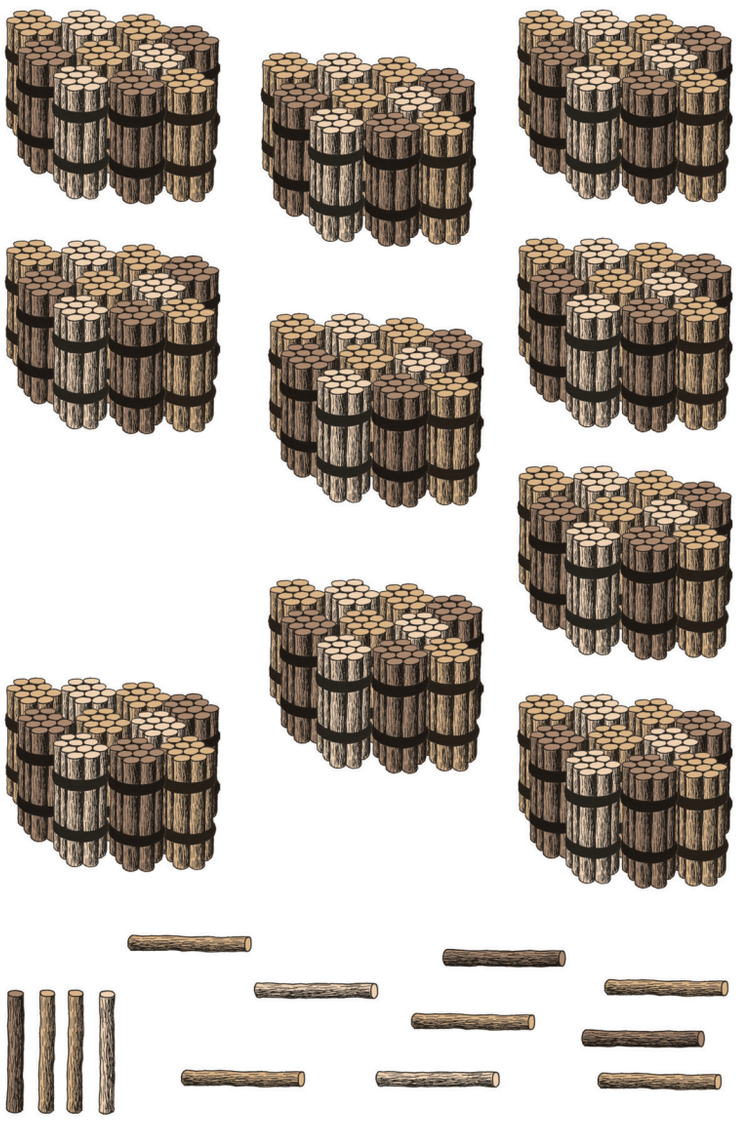
When we are dealing with numbers (rather than a label made up of digits), it is important to say numbers correctly out in full: the way we would write out the number name.

It helps learners to understand place value and the value of numbers better if they say the number out in full, for example “one thousand and fourteen”, “three thousand four hundred and twenty-seven”.

Teaching guidelines

One possible way to organise this section is to use

- pages 119 to 121 for concept development,
- questions 5 and 7 for classwork, and
- question 6 for additional practice.



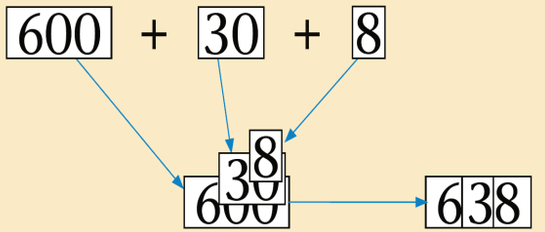
Mathematical notes

What are place value parts? What are place value cards? This is explained in the summary bar and tinted passage on page 12, and repeated below:

> The parts that are mentioned in the name of a number are called the **place value parts**. For example, the place value parts of thirty-seven are 30 and 7.

The cards that you use in school to show numbers can be called **place value cards**. This is because each card that you use to build a number shows one of the place value parts. Look at this example:

six hundred and thirty-eight is



Teaching guidelines

Show learners how to use place value cards to build up the number 998. Demonstrate how to place the 90 over the zeros of the 900, and the 8 over the zero of the 90. Explain that this shows the way we write the symbol 998.

Break down 998 into its place value parts by separating the place value cards. Explain that this is the way we say the number: nine hundred and ninety-eight. Explain that this is the basis of expanded notation and write the expansion 900 + 90 + 8 on the board.

Repeat this process for 1 001, 1 002 and 1 020.

Answers

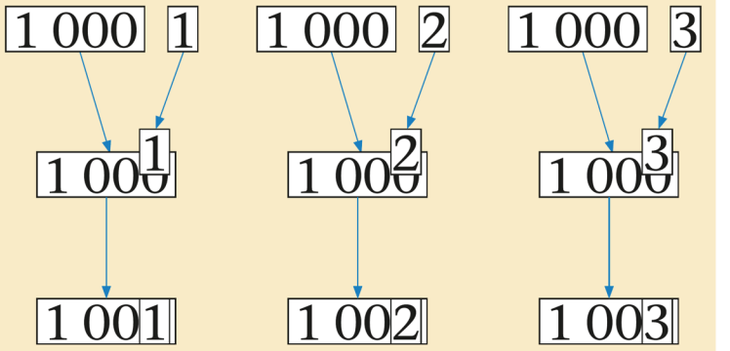
5.

Number symbol	Number name	Expanded notation
1 003	one thousand and three	1 000 + 3
1 004	one thousand and four	1 000 + 4
1 005	one thousand and five	1 000 + 5
1 006	one thousand and six	1 000 + 6
1 007	one thousand and seven	1 000 + 7

The number symbols, number names and expanded notation for some consecutive numbers are given below.

Consecutive numbers are numbers that follow on each other.

Number symbol	Number name	Expanded notation
997	nine hundred and ninety-seven	900 + 90 + 7
998	nine hundred and ninety-eight	900 + 90 + 8
999	nine hundred and ninety-nine	900 + 90 + 9
1 000	one thousand	1 000
1 001	one thousand and one	1 000 + 1
1 002	one thousand and two	1 000 + 2



5. Write the symbols, names and expanded notation for the five numbers that follow on 1 002.

Possible misconceptions

Some learners may write 1 000 1 instead of 1 001; 1 000 13 instead of 1 013.

This shows that learners have not yet mastered our way of writing numbers. They are writing the numbers as we say them.

It also shows that learners have a good sense of the numbers beyond 1 000. These learners are writing the numbers in an unconventional expanded notation: they write 1 000 1 to mean 1 000 + 1 and 1 000 13 to mean 1 000 + 13. Acknowledge that this is the way we *say* numbers and that they understand the parts of the numbers well; also that they can *use* this when writing the expanded notation. However, also use the way we show the numbers with place value cards, i.e. placing the 1 over the last zero of the 1 000 (in the case of 1 001), and the 10 over that last two zeros of 1 000 and the 3 over the zero of the 10 (in the case of 1 013). Explain that this shows how we make and write the number symbol.

Answers

6.

Number symbol	Number name	Expanded notation
1 013	one thousand and thirteen	1 000 + 10 + 3
1 014	one thousand and fourteen	1 000 + 10 + 4
1 015	one thousand and fifteen	1 000 + 10 + 5
1 016	one thousand and sixteen	1 000 + 10 + 6
1 017	one thousand and seventeen	1 000 + 10 + 7
1 018	one thousand and eighteen	1 000 + 10 + 8
1 019	one thousand and nineteen	1 000 + 10 + 9
1 020	one thousand and twenty	1 000 + 20
1 021	one thousand and twenty-one	1 000 + 20 + 1
1 022	one thousand and twenty-two	1 000 + 20 + 2

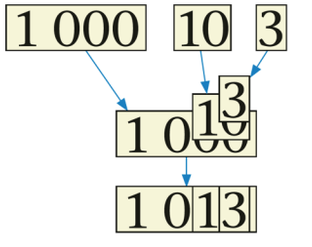
7. (a) 1 001 1 000 + 1
   (b) 1 010 1 000 + 10
   (c) 1 020 1 000 + 20
   (d) 1 100 1 000 + 100

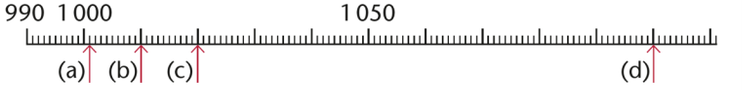
The different ways of writing some other numbers are given below.

Number symbol	Number name	Expanded notation
1 010	one thousand and ten	1 000 + 10
1 011	one thousand and eleven	1 000 + 10 + 1
1 012	one thousand and twelve	1 000 + 10 + 2
...	.....	.....
1 023	one thousand and twenty-three	1 000 + 20 + 3
1 600	one thousand six hundred	1 000 + 600
1 607	one thousand six hundred and seven	1 000 + 600 + 7
1 670	one thousand six hundred and seventy	1 000 + 600 + 70
1 677	one thousand six hundred and seventy-seven	1 000 + 600 + 70 + 7

6. Write the symbols, names and expanded notation for the ten numbers that follow on 1 012.

7. Write the number symbols and expanded notation for the numbers indicated by the arrows on the number line below.





Teaching guidelines for Sections 1.2 to 1.4

One possible way to organise the next three sections is to use:

- page 123 and the tinted passage on page 124 for concept development,
- questions 1, 2, 3, 4 of Section 1.4 for mental mathematics,
- question 3 on page 124, questions 1 and 2 on page 125, and questions 5, 6, 7 and 8(a) to (e) on page 127 for classwork, and
- questions 4 and 5 on page 124, questions 3, 4 and 5 on page 126, and questions 8(f) to (o) on page 127 for additional practice.

1.2 Counting thousands

Mathematical notes

Each small red mark on pages 123 and 124 is called a **stripe.**

An **array** is a systematic arrangement of similar objects, usually in rows and columns. The stripes are arranged in ten rows of 10, to make arrays of 100. There are 10 arrays of 100 across the pages – these make arrays of 1 000.

Notes on questions

The first sentence, first array and question 1 are intended to focus learners' attention on the array with 1 000 stripes. The aim is for learners to use a thousand as a counting unit in question 2, and in question 5 on page 124.

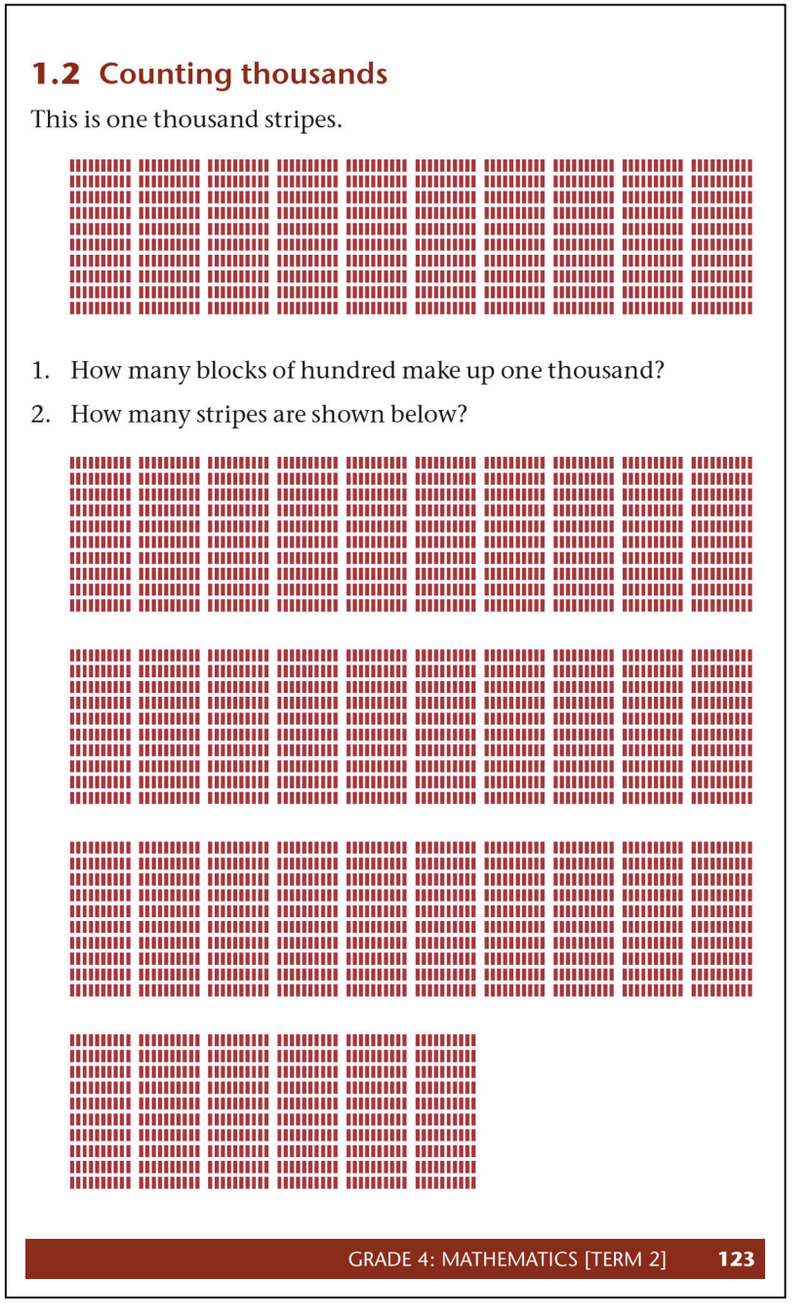
Teaching guidelines

Observe as many learners as possible when they do question 2. There may be learners who do not count in thousands, but in hundreds. Support such learners by making them aware that it is not necessary to count hundred by hundred before they reach the last row in the array.

Some learners may see that there are 3 full arrays of 1 000 and that the last one has 4 hundred arrays less than 1 000. They may calculate that the last row is 1 000 – 400 = 600. So overall there are 3 600 stripes.

Answers

1. 10
2. 3 600



Teaching guidelines

It will be good to ask learners to represent the numbers in question 3 with place value cards as well, and especially to represent their answer for question 5 with place value cards.

Possible misconceptions

Note that some learners may try to represent 2 747 (question 5) with the place value cards 2, 7 and 4, and then be frustrated because they do not have a second place value card 7 available. Such learners may have a number concept that is interpreting a number as only a collection of digits and they may have poor understanding of the meaning of the digits as used in the number symbol. Remind them that every digit in a number has a specific place value with respect to the position of the digit in the number. Further, that the correct way of representing 2 747 with place value cards is with the 2 000, 700, 40 and the 7.

Answers

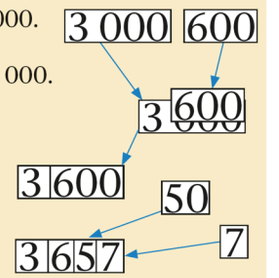
3. (a) 2 700
   (b) 5 300
4. seven thousand two hundred; three thousand eight hundred
5. 2 747

The number symbol for *one thousand* is 1 000.

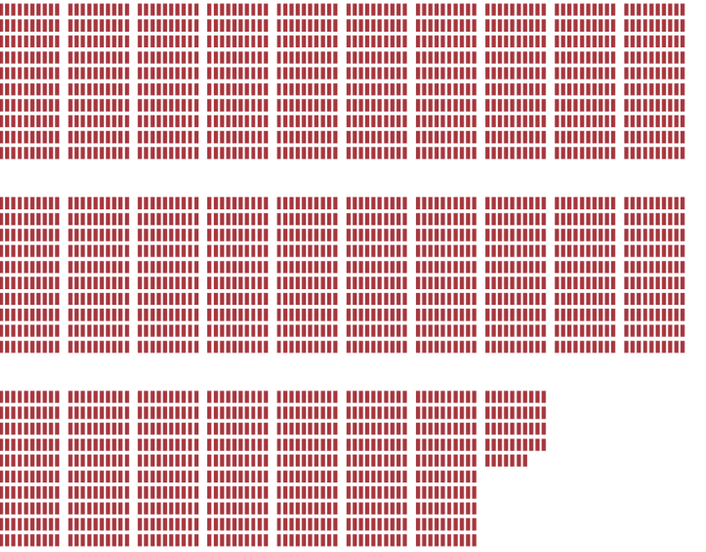
The number symbol for *three thousand* is 3 000.

The number symbol for *three thousand six hundred* is 3 600.

The number symbol for *three thousand six hundred and fifty-seven* is 3 657.



3. Write the number symbols for these numbers:
   (a) two thousand seven hundred
   (b) five thousand three hundred
4. Write the number names for 7 200 and 3 800.
5. How many stripes are shown below?



1.3 Represent numbers in different ways

Teaching guidelines

You may once again demonstrate how numbers are represented with place value cards, and may use the example in the tinted passage for this purpose. After the demonstration you may ask learners to represent different 4-digit numbers with place value cards by packing the cards out on their desks. Make sure that learners know that they can do this in two different ways, for example:

6000 300 80 4

or by putting the cards correctly on top of each other to show what the number symbol looks like.

Possible misconceptions

Again monitor learners to check if there are learners who make a mistake, like using the 6 3 8 4 cards to represent 6 384.

Answers

1. (a) 3 000, 800, 70, 4
   (b) 6 000, 200, 50, 9
   (c) 9 000, 400, 20, 3

2. (a) 3 874 3 000 + 800 + 70 + 4
   (b) 6 259 6 000 + 200 + 50 + 9
   (c) 9 423 9 000 + 400 + 20 + 3

1.3 Represent numbers in different ways

The number *5 thousand 3 hundred and ninety-six* has four place value parts:

5 000 300 90 6

This number can be written in expanded notation:

5 000 + 300 + 90 + 6

With place value cards, the short number symbol for 5 thousand 3 hundred and ninety-six is built by

- putting the 6 on top of the 0 of the 90,
- putting the 96 on top of the 00 of the 300, and
- putting the 396 on top of the 000 of the 5 000.

With all the zeros hidden, the **number symbol** for 5 thousand 3 hundred and ninety-six is 5 396.

5 3 9 6

The **place value parts** of 5 396 are 5 000, 300, 90 and 6.
The **expanded notation** for 5 396 is 5 000 + 300 + 90 + 6.

1. Write down the place value parts of each number:
   (a) three thousand eight hundred and seventy-four
   (b) six thousand two hundred and fifty-nine
   (c) nine thousand four hundred and twenty-three

2. Now write the number symbols and the expanded notation for the numbers in question 1.

Answers

3. (a) 6 000, 200, 80, 5 (b) 5 000, 800, 60, 2
   (c) 2 000, 500, 60, 8 (d) 8 000, 600, 50, 2
   (e) 3 000, 40, 6 (f) 1 000, 500, 4

4. (a) six thousand two hundred and eighty-five 6 000 + 200 + 80 + 5
   (b) five thousand eight hundred and sixty-two 5 000 + 800 + 60 + 2
   (c) two thousand five hundred and sixty-eight 2 000 + 500 + 60 + 8
   (d) eight thousand six hundred and fifty-two 8 000 + 600 + 50 + 2
   (e) three thousand and forty-six 3 000 + 40 + 6
   (f) one thousand five hundred and four 1 000 + 500 + 4

5. (a) four thousand eight hundred and sixty-three 4 863
   (b) three thousand six hundred and eighty-four 3 684
   (c) six thousand three hundred and forty-eight 6 348
   (d) eight thousand four hundred and thirty-six 8 436
   (e) six thousand and sixty-four 6 064
   (f) three thousand and eight 3 008

1.4 Count beyond 1 000

Teaching guidelines

Learners should find questions 1 and 2 (counting in hundreds) fairly easy. In question 3, and in general with counting, the difficulty is crossing the tens, hundreds and thousands barrier. So learners may stumble particularly when they count back and get to 7 000, 6 999.

Answers

1. 900 1 000 1 100 1 200 1 300 1 400 1 500 1 600
   1 700 1 800 1 900 2 000 2 100 2 200 2 300 2 400
   2 500 2 600 2 700 2 800 2 900 3 000

2. 2 000 1 900 1 800 1 700 1 600 1 500 1 400 1 300
   1 200 1 100 1 000 900 800

3. Write down the place value parts of each number:
   (a) 6 285 (b) 5 862
   (c) 2 568 (d) 8 652
   (e) 3 046 (f) 1 504

4. Now write the number names and expanded notation for the numbers in question 3.

5. The place value parts of some numbers are given below. Write the number name and number symbol for each number.
   (a) 3; 4 000; 800 and 60
   (b) 600; 4; 80 and 3 000
   (c) 40; 8; 300 and 6 000
   (d) 8 000; 30; 400 and 6
   (e) 6 000; 60 and 4
   (f) 8 and 3 000

1.4 Count beyond 1 000

1. Start at 900 and count in hundreds up to 3 000. Write the number symbols as you go along:
   900 1 000 1 100 and so on.

2. Start at 2 000 and count backwards in hundreds down to 800. Write the number symbols as you go along:
   2 000 1 900 1 800 and so on.

3. In each case below, first count softly by yourself without writing. When you have finished counting to the specified number, write the number symbols as you did in questions 1 and 2.
   (a) Start at 4 995 and count forwards in twos up to 5 023.
   (b) Start at 7 012 and count backwards in twos down to 6 996.

Answers (*continued*)

3. (a) Learners count on in twos from 4 995 to 5 023:

4 995	4 997	4 999	5 001	5 003	5 005	5 007	5 009	5 011
5 013	5 015	5 017	5 019	5 021	5 023			

(b) Learners count back in twos from 7 012 to 6 996:

7 012	7 010	7 008	7006	7 004	7 002	7 000	6 998	6 996

Teaching guidelines

For question 5 you can make photocopies of templates of the number lines provided in the Addendum (page 447).

In question 8(a) help learners to see that there are 10 small intervals between each larger interval (just like on a ruler). Focus them first on 8(c): once they have seen that this is 4 010, they can go back to (b) and then (a), and then forward to (d) and (e).

Answers

4. (a)

3 850	3 950	4 050	4 150	4 250	4 350	4 450	4 550	
4 650	4 750	4 850	4 950	5 050	5 150	5 250		

(b)

3 950	3 850	3 750	3 650	3 550	3 450	3 350	3 250	3 150
3 050	2 950	2 850	2 750	2 650	2 550	2 450	2 350	2 250
2 150	2 050	1 950	1 850	1 750	1 650			

5. (a) 8 630 (b) 8 640 (c) 8 650
(d) 8 660 (e) 8 670 (f) 8 680
(g) 1 394 (h) 1 385 (i) 1 379

6. 3 365 4 777 5 423 6 152 9 899 9 987

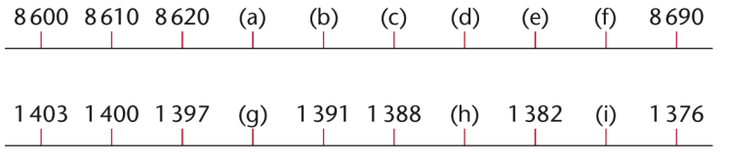
7. 9 356 6 553 5 121 4 001 3 499 2 710

8. The number symbols are given below. Note that the question requires learners to write the number symbols and the number names so do check learners' answers by asking them to read their answers to each other to confirm.

(a) 3 970 (b) 3 990 (c) 4 010 (d) 4 050 (e) 4 080
(f) 3 967 (g) 3 993 (h) 4 012 (i) 4 036 (j) 4 077
(k) 6 700 (l) 6 900 (m) 7 130 (n) 7 400 (o) 7 770

4. In each case below, first count softly by yourself without writing. When you have finished counting to the specified number, write the number symbols.
   (a) Start at 3 850 and count forwards in hundreds up to 5 250.
   (b) Start at 3 950 and count backwards in hundreds down to 1 650.

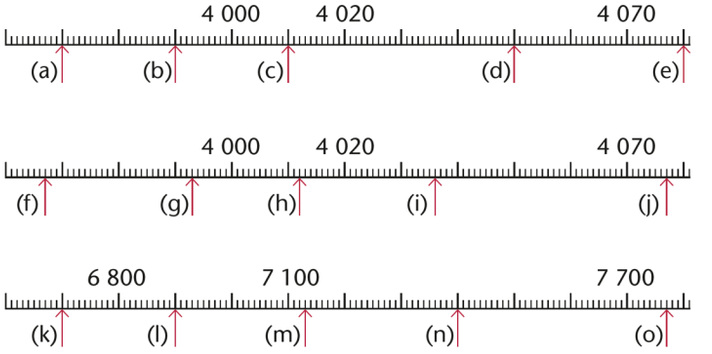
5. In your book, write the numbers that are missing on these number lines, for example (c) = 8 650.



6. Arrange these numbers from smallest to biggest.
   6 152 9 987 5 423 9 899 4 777 3 365

7. Arrange these numbers from biggest to smallest.
   2 710 9 356 6 553 5 121 3 499 4 001

8. Write the number names and number symbols for the numbers indicated by the arrows on the number lines below.



Grade 4 Term 2 Unit 2 Whole numbers: Addition and subtraction

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 Add and subtract with money	Developing basic skills for addition and subtraction	128 to 130
2.2 Methods to add and subtract	Introducing formal methods to add and subtract	131 to 133
2.3 Round off and estimate	Rounding off and adding and subtracting in practical situations	134 to 135

CAPS time allocation	4 hours
CAPS page references	14 to 15 and 69 to 71

Mathematical background

In Term 2, learners continue to engage with the three methods of addition prescribed for Grade 4:

A. Adding by filling up multiples of 10 and 100.
B. Adding by breaking down one number (the smaller one) into place value parts and adding the parts one by one.
C. Adding by breaking down both numbers, rearranging and adding up the answers of the parts.

Methods A and B are addressed in this unit, and method C in Unit 9.

Learners engage with two methods of subtraction in Grade 4:

A. Subtracting by filling up multiples of 10 and 100 (subtraction by addition).
B. Subtracting by breaking down both numbers, rearranging and adding up the answers of the parts.

Method A is addressed in this unit, and method B in Unit 9.

Resources

Learners need sets of place value cards.

You need two sets of large place value cards for teaching purposes.

Play money made from scrap paper.

Using place value cards when doing addition is a powerful way of consolidating understanding of the role of place value parts, and of the commutative and associative properties of addition.

2.1 Add and subtract with money

Mathematical notes

In this section learners break down and build up numbers in multiples of 100 and 10. The use of money as a context helps to make this practical.

Learners continue to use calculation strategies from Term 1, but the number range has increased to include calculating with thousands.

Teaching guidelines

Aim to complete Section 2.1 in 1 hour. One possibility is to use

- questions 1, 2, 3, 4(c) to (h), and 8 for concept development,
- questions 4(a), (b), (i) and (j), 5, 6 , 7, 9, 11(d) and (h), and 12 for classwork, and
- questions 4(k) to (n), 10, and 11(a), (b), (c), (e), (f) and (g) for additional practice.

Resources

When doing the concept development, it might be useful to have play money that learners can use to demonstrate other ways in which R460 (question 2) can be broken up. This does not need to be commercially made play money. Learners can quickly make 10 copies each of R200, R100, R50, R20 and R10 notes by writing the amounts on pieces of scrap paper.

Notes on questions

Question 1 requires learners to count on or add on in multiples of 100, 10 and 1, i.e. in place value parts. In question 2 learners are only asked to describe their answers. In question 3 they are shown how to write their descriptions down as number sentences.

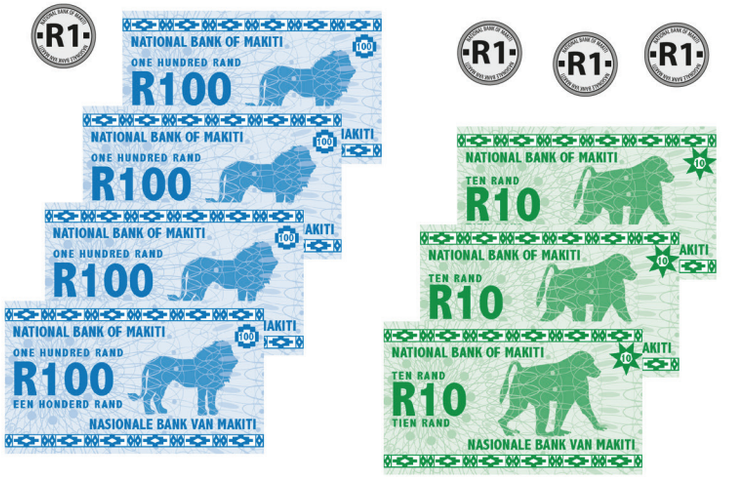
Answers

1. (a) R812
   (b) R1 212
2. Some, but not all possibilities are provided below. Learners' answers may differ. Check whether they are correct.
   One R200 note, four R50 notes, three R20 notes
   One R100 note, six R50 notes, six R10 notes
   One R200 note, four R50 notes, one R20 note, four R10 notes
   Four R100 notes, three R20 notes
   One R200 note, one R100 note, three R50 notes, one R10 note
   Eight R50 notes, three R20 notes

UNIT 2 WHOLE NUMBERS: ADDITION AND SUBTRACTION

2.1 Add and subtract with money

1. Simanga has saved R378.
   (a) He adds the money below to his savings. How much has he now saved in total?



   (b) Simanga adds another R400 to his savings. How much has he now saved in total?

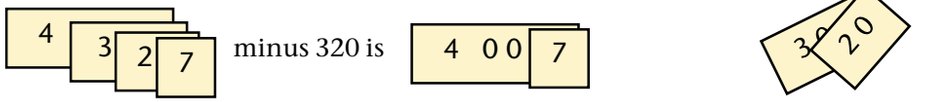
2. The school fee at a certain school is R460.
   Manare's mother pays with one R200 note, two R100 notes and three R20 notes.
   Elizabeth's father pays with four R100 notes, one R50 note and one R10 note.

   Describe three other ways in which R460 can be made up from different banknotes.

Teaching guidelines

You can use the worked examples in question 3 to show learners how to write their answers as number sentences. You can ask learners: *"How much more than 1 000 is 1 200?"* You can do at least one example to show learners how to use their answers in question 3(a) to get answers in 3(b) by adding 200 to their answers in 3(a). In general it is important that you help learners to try to think about what they already know that can help them with what they are doing. Learners think about: *"What do I already know that can help me here?"*, particularly in questions 4, 7(b) and (c), 8, 9, 10, 11 and 12.

You may suggest to learners that are challenged by question 4 to use place value cards to make the initial numbers in questions 4(b), (c) and (e) to (k). Learners can then remove the cards that show the subtracted amounts. For example, in 4(e):



Once learners have completed question 4(c): 4 327 – 4 000 = 327, they can use this to find 4(d): 4 327 – 2 000 = 2 327. In 4(d) they subtracted 2 000 less than in 4(c), so the answer must be 2 000 more. They will use this kind of reasoning with pairs of questions in questions 9 and 10.

Possible misconceptions

If some learners find question 3 difficult, let them first write number sentences that add up to 10 and 12, and then adapt their answer to 1 000 and 1 200.

Check to see that learners are using zero as a placeholder to correctly write the answers to questions 4(b) and (e) to (k). For example, in 4(e) the answer is 4 007 and not 47. Working with the place value cards will help learners to understand this.

Answers

3. (a) Example: 100 + 200 + 700; 100 + 400 + 500
   (b) Example: 700 + 300 + 200; 600 + 500 + 100; 500 + 400 + 300
4. (a) 283 (b) 503 (c) 327 (d) 2 327
   (e) 4 007 (f) 6 024 (g) 6 004 (h) 724
   (i) 6 020 (j) 900 (k) 5 032 (l) 1 111
   (m) 10 (n) 6 997
5. (a) No, it adds up to R900.
   There are many possible answers to 5(b) and (c). One solution to each is provided below.
   (b) Three R200 notes, five R20 notes, five R10 notes
   (c) Two R200 notes, nine R20 notes, seventeen R10 notes

3. 1 000 can be formed by adding up three different multiples of 100. For example:
   1 000 = 300 + 200 + 500 and 1 000 = 600 + 100 + 300.
   (a) Describe two other ways in which 1 000 can be formed by adding up three different multiples of 100.
   (b) Describe three different ways in which 1 200 can be formed by adding up different multiples of 100.
4. How much is each of the following?
   (a) 583 – 300 (b) 583 – 80
   (c) 4 327 – 4 000 (d) 4 327 – 2 000
   (e) 4 327 – 320 (f) 6 724 – 700
   (g) 6 724 – 720 (h) 6 724 – 6 000
   (i) 6 724 – 704 (j) 5 932 – 5 032
   (k) 5 932 – 900 (l) 5 932 – 4 821
   (m) 6 287 – 6 277 (n) 7 023 – 26
5. (a) Is it true that three R200 notes, two R100 notes, one R50 note, two R20 notes and one R10 note make up R1 000?
   (b) How can R750 be made up with R200 notes, R20 notes and R10 notes?
   (c) Describe a different way to make up R750 with R200 notes, R20 notes and R10 notes.
6. Daniel has R200 notes, R100 notes, R50 notes, R20 notes and R10 notes.
   Describe two different ways in which Daniel can make up R800 with the notes, using at least one of each kind of note he has. Use number sentences to describe the two ways.
7. (a) Willem has R1 000. He spends R300. How much is left?
   (b) Sarah has R7 000. She spends R3 000. How much is left?
   (c) Bongi has R3 000. She needs R7 000. How much money is she short of?

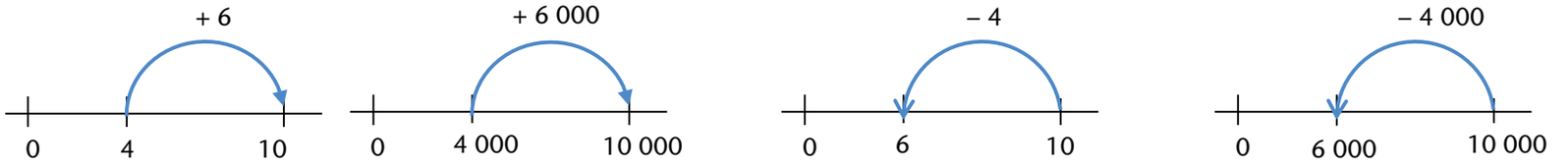
Answers (*continued*)

6. A range of solutions are possible. Two sample solutions are:
   R200 + R200 + R200 + R100 + R50 + R20 + R20 + R10 = R800
   R200 + R200 + R100 + R100 + R50 + R50 + R50 + R20 + R10 + R10 + R10 = R800
7. (a) R700 (b) R4 000 (c) R4 000
   Note that learners can use the answer in 7(b) to get the answer to 7(c).

Teaching guidelines

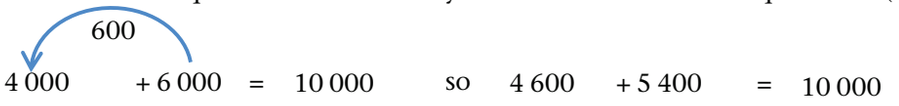
Most of the strategies required on page 130 were developed in Term 1 Unit 3 Section 3.4. Learners are now applying what they know to a higher number range. You can refer back to the tinted passages on pages 34 and 35.

Encourage learners to ask themselves: "*What do I already know that can help me to answer this?*"
For example, in 8(a) learners know that 4 + 6 = 10 and 10 – 4 = 6, therefore 4 *thousand* + 6 *thousand* = 10 *thousand*, and 10 *thousand* – 4 *thousand* = 6 *thousand*.



This reasoning will help learners with questions 9(a), (c) and (g), and questions 10(a) and (c).

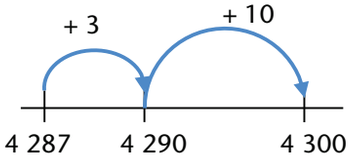
Check that learners understand how much each small space represents on the number lines in question 8 (each small space represents 100). You can help them to see that the two detailed number lines in question 8 show how you can use the answer to question 8(a) to get the answers to question 8(b). This can be done using transfer, i.e.



The idea of transfer can be used as a support for answering questions 9(b), (d), (h), (i) and (j), and questions 10(b), (d) and (f).

In question 11 learners can use filling up to the nearest multiples of 10, 100 and 1 000.
They can show this with number lines or arrow diagrams. For example:

$4\,287 + 3 \rightarrow 4\,290 + 10 \rightarrow 4\,300$ or



This strategy is further developed in Section 2.2. You can read the tinted passage on page 131.

In question 12(b) learners should look back to see which sub-questions in question 11 can help them to see the answer immediately without calculating. If learners struggle to find these sub-questions, ask them to rewrite questions 12(a) and (b) as addition number sentences.

Answers

8. (a) 6 000 m (b) 5 400 m

9. (a) 6 000 (b) 5 400
   (c) 4 000 (d) 4 000
   (e) 4 000 (f) 3 365
   (g) 4 000 (h) 4 010
   (i) 3 990 (j) 2 990

10. (a) 10 000 (b) 10 000
    (c) 10 000 (d) 10 000
    (e) 9 000 (f) 10 000
    (g) 7 000 (h) 6 100

11. (a) 13 (b) 310
    (c) 376 (d) 2 376
    (e) 937 (f) 2 937
    (g) 3 337 (h) 3 374

12. (a) 2 376 (b) 3 374

8. (a) Lea has already run 4 000 m of a 10 000 m race. How far does she still have to run?



   (b) Ben has already run 4 600 m of a 10 000 m race. How far does he still have to run?



9. How much is each of the following? Try to be smart and find the answers with as little work as possible.
   (a) 10 000 – 4 000 (b) 10 000 – 4 600
   (c) 7 000 – 3 000 (d) 7 500 – 3 500
   (e) 7 267 – 3 267 (f) 8 365 – 5 000
   (g) 6 000 – 2 000 (h) 6 000 – 1 990
   (i) 6 000 – 2 010 (j) 6 000 – 3 010

10. Calculate each of the following.
    (a) 4 000 + 6 000 (b) 4 600 + 5 400
    (c) 7 000 + 3 000 (d) 6 800 + 3 200
    (e) 5 000 + 4 000 (f) 5 300 + 4 700
    (g) 3 000 + 4 000 (h) 2 060 + 4 040

11. Find the missing number in each case. You can do it in steps, and use arrows to show your thinking if you wish.
    (a) 4 287 + . . . = 4 300 (b) 4 690 + . . . = 5 000
    (c) 5 624 + . . . = 6 000 (d) 5 624 + . . . = 8 000
    (e) 3 063 + . . . = 4 000 (f) 3 063 + . . . = 6 000
    (g) 3 063 + . . . = 6 400 (h) 3 063 + . . . = 6 437

12. How much is each of the following? You can use the work you did in the previous question.
    (a) 8 000 – 5 624 (b) 6 437 – 3 063

2.2 Methods to add and subtract

Mathematical notes

Three calculation strategies are shown in this section:

- Adding on to reach the next multiple of 10, 100 and then 1000.
- Adding to subtract (this uses the inverse relationship between addition and subtraction.
- Adding on by breaking down the number added into place value parts.

Teaching guidelines

Aim to complete Section 2.2 in 2 hours.

Focus on adding by filling up and subtraction by adding on in one lesson. In the second lesson you can do addition by adding on place value parts.

One possibility is to use

- the tinted passage on page 131, along with question 1, and the first tinted passage on page 132, along with question 3(a), for concept development during the *first* lesson,
- questions 2 and 3(b) to (f) for classwork during the *first* lesson,
- the second tinted passage on page 132 for concept development during the *second* lesson, and
- questions 4, 5, 6, 7 and 8 for classwork during the *second* lesson.

Explain how to use both arrow number sentences and number lines when recording how to fill up to multiples of 10, 100 and 1 000. In the example in question 1, check that learners understand that each small space represents 10 on the number line.

Answers

1. (a) 2 (b) 30 (c) 200 (d) 244
   (e) The red lines represent 6 768 and 7 244. They are 476 apart.
   [Step 1 (+**2**) + Step 2 (+**30**) + Step 3 (+**200**) + Step 4 (+**244**) = **476**]

2. (a) 5 876 + 4 → 5 880 + 20 → 5 900 + 100 → 6 000 + 344 = 6 344
   (b) 7 783 + 7 → 7 790 + 10 → 7 800 + 200 → 8 000 + 456 = 8 456
   (c) 3 364 + 6 → 3 370 + 30 → 3 400 + 411 → 3 811
   (d) 4 849 + 1 → 4 850 + 50 → 4 900 + 100 → 5 000 + 482 = 5 482

2.2 Methods to add and subtract

Add by filling up multiples of 10 and 100 and 1 000

This method is useful when one number is smaller than the other one, for example when 6 768 + 476 has to be calculated.

In this case, part of the second number is added to reach the next multiple of 10:
6 768 + 2 → 6 770 (**Step 1**)

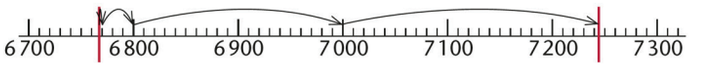
Since 476 = 2 + 474, we must still add 474.
Part of the 474 is now added on to reach the next multiple of 100:
6 768 + 2 → 6 770 + 30 → 6 800 (**Step 2**)

Since 474 = 30 + 444, we must still add 444.
Part of the 444 is added on to reach the next multiple of 1 000:
6 768 + 2 → 6 770 + 30 → 6 800 + 200 → 7 000 (**Step 3**)

Since 444 = 200 + 244, there is still 244 to be added:
6 768 + 2 → 6 770 + 30 → 6 800 + 200 → 7 000 + 244 = 7 244 (**Step 4**)

If you look back at the steps, you will notice that the 476 was broken down into four parts:
2 + 30 + 200 + 244 = 476

1. (a) What part of 476 was added on to 6 768 in Step 1?
   (b) What part of 476 was added on in Step 2?
   (c) What part of 476 was added on in Step 3?
   (d) What part of 476 was added on in Step 4?
   (e) The steps are shown on this number line. What are the numbers at the two red lines, and how far apart are they?



2. Use the filling-up method to calculate the following.
   (a) 5 876 + 468 (b) 7 783 + 673
   (c) 3 364 + 447 (d) 4 849 + 633

Teaching guidelines

When you buy goods at a shop without an electronic till, such as a spaza shop, mobile shop or school tuck shop, you will experience *adding on to subtract* in real life.

You can start by role-playing this kind of situation before showing learners how to record it with arrow number sentences or on number lines. You can give one learner a script that explains she should give her daughter R20 (play money on scrap paper) and ask her to buy a litre of milk at the spaza shop. Have some play money coins (circles cut from scrap paper) ready.

At the shop

Learner: 1 litre of milk, please.

Learner gives R20.

Shopkeeper: That will be R12.

Shopkeeper: *Gives R1. Says:* R13

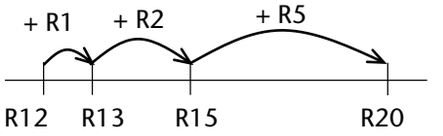
Shopkeeper: *Gives R2. Says:* R15

Shopkeeper: *Gives R5. Says:* R20

Recording with an arrow number sentence:

R12 + R1 → R13 + R2 → R15 + R5 → R20

Recording on a number line:



Then demonstrate these ways of recording using question 3(a) or other 4-digit numbers.

Answers

Learners should first check their answers to question 3 by doing question 5 before you mark them.

3. (a) 2 875 + 5 →
2 880 + 20 →
2 900 + 100 →
3 000 + 4 236
= 7 236

5 + 20 + 100 + 4 236 = 4 361

7 236 – 2 875 = 4 361

(b) 2 218 (c) 7 766 (d) 2 595 (e) 4 223 (f) 3 123

Subtract by adding on

Subtraction can also be done by filling up multiples of 10, 100 and 1 000.

For example, 7 234 – 4 876 can be calculated by finding out how much should be added to 4 876 to reach 7 234, as shown below.

We add on in steps from 4 876 until we reach 7 234, and then check how much we had to add on in total:

4 876 + **24** → 4 900 + **100** → 5 000 + **2 234** = 7 234.

In total we added on 24 + 100 + 2 234 which is 2 358.

So, 4 876 + 2 358 = 7 234 and 7 234 – 4 876 = 2 358.

3. Use the add-on method of subtraction to calculate the following.
   (a) 7 236 – 2 875 (b) 6 721 – 4 503
   (c) 9 000 – 1 234 (d) 8 187 – 5 592
   (e) 7 386 – 3 163 (f) 8 396 – 5 273

Add by adding on place value parts

To calculate 3 465 + 4 574 the number that is to be added can be broken down into its place value parts. The place value parts can then be added one by one to the first number:

4 574 = 4 000 + 500 + 70 + 4

So, 3 465 + 4 574 = 3 465 + 4 000 + 500 + 70 + 4 and it can be calculated like this:

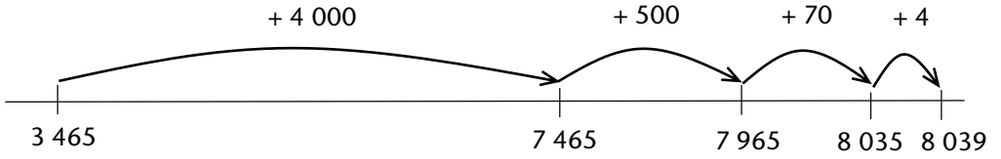
3 465 + 4 000 = 7 465
7 465 + 500 = 7 965
7 965 + 70 = 8 035
8 035 + 4 = 8 039

The work can also be shown like this:

3 465 + 4 000 → 7 465 + 500 → 7 965 + 70 → 8 035 + 4 = 8 039

Teaching guidelines

You can use the second tinted passage on page 132 and a number line to show learners how to break the number that is to be added down into place value parts and add them on one by one.



Questions 3 and 5 allow learners to think about and reflect on the calculation strategies. Once learners have done these questions they should discuss their answers and methods.

Learners have already learnt that the order in which you add two numbers does not affect the answer. This is also true if more than two numbers are added. Questions 6 and 8 will demonstrate this to learners. Consolidate this through discussion and reflection.

Answers

4. (a) 4 628 + 2 000 → 6 628 + 700 → 7 328 + 70 → 7 398 + 5
   = 7 403
   (b) 4 775 + 2 000 → 6 775 + 600 → 7 375 + 20 → 7 395 + 8
   = 7 403
5. Learners check their answers for question 3 by adding on place value parts.
6. No, it does not matter.
   (a) 3 465 + 4 → 3 469 + 70 → 3 539 + 500 → 4 039 + 4 000
   = 8 039
   (b) 3 465 + 70 → 3 535 + 4 → 3 539 + 4 000 → 7 539 + 500
   = 8 039
   (c) 3 465 + 500 → 3 965 + 4 000 → 7 965 + 4 → 7 969 + 70
   = 8 039
7. Learners write the instruction: 3 465 + 70 → . . . + 500 → . . . + 4 → . . . + 4 000 = . . .
8. Learners may choose different orders. The answers should, however, be the same.
   (a) Example: 5 374 + 4 → 5 378 + 20 → 5 398 + 800 → 6 198 + 2 000 = 8 198
   (b) Example: 5 374 + 2 000 → 7 374 + 20 → 7 394 + 800 → 8 194 + 4 = 8 198
   (c) Example: 6 785 – 3 000 → 3 785 – 200 → 3 585 – 40→ 3 545 – 1 = 3 544
   (d) Example: 6 785 – 200 → 6 585 – 1→ 6 584 – 40 → 6 544 – 3 000 = 3 544

4. Add on the place value parts of the second number to calculate the following:
   (a) 4 628 + 2 775 (b) 4 775 + 2 628
5. Check your answers for question 3 by adding in the way you just did in question 4.
6. When 3 465 + 4 574 is calculated, do you think it matters which of 4 000, 500, 70 and 4 is added to 3 465 first and which part is added next?

   Complete these calculations to check your answer:

   (a) 3 465 + 4 → . . . . + 70 → . . . . + 500 → . . . . + 4 000 = ...
   (b) 3 465 + 70 → . . . . + 4 → . . . . + 4 000 → . . . . + 500 = ...
   (c) 3 465 + 500 → . . . . + 4 000 → . . . . + 4 → . . . . + 70 = ...

In question 6(a) the instructions are to add 4 first, then 70, then 500 and then 4 000.

In question 6(c) the instructions are to add 500 first, then 4 000, then 4 and then 70.

7. Write like in question 6 to give the following instructions:
   Calculate 3 465 + 4 574 by first adding 70, then 500, then 4 and then 4 000.
8. (a) Calculate 5 374 + 2 824 by adding the second number in parts. You can add the parts in any order you prefer.
   (b) Calculate 5 374 + 2 824 again but now add the parts of the second number in a different order.
   (c) Calculate 6 785 – 3 241 by subtracting the place value parts of 3 241 one by one.
   (d) Calculate 6 785 – 3 241 by subtracting the place value parts of 3 241 in a different order.

2.3 Round off and estimate

Mathematical notes

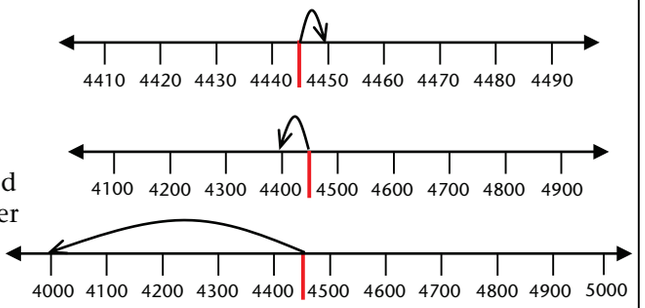
In this section learners use rounding off and estimating as a calculation strategy. In Term 1 Unit 3 Section 3.7 learners rounded off 3-digit numbers to the nearest 10 and 100. (You might like to read those "Teaching guidelines".) Now for the first time they are rounding off to the nearest thousand.

Teaching guidelines

As in Term 1 Unit 3, rounding off is used as a calculation aid. Aim to complete Section 2.3 in 1 hour. One possibility is to use

- the tinted passage and questions 1(a), 2(a), 3(a) and 4 for concept development,
- questions 1(b) to (g), 2(b) to (g), 3(b) to (g), 5(a) and (b), and related parts of questions 6, 7, 8 and 9 for classwork, and
- questions 5(c) and (d), and related parts of questions 6, 7, 8 and 9 for additional practice.

Continue to stress that rounding to the nearest multiple involves seeing which multiple is closest to that number, but that numbers halfway between are rounded up. Number lines show which multiple is closest. You can use the tinted passage on page 134, but also use number lines. For example, the number lines alongside show 4 445 rounded to the nearest 10, 100 and 1 000.



Answers

1. (a) 6 000 (b) 7 000 (c) 4 000 (d) 4 000
   (e) 3 000 (f) 3 000 (g) 7 000 (h) 7 000
   (i) 8 000 (j) 7 000 (k) 6 000 (l) 6 000
2. (a) 6 500 (b) 6 500 (c) 4 450 (d) 4 450
   (e) 3 240 (f) 3 230 (g) 7 250 (h) 7 250
   (i) 7 500 (j) 7 500 (k) 6 010 (l) 6 020
3. (a) 6 500 (b) 6 500 (c) 4 500 (d) 4 400
   (e) 3 200 (f) 3 200 (g) 7 200 (h) 7 300
   (i) 7 500 (j) 7 500 (k) 6 000 (l) 6 000
4. (a) R4 000 – R2 000 = R2 000 (b) R4 400 – R2 400 = R2 000

2.3 Round off and estimate

Numbers can be rounded off to make estimates of the answers for calculations before you do the calculations accurately.

3 567 rounded off to the nearest 10 is 3 570
3 567 rounded off to the nearest 100 is 3 600
3 567 rounded off to the nearest 1 000 is 4 000

3 565 rounded off to the nearest 10 is 3 570
3 564 rounded off to the nearest 10 is 3 560

3 550 rounded off to the nearest 100 is 3 600
3 549 rounded off to the nearest 100 is 3 500

3 500 rounded off to the nearest 1 000 is 4 000
3 499 rounded off to the nearest 1 000 is 3 000

1. Round each of these numbers off to the nearest 1 000, in the same way as in the above examples.
   (a) 6 499 (b) 6 500
   (c) 4 450 (d) 4 449
   (e) 3 235 (f) 3 234
   (g) 7 249 (h) 7 250
   (i) 7 500 (j) 7 499
   (k) 6 008 (l) 6 015
2. Round each number in question 1 off to the nearest 10.
3. Round each number in question 1 off to the nearest 100.
4. Jana has already paid back R2 386 of the R4 437 she borrowed from Petra.
   (a) Round the numbers off to the nearest 1 000 and make an estimate of how much Jana still has to pay back.
   (b) Also make an estimate by rounding off to the nearest 100.

Mathematical notes

When we round off numbers before calculating, our answers will not be accurate. The difference between the accurate answer and the approximate answer is called the estimation error.

If you round off to the nearest 1 000, you will have a larger estimation error than if you round off to the nearest 100. If you round off to the nearest 100, you will have a larger estimation error than if you round off to the nearest 10.

Answers

5. (a) 4 000 litres + 5 000 litres = 9 000 litres
   (b) 9 000 packets – 2 000 packets = 7 000 packets
   (c) 9 000 bricks – 2 000 bricks = 7 000 bricks
   (d) 7 000 metres – 6 000 metres = 1 000 metres

6. (a) 3 500 litres + 4 900 litres = 8 400 litres
   (b) 8 600 packets – 2 000 packets = 6 600 packets
   (c) 8 700 bricks – 2 400 bricks = 6 300 bricks
   (d) 7 300 metres – 5 700 metres = 1 600 metres

7. (a) 3 520 litres + 4 850 litres = 8 370 litres
   (b) 8 560 packets – 2 050 packets = 6 510 packets
   (c) 8 680 bricks – 2 360 bricks = 6 320 bricks
   (d) 7 350 metres – 5 680 metres = 1 670 metres

8. (a) 8 376 (b) 6 516 (c) 6 318 (d) 1 666 m

9.

Question 5	(a)	(b)	(c)	(d)
Error when rounding to the nearest 1 000	624 more	484 more	682 more	666 less

Question 6	(a)	(b)	(c)	(d)
Error when rounding to the nearest 100	24 more	84 more	18 less	66 less

Question 7	(a)	(b)	(c)	(d)
Error when rounding to the nearest 10	6 less	6 less	2 more	4 more

5. Give *approximate answers* for the questions below, by first rounding off the numbers to the nearest 1 000.
   (a) Selina uses 3 524 litres of water from the tank to water her small field of maize. If there is then 4 852 litres water left, how much water was in the tank?
   (b) A store sold 8 563 packets of chips during a month. Of these, 2 047 were sold during the last week. How many packets were sold during the first three weeks of the month?
   (c) Willem has to lay 8 675 bricks. So far he has laid 2 357 bricks. How many bricks must he still lay?
   (d) Lerato walked 5 683 m. In the same period of time, Sipho walked 7 349 m. How much further than Lerato did Sipho walk?
6. Give *better approximate answers* to the above questions, by rounding off the numbers to the nearest 100.
7. Give *even better approximate answers* to the above questions, by rounding off the numbers to the nearest 10.
8. Calculate the *accurate answers* to the above questions.
9. Find out by how much your estimates in question 5 differed from your accurate answers. Report your errors in a table like the one below.

   These differences are called the **estimation errors**.

   Also find the estimation errors for your estimates in question 6 and in question 7, and report them in your table.

Question	(a)	(b)	(c)	(d)
5. Error when rounding to the nearest 1 000				
6. Error when rounding to the nearest 100				
7. Error when rounding to the nearest 10				

Grade 4 Term 2 Unit 3 Common fractions

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Equal sharing into fractions	Dividing into equal parts, fraction names, representing fractions with sketches	136 to 138
3.2 Naming fractions	Naming fractions	139 to 141
3.3 Comparing fractions	Comparing and ordering fractions	142 to 143
3.4 Using fractions to measure	Fractions as measurements. Introducing equivalent fractions	144 to 146

CAPS time allocation	6 hours
CAPS page references	16 and 71 to 72

Mathematical background

It is widely assumed that fractions were invented for accurate measurement, to facilitate the measuring of objects or parts of objects smaller than the commonly used unit of measurement. This is reflected in the Latin names of our current units of measurement, for example centimetres (hundredths of a metre) and millimetres (thousandths of a metre). If the brown strip below is measured with the yellow strip as a unit, its length is 3 and 3 fifths of the yellow unit (5 green units are equal to 1 yellow unit). This example demonstrates how **fractions are used as measures**.



Mathematically, the fraction concept is critical to the understanding of decimals, because the place value parts after the decimal comma are fractions. The expanded notation for the number 23,47 is $20 + 3 + \frac{4}{10} + \frac{7}{100}$ or 2 tens + 3 units + 4 tenths + 7 hundredths. A good understanding of common fractions will lay a sound basis for decimal fractions when learners work with them in Grade 6. Decimal fractions are not a topic in Grade 4. Grade 4 learners only work with decimals in the context of measurement.

Fractions are also used to describe **parts of collections** and parts of non-physical quantities, for example “3 eighths of the learners in a school” or “63 hundredths of the available marks”. In a case like the latter, the percentage notation (% for hundredths) is normally used.

In everyday life and language certain fractions, such as “half” and “quarter”, are sometimes used to indicate approximate **parts of whole objects**. People may, for example, refer to a “quarter of an apple” or “a half-loaf of bread”. Although this everyday use of fraction language differs from the mathematical use in the sense that the fraction words are not used to indicate precise parts, the everyday use provides a starting point for learning about fractions. You can also use what learners know about fractions of time, for example the position of the clock hands at a quarter past the hour, half past the hour, and quarter to the hour.

A fraction is a number of equal parts of the same object or measurement unit, for example 7 hundredths of a metre.

Learners’ concept of fractions is often undermined by the idea that a fraction consists of two numbers, for example that “3 fifths” is made up of the numbers 3 and 5, without understanding the entirely different roles of the two numbers. This misconception is often supported by misleading language, such as referring to $\frac{3}{5}$ as “three over five” instead of “3 fifths”. The use of the proper fraction name, for example “3 fifths”, is essential.

3.1 Equal sharing into fractions

Teaching guidelines

This is a long unit. You could plan to spend 2 hours on Section 3.1. One possible way to organise the section is to use

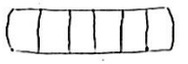
- questions 1, 2, 3, 11 and 12 for concept development (you can use questions 1 and 2 to assess whether learners remember how to name fractions),
- questions 4, 5 and 7 for classwork, and
- questions 6, 8, 9 and 10 for additional practice.

In Grade 4 learners are *not* expected to learn arithmetic rules and procedures for how to add or subtract fractions, how to make equivalent fractions or how to divide wholes into fraction parts. The expectation is rather that learners work with apparatus, diagrams and visual images. This is reflected in the solutions modelled in this unit.

Notes on questions

It is really important that each learner makes their own drawing in question 2 to show how a sausage can be cut into six equal parts. Making the drawing and calling each part a sixth provides the learner with an opportunity to consolidate understanding of the relationship between fractions and dividing into equal parts.

While it is important that learners think about dividing into six equal parts when they make the drawing, there is no need to make an accurate drawing. It is best to suggest to learners that they do not use a ruler, but just try to make a neat drawing.



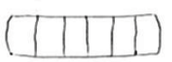
Critical knowledge and skills

It is critical that learners acquire the skill to demonstrate their understanding of fractions with simple drawings that they can make quickly (fraction strips).

This skill will prove valuable when learners work at tasks relating to fractions, for example to find equivalent fractions. Making their own drawings to illustrate fractions will also empower learners to follow your explanations, or the explanations in the textbook, where fractions concepts are explained through different kinds of diagrams.

Answers

1. One fifth of the loaf
2. One sixth of the sausage



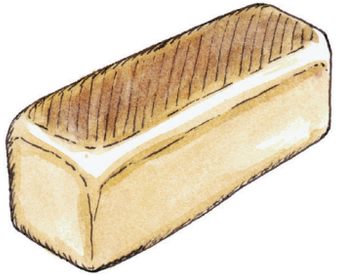
UNIT 3 COMMON FRACTIONS

3.1 Equal sharing into fractions

In this section, we will look at sharing bread, oranges and peanuts in different ways.

Note: Do not use symbols to write the answers. Use *words* to describe the parts in your answers.

1. Five children share this loaf of bread equally.
   "Equally" means that every child gets the same.



What can we call the part that each child gets?

To share the loaf of bread in this way, it is divided into **five equal parts**.

Each of these parts is called **one fifth** of the loaf.

If a loaf of bread or some other object is divided into seven equal parts, each part is called **one seventh** of the whole.

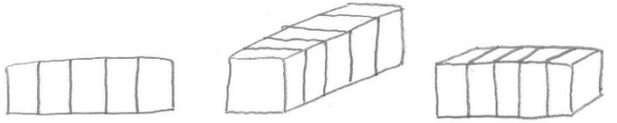
2. A whole sausage is shared between six children so that every child gets the same.
   What part of the sausage does each child get?
   Make a drawing to show how the sausage must be cut.

Teaching guidelines

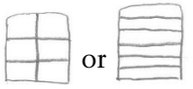
Ensure that learners do not spend inordinate time to try to make neat, pretty sketches. The making of sketches here is just a vehicle for thinking. Sketches like the ones below are good enough.

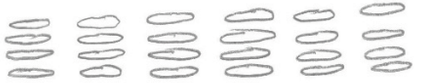
Answers

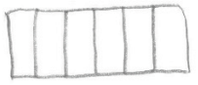
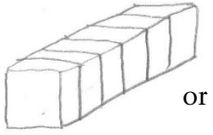
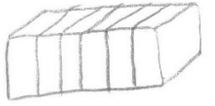
3. Learners may draw it in different ways, for example:

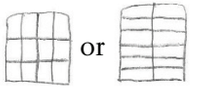


4. Learners may draw it in different ways, for example:

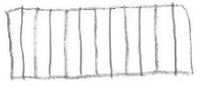
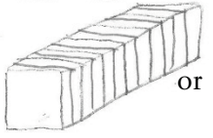
(a)  or

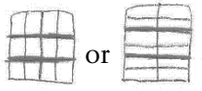
(c) 

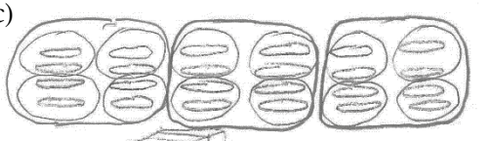
(b)  or  or 

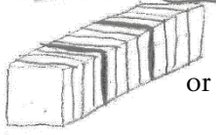
5. (a)  or

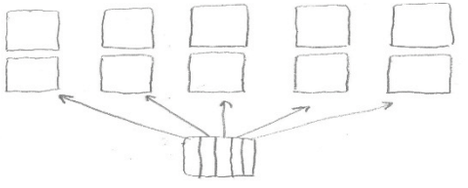
(c) 

(b)  or  or 

6. (a)  or

(c) 

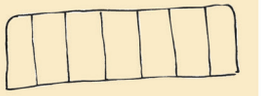
(b)  or  or 

7. 

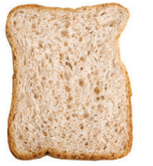
3. Five children share a loaf of bread equally. Nothing is left over. Make a *rough sketch* to show how they can do it.

Remember, to share **equally** means that everyone gets the same.

This rough sketch shows how a loaf of bread can be cut to share it equally between seven people.



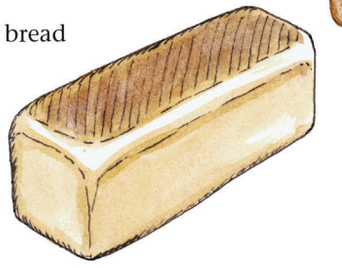
4. Make *rough sketches* to show how each of the following can be shared equally between six children:



(a) one slice of bread

(b) one whole loaf of bread

(c) 24 peanuts





5. Change the sketches you have made to now show how each of the following can be shared equally between 12 children:

(a) one slice of bread (b) one whole loaf of bread

(c) 24 peanuts

6. Change your sketches again to show how each of the following can be shared equally between 3 children:

(a) one slice of bread (b) one whole loaf of bread

(c) 24 peanuts

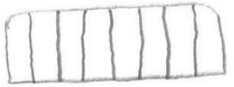
7. Five people share 11 slices of bread. Every person must get the same. Nothing is left over. Make a rough sketch to show how they can do it.

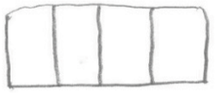
Teaching guidelines

Suggest to learners that for question 8 they make simple sketches like the example in the tinted passage on page 137 of the Learner Book.

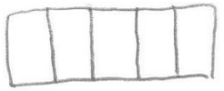
Answers

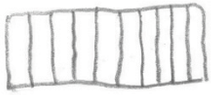
8. (a) 8 equal parts (b) 4 equal parts





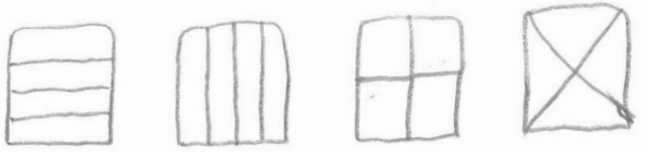
(c) 5 equal parts (d) 10 equal parts





9. Photograph B: quarters Photograph D: eighths

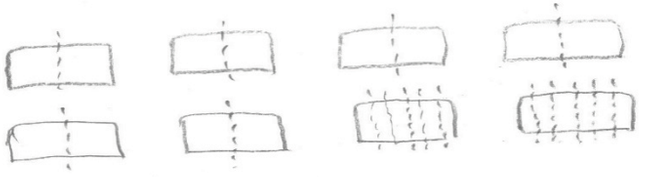
10.



11. Half a loaf and another eighth of a loaf



12. Half a loaf and another sixth of a loaf



Some learners may divide each of the last two loaves into 12 slices and give each person 2 out of 12 slices. The answer: half a loaf and 2 twelfths of a loaf is also acceptable.

8. Make rough sketches to show how each of these equal sharings can be done:

   (a) 8 children share a loaf of bread.

   (b) 4 children share a loaf of bread.

   (c) 5 children share a loaf of bread.

   (d) 10 children share a loaf of bread.

9. In Photograph A an orange is cut into halves. In Photograph C an orange is cut into eighths.

   Into what fraction parts is the orange cut in Photographs B and D?



Photograph A



Photograph B



Photograph C



Photograph D

10. Make rough sketches to show different ways in which a slice of bread can be cut into quarters.

11. 8 people share 5 loaves of bread equally. How much bread does each person get? You can make a drawing to help you.

12. 12 people share 8 loaves of bread equally. How much bread does each person get?

3.2 Naming fractions

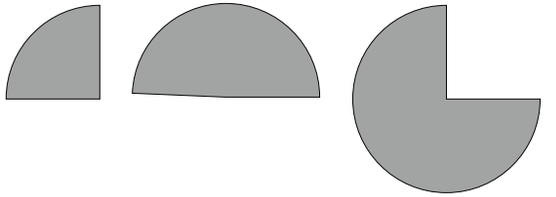
Critical knowledge

It is critical that learners know that 1 whole equals 2 halves, 3 thirds, 4 quarters, 5 fifths, 6 sixths, 7 sevenths, 8 eighths, etc.

It is critical that learners learn to pronounce the fraction names. Taking verbal feedback from many learners for question 1 will promote this.

Possible misconceptions

Learners sometimes acquire the damaging misconception that a fraction is a name for a certain shape. For example, learners may have the idea that a quarter, a half and three quarters are always shapes like those shown alongside.



This misconception arises when there is an overemphasis on cutting up circular objects such as cakes or pizzas when learners are introduced to fractions. To combat this misconception, a variety of other contexts for fractions are used in this unit, including loaves of bread, spherical objects (oranges), collections of objects and measurement (fractions as smaller units of measurement). The photographs on page 138 show eighths in different ways, and the photographs on page 140 show halves and quarters in different ways. The solutions in Section 3.1 questions 3, 4, 5 and 6 also show different representations of common fractions.

Teaching guidelines

In question 1 you could draw the loaves on the board.

You could ask learners to count in halves as they look at a diagram showing halves: 1 half, 2 halves. Stress that 2 halves is 1 whole. Repeat this for thirds, quarters, fifths, sixths, sevenths and eighths.

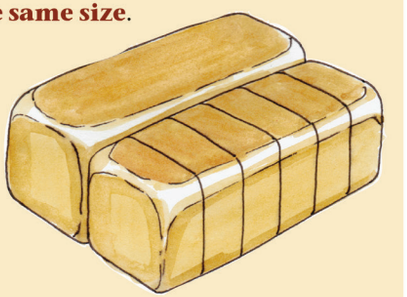
Answers

1. (a) fifths (b) thirds (c) tenths
2. 7 pieces
3. One fifth. If the cake is cut into 5 equal pieces, each piece will be larger than when it is cut into 6 pieces. If 5 people share a cake equally, the pieces will be bigger than when 6 people share the same cake.

3.2 Naming fractions

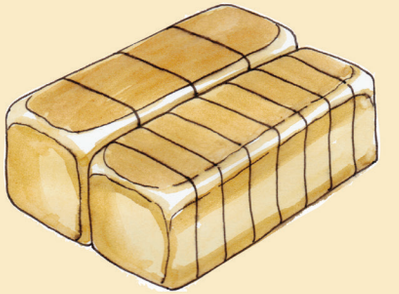
Equal parts are parts of **the same size**.

If we cut a loaf of bread into six equal parts, each part is called **one sixth** of a loaf.



If we divide something into four equal parts, each part is called **one fourth** (or **one quarter**) of the whole.

If we divide something into eight equal parts, each part is called **one eighth** of the whole.



1. What can we call each part in the following cases?
   (a) The loaf is divided into five equal parts.
   (b) The loaf is divided into three equal parts.
   (c) The loaf is divided into ten equal parts.
2. A cake is divided into equal pieces and each piece is a seventh of the whole cake. How many pieces are there?
3. Which is greater, one sixth of a cake or one fifth of the same cake? Do not use a drawing to explain your answer.

GRADE 4: MATHEMATICS [TERM 2] 139

Mathematical notes

Questions 4 and 5, in conjunction with questions 6, 7 and 8 on the next page, are intended to promote the understanding that an object can be divided in different ways into the same fraction parts.

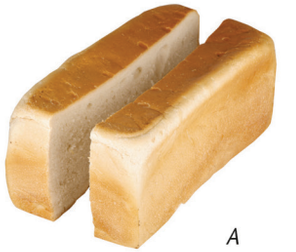
Teaching guidelines

Stress that we get halves when we divide a whole into two equal parts and that there are many ways to show halves. Stress that we get quarters when we divide a whole into four equal parts and that there are many ways to show quarters. Give each learner three pieces of scrap paper and ask them to show quarters in different ways by folding or cutting the paper.

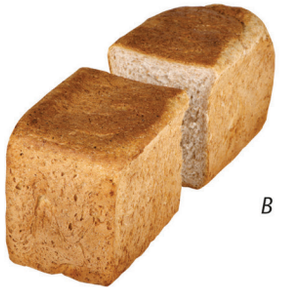
Answers

4. No
5. Yes

Three different ways of halving a loaf of bread are shown by Photographs A, B and C.



A



B

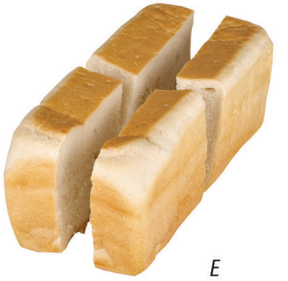


C

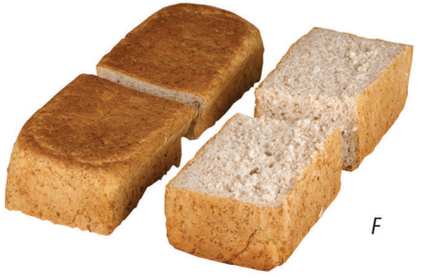
4. Can the halves in Photograph B be cut to form smaller parts such as those shown in Photograph D?
5. Can the halves in Photograph B be cut to form smaller parts such as those shown in Photographs E and F?



D



E



F

Teaching guidelines

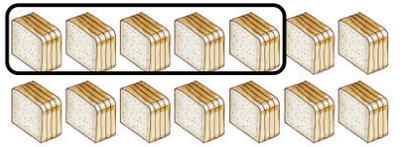
The stacks in question 9 refer to the groups of 4 slices of bread.

In question 9(a) you can ask learners to count in fifths to 5 fifths as they point to each successive stack of 4 slices of bread. In question 9(b) learners are only expected to add (1 stack plus 2 stacks is 3 stacks) or count (1 stack, 2 stacks, 3 stacks), and translate this into 3 fifths.

Many learners do not connect different parts of mathematics. As a teacher it is important that you train learners to do this. Help learners to develop the practice of thinking: "*What have I learnt before that can help me here?*" Some learners do not even connect different questions with each other. Question 9 on page 141 lays the basis for questions 4 and 5 on page 143.

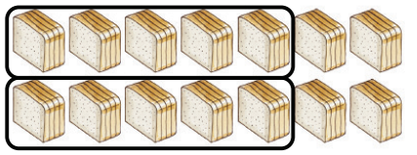
Answers

6. quarters
7. Photograph A or Photograph B
8. Photograph B
9. (a) Learners can count the stacks as fifths:
   1 fifth, 2 fifths, 3 fifths, 4 fifths, 5 fifths *or*
   1 whole is equal to 4 slices, 8 slices,
   12 slices, 16 slices, 20 slices

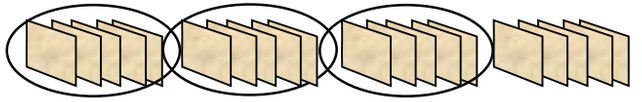


   (b) 3 fifths

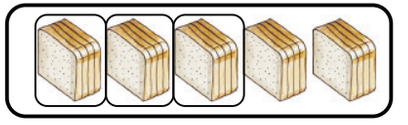
   (c) 2 loaves with 4 fifths left over



   (d) From question (a), the whole loaf is 20 slices. So 1 quarter is 5 slices.
   So 3 quarters is 15 slices.



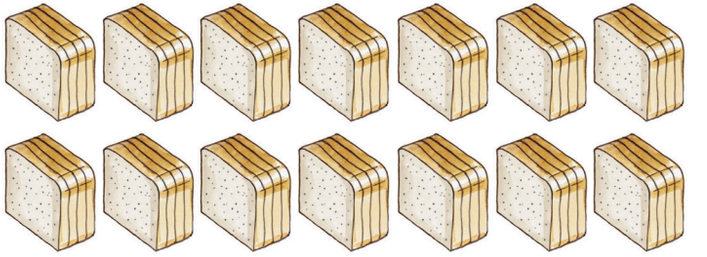
   (e) From question (a), the whole loaf is
   20 slices, and 1 fifth is 4 slices.
   So 3 fifths is 3 × 4 slices = 12 slices.
   From question (d), 3 quarters is 15 slices.
   So 3 quarters is more than 3 fifths.



   (f) 4 quarters (of a loaf) makes 1 whole (loaf), so 8 quarters is 2 whole loaves.

   (g) 4 quarters (of a loaf) makes 1 whole (loaf), so 20 quarters is 5 whole loaves.

6. What fraction parts of a whole loaf of bread are shown in Photographs D, E and F?
7. Which halves can be cut to form smaller parts such as those shown in Photograph E?
8. Which halves can be cut to form smaller parts such as those shown in Photograph F?
9. (a) How many slices are there in one whole loaf, if each of these stacks is one fifth of a whole loaf?



   (b) Susan ate one of the above stacks of slices in the morning and two of them in the evening. What part of a loaf did she eat, altogether?

   (c) How many full loaves can be made up by putting these stacks together, and what part of a full loaf will be left over?

   (d) How many slices is 3 quarters of the whole loaf?

   (e) Which is more, 3 quarters of the whole loaf or 3 fifths of the whole loaf?

   (f) How many whole loaves can be made by putting 8 quarters together?

   (g) How many whole loaves can be made by putting 20 quarters together?

3.3 Comparing fractions

Mathematical notes

This section focuses on fractions as parts of collections. It also addresses the connection between fractions and division as sharing. Note that the connection between fractions and division as grouping is more complicated and is not addressed here.

Critical knowledge

If we divide a whole into more parts, each part is smaller. If we take a smaller fraction of the *same whole*, then we get less (see Section 3.2 question 3: If a cake is cut into 5 pieces, each piece will be bigger than if the same sized cake is cut into 6 pieces).

However, if the wholes are *not* the same then the smaller fraction is not necessarily less. For example, in question 2(b) of this section: a quarter of R100 is more than half of R40.

Answers

1. (a) The whole group is 15 people.



1 fifth of 15 people is 3 people.



3 fifths of 15 people is 9 people.



(b) Make 3 groups from 15, i.e. 15 ÷ 3 = 5



(c) From question 1(b), 1 third of 15 people is 5 people.

(d)  5 groups

(e) 4 people. In question 1(d), 20 people were divided into 5 groups. Each group of 4 people is 1 fifth of the group of 20 people.

2. (a) A quarter of R40 is R10, half of R100 is R50. Lizzie spent more money.
   (b) A quarter of R100 is R25, half of R40 is R20. Maggie spent more money.
   (c) A quarter of R40 is R10, half of R40 is R20. Lizzie spent more money.

3.3 Comparing fractions

If the teacher divides a group of 15 learners into five smaller, equal groups, each small group is **one fifth** of the whole group.

Can you see it in the picture?



1. (a) How many are three fifths of 15 people?
   (b) How many are one third of 15 people? Show how you can use division to find the answer.
   (c) 15 people are divided into five equal groups. How many people are in each group? Show how you can use a fraction to find the answer.
   (d) 20 people are divided into groups of 4 each. How many groups are there?
   (e) How many are one fifth of 20 people?

When you **divide** something into **5 equal parts**, each part is **one fifth** of the whole.

2. Maggie spent a quarter of her money, and Lizzie spent half of her money. Who spent the most money in each case below?
   (a) Maggie had R40 and Lizzie had R100.
   (b) Maggie had R100 and Lizzie had R40.
   (c) Maggie and Lizzie both had R40.

Notes on questions

Questions 3, 4 and 5 address fractions of a whole (the loaf), as well as fractions of collections (the 20 slices).

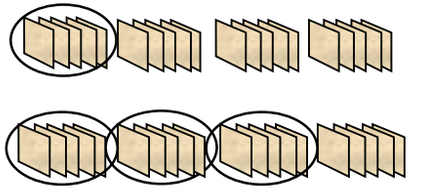
The illustrations of loaves of bread between questions 3 and 4 relate to question 4. In question 4, learners may say many things about the loaves of bread, for example: *"They are loaves of white bread."*; *"They are sliced loaves of bread."* Focus learners' attention on the slices as parts of the whole loaf, and how these slices are grouped and what fraction each group of slices is of the whole.

Help learners to connect what they did in Section 3.2, especially question 9 on page 141, with questions 4 and 5 on page 143.

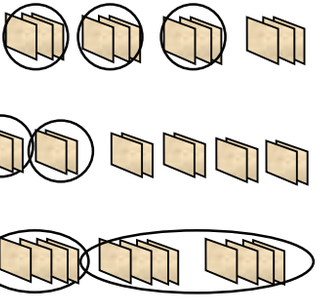
Answers

3. 

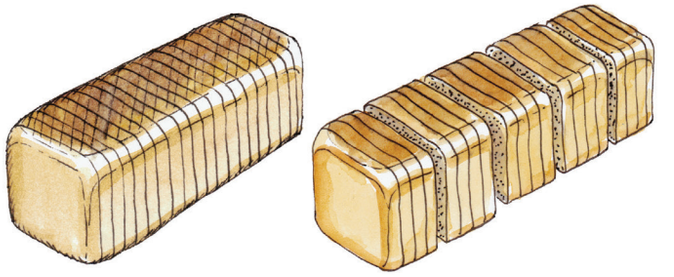
4. (a) The loaf is cut into 20 slices. Slices are grouped into 5 groups with 4 slices in a group. Each group is one fifth of the loaf.
   (b) 20 slices (c) 4 slices (d) 3 groups of 4 slices = 12 slices
   (e) The whole loaf is 20 slices. If you divide the loaf into 4 equal groups, each group is a quarter. 1 quarter of the loaf is 5 slices.
   (f) From question (e), 1 quarter of the loaf is 5 slices. So 3 quarters of the loaf is 15 slices.
   (g) Each loaf can be made into 4 quarters. So, 5 loaves give 5 × 4 quarters = 20 quarters.
   (h) 1 loaf is 4 quarters, 2 loaves are 8 quarters, 3 loaves are 12 quarters.
   (i) 3 quarters, because a quarter loaf is 1 of four equal parts, and 1 fifth of a loaf is one of five equal parts. If you divide the same thing into more parts, each part is smaller.



5. (a) 9 slices. If you divide this loaf into 4 equal parts each part has 3 slices. A quarter of this loaf is 3 slices, so 3 quarters of this loaf is 9 slices.
   (b) 4 slices. If you divide this loaf into 6 equal parts each part has 2 slices. A sixth of this loaf is 2 slices, so 2 sixths of this loaf is 4 slices.
   (c) 4 slices. If you divide this loaf into 3 equal parts each part has 4 slices. A third of this loaf is 4 slices.
   (d) 2 thirds. 1 third is 4 slices, so 8 slices are 2 thirds.



3. Make a rough drawing of a loaf of bread that is cut into slices so that each slice is one tenth of the whole loaf.



4. (a) Write three sentences to say what you see in the picture.
   (b) How many slices are there in the whole loaf?
   (c) How many slices are there in each fifth of the loaf?
   (d) How many slices are there in 3 fifths of the whole loaf?
   (e) How many slices are there in a quarter of the whole loaf?
   (f) How many slices are there in 3 quarters of the whole loaf?
   (g) How many quarter-loaves can you make up from 5 full loaves?
   (h) How many full loaves do you need to make up 12 quarter-loaves?
   (i) Which is more, 3 fifths or 3 quarters of a loaf?
5. A different loaf of bread is cut into 12 equal slices.
   (a) How many slices are three quarters of this loaf?
   (b) How many slices are two sixths of this loaf?
   (c) How many slices are one third of this loaf?
   (d) Peter eats 8 slices of this loaf. What part of the loaf does he eat?

   You can make a rough drawing if it will help you.

3.4 Using fractions to measure

Mathematical notes

Fractions as parts of wholes and collections was addressed in Sections 3.1 and 3.2.

This section is about fractions as parts of measuring units, to facilitate accurate measurement. It also introduces equivalent fractions. This is further explored in Term 3 Unit 1 Section 1.2.

Teaching guidelines

You could plan to spend 2 hours on Section 3.4. One possible way to organise this section is to use

- questions 1, 2, 3, 11, 12 and 13 for concept development,
- questions 4, 5 and 6 for classwork, and
- questions 7, 8, 9 and 10 for homework.

You might like to read Section 1.2 of Term 3 Unit 1 in this Teacher Guide, which deals with equivalent fractions, before you teach questions 9 to 12.

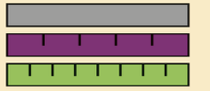
Note that question 3 raises the issues that are dealt with in questions 4, 5, 6, 7 and 8.

Answers

1. (a) Yes
   (b) The green and purple sticks are subdivided into smaller equal parts, the grey stick is not.
   (c) The green stick is divided into 8 equal parts and the purple stick into 5 equal parts.
   (d) 5
   (e) fifths
   (f) eighths
2. (a) Yes, I can say it is 2 sticks long.
   (b) No, I cannot say exactly. I can only say it is 2 and a bit sticks long.
   (c) No, I cannot say exactly. I can only say it is about 2 and a half sticks long.
   (d) No, I cannot say exactly. I can only say it is a little bit more than 2 sticks long.
3. Accept any reasonable plans. Some learners may say that we can divide the grey stick into smaller parts of equal size.

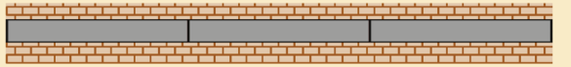
3.4 Using fractions to measure

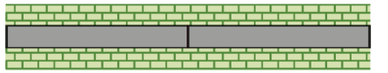
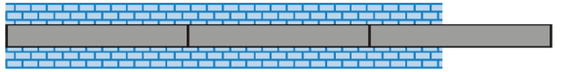
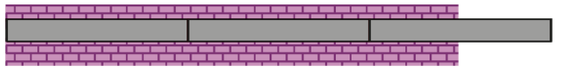
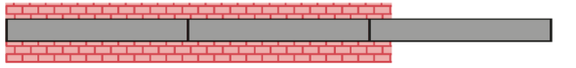
Billy uses these measuring sticks to take measurements on a building site.



1. (a) Are the three sticks the same length?
   (b) How do the green and purple sticks differ from the grey stick?
   (c) How do the green and purple sticks differ from each other?
   (d) How many equal parts can you see on the purple stick?
   (e) What name can you give to these parts?
   (f) What name can you give to the parts on the green stick?

This brick wall is exactly **3 sticks long**:



2. Can you say exactly how long each brick wall below is? Give reasons for your answers.
   (a)
   
   (b)
   
   (c)
   
   (d)
   
3. Think of a plan to measure the blue and red walls accurately.

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Answers

4. (a) Yes, 2 and 1 eighth of a stick
   (b) Yes, 2 and 2 fifths of a stick
5. 5 sixths
6. (a) Two and 1 eighth of a stick
   (b) Two and 2 fifths of a stick
7. Wall (d)
8. (a) One and 3 fifths of a stick long
   (b) One and 3 sixths of a stick long
   (c) One and 3 tenths of a stick long
9. Yes, because 3 sixths is the same length as 1 half.
   3 is half of 6, so 3 sixths is "halfway", or half of a stick divided into sixths.

Notes on questions

Questions 9 to 13 serve to develop awareness of the fact that the same quantity can be described as a fraction in different ways, in other words the concept of equivalent fractions. This is critically important work.

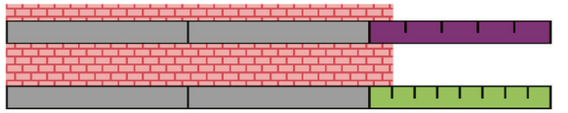
Critical knowledge

Equivalence is probably the most important topic in fractions in the Intermediate Phase. It is the basis of many of the calculations done with fractions in the Intermediate Phase and beyond.

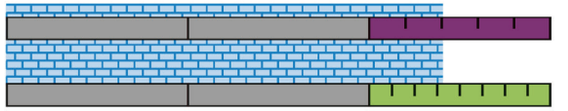
Recognising equivalent fractions is the focus of fractions in Term 3 Unit 1 Section 1.2. It is also the focus of a large part of the work on fractions in Grade 5 and 6.

4. Here you can see the blue and red walls again.
   Can you now say exactly how long they are?

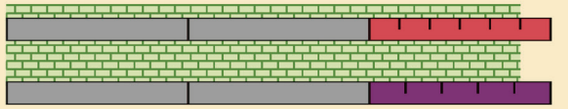
(a)



(b)

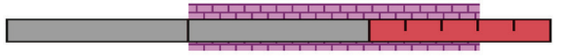


The green wall below is **two and 5 sixths of a stick** long.

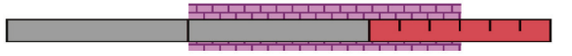


5. Which is longer, 5 sixths of Billy's measuring stick or 4 fifths of Billy's measuring stick?
6. Check your answers for question 4, and write them in the same way that the length of the green wall is written above.
7. Which of the walls below is one and 3 eighths of a stick long?

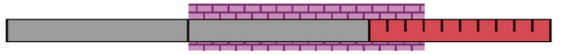
(a)



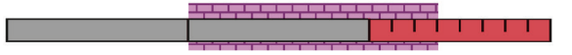
(b)



(c)



(d)



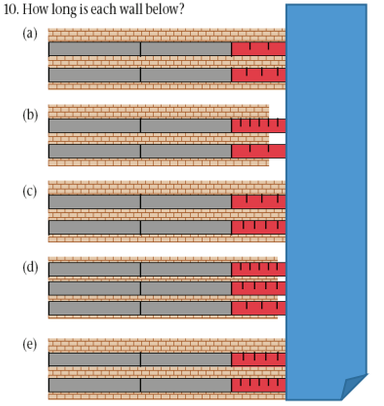
8. How long is each of the other walls in question 7?
9. Billy says the wall in 7(b) is one-and-a-half sticks long.
   Do you agree?
   Explain in writing why you agree or why you disagree.

Teaching guidelines

Grade 4 learners are *not* expected to use arithmetic rules to find equivalent fractions (or to do calculations). They should rather work with diagrams and apparatus when comparing, ordering and finding equivalent fractions.

For question 11 you can draw two bars of the same length on the board. Divide one into 4 equal parts and shade 3 of these parts. Also divide the second bar into 4 equal parts and shade 3 of these parts. Then halve each of the 4 parts in the second bar. Ask learners: "*How many parts are shown now?*", "*What do we call each part?*", "*How many parts are shaded?*", "*What fraction of the second bar is shaded?*", "*Can you see that 3 quarters is the same amount as 6 eighths?*"

In question 12 learners are asked to find a wall that is two and 3 fifths of a stick long. Here they need to find a stick divided into parts equivalent to fifths. Let them first find a stick that is divided into fifths that is not at a wall that is 2 and 3 fifths of a stick long. The top stick at Wall (a) and the bottom stick at Wall (b) are divided into fifths. Then learners need to find a wall that is 2 and 3 fifths long. The easiest way for them to do this is to use a ruler or any object with a straight edge (a piece of paper will do). They can place the straight edge against 3 fifths on the stick in Wall (a), and look down to see which wall has this length (2 and 3 fifths of a stick). They will see that Wall (e) has this length and that its length coincides with 2 sticks and 6 tenths.

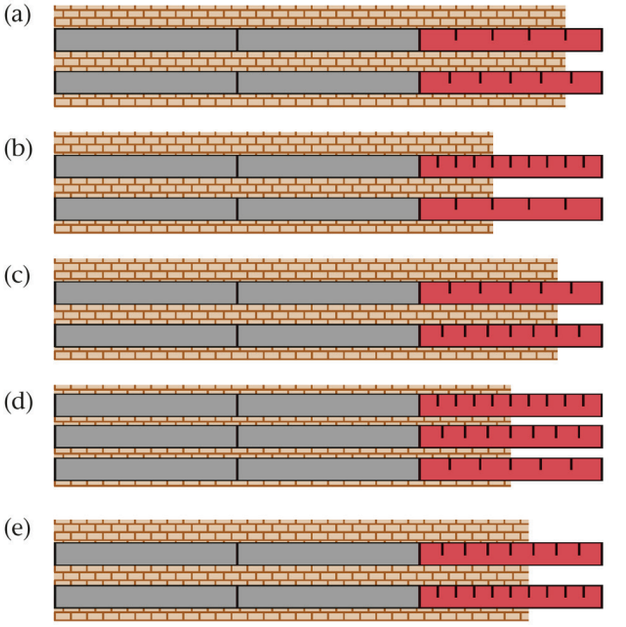


In question 13 learners are only expected to know that double 5 is 10 or 5 + 5 = 10 or 2 × 5 = 10. It does not matter whether they are adding 5 sweets and 5 sweets, 5 cm and 5 cm or 5 eighths and 5 eighths, all they need to do is say 5 + 5 or double 5 or 2 × 5 = 10.

Answers

10. (a) Two and 4 fifths of a stick long
    (b) Two and 4 tenths or two and 2 fifths of a stick long
    (c) Two and 6 eighths of a stick long
    (d) Two and 5 tenths or 4 eighths or 3 sixths of a stick long
    (e) Two and 6 tenths of a stick long
11. (a) Wall (c) (b) Wall (c)
12. Wall (e) is two and 6 tenths long. 6 tenths is the same length as 3 fifths.
13. 10 eighths

10. How long is each wall below?



11. (a) Which of the walls is two and 3 quarters of a stick long?
    (b) Which of the walls is two and 6 eighths of a stick long?

3 quarters and 6 eighths are called **equivalent fractions**, because they are just different ways to describe the same fraction part of a whole object.

12. Which of the walls is two and 3 fifths of a stick long? Explain how you know that.
13. One section of a wall is 5 eighths of a stick long, and another section is 5 eighths of a stick long. How long are the two sections together?

Grade 4 Term 2 Unit 4 Length

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Comparing and measuring length	Comparing lengths directly and with informal units of length	147 to 148
4.2 Standard units of measurement	Choosing appropriate units of length, and estimating in centimetres	149 to 151
4.3 Measuring short lengths accurately	Using a ruler to measure in centimetres and millimetres	151 to 154
4.4 Writing lengths in different units	Converting between millimetres, centimetres, metres and kilometres	155 to 156
4.5 Measuring distances accurately	Measuring using measuring tapes, builder's tape measures and trundle wheels	156 to 157
4.6 Rounding off	Rounding off to the nearest 10, 100, 1 000	157 to 159
4.7 Apply your skills	Arranging lengths in order. Solving problems	159 to 160

CAPS time allocation	7 hours
CAPS page references	25 and 73 to 75

Mathematical background

Length, mass, capacity/volume, perimeter and area are different properties of objects. When we measure these properties we are describing how much of the property objects have; we are describing the property in terms of a numerical value.

When we measure the length of objects or spaces we are allocating a number value to how long or wide the object or space is. This allows us to compare and order objects in terms of their length. For example, the teacher's desk is wider than the classroom door. It also allows us to do calculations. For example, if a roll of string is 500 m, is this enough string to give each Grade 4 learner 2 m of string if there are 4 classes of 40 learners?

Learners go through four stages when learning to measure:

1. Identifying and understanding the property they are measuring.
2. Comparing and ordering examples of a particular measure (see question 1 of Section 4.1).
3. Using informal or non-standard units to measure (see question 6 of Section 4.1).
4. Using formal or standard units to measure (see Sections 4.2, 4.3 and 4.5).

When you use an informal unit to measure length you can compare the lengths of many objects to the length of the same object. For example, using a match as your informal unit means that you can compare the lengths of different objects by saying how many match lengths each object is. A standard unit allows many people to compare the lengths of many objects to the same lengths, for example metric units of length. A difficulty with standard units is that the instruments are often complex to read (see Sections 4.3 and 4.5).

Resources

String; matches; new pens or pencils all of the same length; rulers; measuring tapes; builder's tape measures; trundle wheel.

4.1 Comparing and measuring length

Mathematical notes

In this section learners first compare lengths without any units, then they use an informal unit (a match) to measure. The section ends by motivating the need for standard units.

Resources

A piece of string about 2,5 m for each pair of learners; matches; new pencils or pens all of the same length.

Teaching guidelines

All the principles of measuring can be learnt from informal measuring. In this section learners are not expected to use standard units and formal measuring instruments.

In question 1 learners can first just estimate which of the pairs of measurements are longer. Then they can check using a piece of string for 1(a), (b), (c) and (f). In 1(d) learners can pace out the distance between the classroom and the office. While they will agree on which classroom is furthest from the office, they may not agree on the number of paces or strides. This is because the length of their legs will differ. You can use this, and the tinted passages on pages 147 and 148, to motivate the need for standard units.

Learners might find measuring curved lines (see question 4) more complicated than measuring straight lines. This might only become an issue when they measure with rulers.

Answers

1. (a) Height of the classroom door (b) Width of the chalkboard
   (c) Height of the teacher, unless the teacher is very short.
   (d) Answers will differ from school to school.
   (e) Length of foot (f) Distance between ears
2. Learners' methods will differ: accept all reasonable methods. For 1(a), (b), (c) and (f) they might use a piece of string, as outlined in question 4. They might put their hand directly against their foot in 1(e). They might pace out the distances in 1(d).
3. Accept any reasonable answers. Also see the answer to question 4.
4. Learners place a piece of string along the line and then measure the length of the string. They may measure it with finger or thumb widths, or use matches: the line is just less than $2\frac{1}{2}$ match lengths. Learners have measured in centimetres with rulers in Grade 3. Some learners may use rulers: accept answers from $9\frac{1}{2}$ cm to 11 cm.

UNIT 4 LENGTH

4.1 Comparing and measuring length

People need to be able to agree on how big things are. If this cannot happen, it will be very difficult to build houses or roads and many other things we need. This means we have to be able to measure things.

1. Which of the following pairs of things is longer?
   (a) the height of a classroom door or the length of a desk
   (b) the width of a classroom door or the width of a chalkboard
   (c) the height of your teacher or the length of your teacher's table
   (d) the distance from your classroom door to the office or the distance from the Grade 6 classroom door to the office
   (e) the length of your hand or the length of your foot
   (f) the distance from your chin to your ear or the distance between your ears
2. What did you do to find the answers to question 1? Write down your answers.
3. How can you find out what the length of this blue line is? Discuss your idea with one or two classmates.



4. Senzo took a piece of string. He placed it carefully all along the curved line above. He marked the end of the line on the string and then measured the length of the string. Use Senzo's method to measure the length of the curved line.

GRADE 4: MATHEMATICS [TERM 2] 147

Answers

5. There are too many possible answers to list all of them. Accept any reasonable answers.
   (a) Length: length of a room; length of a skirt, etc.
   (b) Distance: distance between towns; distance from home to school, etc.
   (c) Height: own height; height of a window (before buying curtains), etc.
   (d) Width: width of a room; width of a window (before buying curtains), etc.
   (e) Depth: depth of hole; pool; cupboard; drawer, etc.
6. Pencil
7. (a) Themba's answer is an estimate.
   (b) Take a matchstick and measure the pencil with it.
   (c) Measure the length of the pencil and the length of the matchstick. Divide the length of the matchstick into the length of the pencil to get your answer.

Possible misconceptions

Sometimes when learners measure with informal units they leave gaps between the units.



Explain to learners that units, in this case matches, need to be placed end to end, as shown in the second photograph on page 148.

5. Give two examples from everyday life where you would measure:
   (a) length (b) distance (c) height
   (d) width (e) depth

In all of the comparisons you made in question 1, you could only tell which object or distance is longer. You were not able to tell *how much longer* it was.

6. Which is longer, the pencil or the matchstick?



It is quite easy to see that the pencil is longer than the matchstick. However, we need another way to compare them if we want to be more exact.

Themba says: "The pencil is 4 times as long as the matchstick."

7. (a) Is Themba correct?
   (b) How can you find out if Themba is correct, without using a ruler?
   (c) How can you find out if Themba is correct by using a ruler?

Themba did this:



Matchsticks are used above to measure the length of the pencil. We say that the matchstick is the **unit of measurement**. However, not all people will use a matchstick as a unit of measurement, or some people's matchsticks may be shorter or longer than others. This means that different people will get different measurements for the same object. So we need a *standard* unit of measurement, which is a unit that everyone can use and that is always the same length.

4.2 Standard units of measurement

Mathematical notes

This section introduces the four standard units that we commonly use to measure length in everyday life, namely millimetres, centimetres, metres and kilometres.

Learners choose appropriate units to measure different lengths. They also find common referents of particular lengths in centimetres and use these to estimate other lengths in centimetres. Learners will focus on using instruments for measuring in later sections.

Resources

Rulers. If you don't have rulers you can photocopy the examples provided in the Addendum (pages 414 and 415). Learners can also use a piece of string and mark off centimetres using the illustration in the tinted passage at the bottom of page 150.

Possible misconceptions

In the Intermediate Phase, learners measure and calculate length using only millimetres, centimetres, metres and kilometres. This may lead to the misconception that these are the only units of length. In Section 4.4 of this Teacher Guide we introduce more units of measurement (also see "Mathematical Notes" on the next page) and show how these can help learners to convert between units.

Teaching guidelines

You can use the first tinted passage on page 149 to introduce the notion of standard units. Learners should already have measured in metres and centimetres in Grade 3, but they were not required to know the relationship between the two. You can check with learners which standard units of length they know and whether they have a sense of how long a metre and a centimetre is. Use the summary bar to explain the relationship between millimetres, centimetres, metres and kilometres. You can demonstrate the size of a metre and ask learners to imagine how far 1 000 metres, or 1 kilometre, is. Let learners see the size of a millimetre and centimetre by looking at their rulers or at the illustration in the tinted passage on page 150.

Ask learners to choose the appropriate units for the lengths given in question 2. They should motivate their answers.

In questions 3 to 6 learners name objects with specific lengths. They use these to estimate other lengths. We call these informal referents. Learners can check their answers to questions 3 to 7 by measuring with a ruler.

Answers

1. Standard units are the same all over the world. Non-standard units are not the same.

4.2 Standard units of measurement

Today, in many parts of the world, we use the same system of units to measure length. That makes it possible for us to tell other people how long an object is. The system is called the International System of Units (SI).

We call these units the **standard units**. They are the same everywhere in the world where they are used.

1. What is the difference between a standard unit and a unit that is not standard? Talk about this with a classmate.

We use different instruments that are marked in standard units to measure the length of objects. Here are two of the instruments that we use to measure shorter lengths.



Ruler

Measuring tape



Your teacher can show you another commonly used ruler. It is 1 m long and it is called a *metre stick*.

In 1793, people in France decided that they would use the **metre (m)** as the **standard unit for length**. *One metre* is the standard and the other units are named for how they relate to the metre.

A **centimetre (cm)** is one of the parts if 1 m is divided into 100 equal parts.

There are 100 cm in 1 m.
Centi- in centimetre means hundredth.

100 cm = 1 m

Mathematical notes

Although the relationship between metres and millimetres, centimetres and kilometres is stated in this section, learners are *not* expected to do conversions between units here.

As mentioned before, there are many standard units of length that we do not commonly use to measure in everyday life. A more complete list ranging from kilometres to millimetres is kilometres, hectametres, decametres, metres, decimetres, centimetres and millimetres. This is described further in Section 4.4 of this Teacher Guide.

Answers

2. (a) centimetres
   (b) metres
   (c) millimetres
   (d) kilometres

A **millimetre (mm)** is one of the parts that is formed when 1 m is divided into 1 000 equal parts.

1 000 mm = 1 m

There are 1 000 mm in 1 m. *Milli-* in millimetre means thousandth.

There are 10 mm in 1 cm.

10 mm = 1 cm

A **kilometre (km)** is 1 000 times as long as 1 m. *Kilo-* in kilometre means thousand.

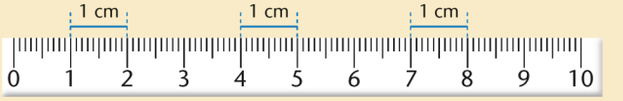
1 000 m = 1 km

2. Which of the units will you use if you have to measure the length of each of these objects?
   (a) the length of your textbook
   (b) the length of the classroom
   (c) the thickness of your pencil
   (d) the distance between two towns

Most **rulers** have centimetres (cm) and millimetres (mm) as their units. We use rulers to measure shorter lengths such as the length of a book or the length in a geometric figure.

On **measuring tapes** you will see millimetres, centimetres and metres (m). We use measuring tapes to measure longer lengths, such as the height of a person or the length of a skirt. For even longer distances, such as the length of a wall in a building, there are builder's tape measures and surveyor's tape measures.

Now look at the ruler below. There are 1 cm spaces on the ruler.



Answers

3. Answers will differ. Examples include: the width of some learners' fingers, width of some pens, width of the ear of a cup, width of an AAA battery.
4. Answers will differ. Examples include: a cellphone, width of an envelope, width of a sheet of A4 paper folded lengthwise, width of some learners' palms.
5. Answers will differ. Examples include: length of a ruler, length of an A4 sheet of paper, width of a chopping board.
6. Answers will differ. One answer is an A4 sheet of paper.
7. Estimates will differ.

4.3 Measuring short lengths accurately

Mathematical notes

In the previous section learners estimated lengths and chose suitable instruments. The focus here is on measuring in centimetres and millimetres using a ruler, and on how to place the object measured against the ruler.

Possible misconceptions

Learners may think that they simply read the length from where the end of the object aligns with the ruler. This is only true if they place one end of the object against zero on their ruler.

If learners do not place the end of the object against zero, they need to subtract the number at the beginning of the object from the number at the end of the object: see questions 1 and 5.

Resources

Rulers that are at least 15 cm long (you can photocopy the examples provided in the Addendum on pages 414 and 415); lengths of string or cotton, about 15 cm long.

Teaching guidelines

In the first eight questions, learners use rulers to measure straight lines or straight edges. In question 9 learners measure curved lines and curved edges; this is more complicated than measuring straight lines. Question 4 on page 147 explains how to do this. You can cut lengths of string or cotton of about 15 cm long for each learner.

Answers

1. (a) No
   (b) 3 cm. She should have started measuring from 0 or she should have subtracted 1 cm (the beginning of the bar) from 4 cm (the end of the bar).

Each centimetre space is divided into 10 equal smaller spaces. So every centimetre is divided into tenths of a centimetre. This means that the space of 1 cm is the same length as the space that is 10 mm long. You already know that **1 cm = 10 mm**.

3. Name three objects that are about the length of a centimetre. (Hint: look at your hands or look around in the classroom.)
4. Name three objects that are about 10 cm long or wide.
5. Name three objects that are about 30 cm long or wide.
6. Name an object that is about 30 cm long and 20 cm wide.
7. Now use some of the objects that you named in questions 3 to 6 to help you estimate the following:
   (a) Estimate how long your thumb is.
   (b) Estimate how long your desk is.
   (c) Estimate how wide the distance is between two desks in your classroom.

4.3 Measuring short lengths accurately

1. Kholeka measures the length of the blue bar below.



   (a) Kholeka says the blue bar is 4 cm long. Is she correct?
   (b) If 4 cm is wrong, give the correct answer and explain to Kholeka what she did wrong.

Teaching guidelines

Allow time for learners to share their answers and explanations of their answers.

Answers

2. (a) No
   (b) It is 3 cm long. The blue bar extends slightly past the 0 on the left. It stops just before the 3 cm mark.
3. Put one end of the blue bar on the 0 mark of the ruler. Read the measurement where the end of the bar is.
4. 7 cm
5. (a) 5 cm
   (b) Subtract 2 cm from 7 cm.

2. Luke measured the length of the blue bar like this:



He says it is almost 3 cm long.

(a) Is Luke correct?

(b) Explain your answer.

3. Explain in your own words how you will measure the length of an object like the blue bar accurately.

4. What is the length of the green bar?



The easy correct way to use the ruler is to put one end of the object that you measure on the 0 mark of the ruler. Read the measurement where the other end of the object is.

5. (a) What is the length of this red bar?



(b) How did you find the answer?

Mathematical notes

On page 152 learners only read measurements in centimetres.

The focus here is on measuring in centimetres and millimetres using a ruler. In Grade 4 learners do not work with decimal fractions. This means that they can only state their measurements in millimetres, centimetres and millimetres, or centimetres and fractions of centimetres (expressed as common fractions) – see the tinted passage. Page 73 of the Intermediate Phase Mathematics CAPS states the following about Grade 4 learners:

"Once learners have learned from reading commercial mass and capacity that $2\frac{1}{2}$ is the same as 2,5 they will also be able to use the decimal 5 in their recordings i.e. 2,5 cm long."

This means that from early on in Term 3 learners can use the decimal 5. While learners are not required to use the decimal form or teachers expected to teach it, if learners happen to know how to write a decimal half it should not be marked as incorrect.

Answers

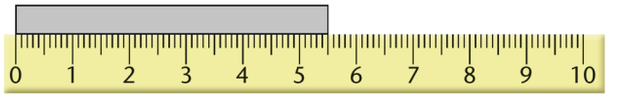
6. (a) 55 mm (b) $5\frac{1}{2}$ cm or 5,5 cm

7. Learners will express their reasoning differently but essentially 5 cm = 50 mm, so 5 cm and 7 mm = 50 mm + 7 mm = 57 mm.

8.

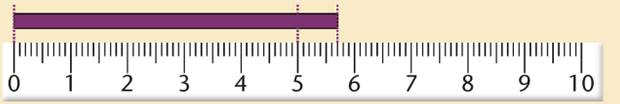
Bar	Estimated lengths	Measured lengths
Red	Estimated lengths will differ	10 cm
Blue		6 cm and 3 mm
Purple		2 cm and 2 mm
Light green		6 cm and 3 mm
Dark green		4 cm and 7 mm
Grey		8 cm and 5 mm or $8\frac{1}{2}$ cm

6. (a) What is the length of this grey bar in millimetres?



(b) What is the length of the grey bar in centimetres?

We can also record the length of an object with a combination of two different units. The length of the grey bar in question 6 can be given as 5 cm and 5 mm, or $5\frac{1}{2}$ cm.



The bar above is more than 5 cm but less than 6 cm long.
It is 7 mm longer than 5 cm. We can therefore say that the bar is 5 cm and 7 mm. Or we can record it as 57 mm.

7. Discuss with one or two classmates why we can also record the length of the purple bar as 57 mm.

8. First estimate the centimetre length of each of the bars on the next page. Then measure each bar with your ruler. Complete this table.

Bar	Estimated length	Measured length
Red		
Blue		
Purple		
Light green		
Dark green		
Grey		

Notes on questions

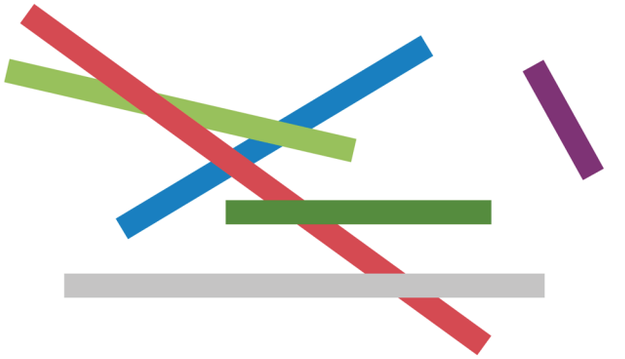
In question 9, learners measure curved lines and curved edges. This is more complicated than measuring straight lines. Question 4 on page 147 explains how to do this. You can cut lengths of string or cotton of about 15 cm long for each learner.

Answers

9. (a) Answers will differ.

(b)

Object	Estimated length	Measured length
The length of the purple curved bar	Estimated lengths will differ	9 cm and 8 mm or $9\frac{8}{10}$ cm
The length of the green curved bar		15 cm
The distance around the red circle		12 cm



9. Estimate the lengths below and then measure them.

(a) First discuss with a friend how you can use a piece of string to measure the lengths.

(b) Complete this table.

Object	Estimated length	Measured length
The length of the purple curved bar		
The length of the green curved bar		
The distance around the red circle		



4.4 Writing lengths in different units

Mathematical notes

Here learners convert between commonly used standard units of length. Learners can learn these conversion factors off by heart. However, as with everything learnt off by heart, learners will sometimes forget it and use the wrong conversion factor. It may be better for them to understand how the relationship between metric units works in general – see next page.

Possible misconceptions

Learners might think that millimetres, centimetres, metres and kilometres are the only units of length. On the following page we introduce more units of measurement and show how these can help learners to convert between units.

Answers

1. (a) 50 mm (b) 2 cm
   (c) 2,5 cm or $2\frac{1}{2}$ cm (d) You multiply by 10.
   (e) You divide by 10. (f) Learners' own work
2. (a) 6 000 m (b) 3 km
   (c) 7 km and 500 m or 7 $\frac{500}{1\,000}$ km or $7\frac{1}{2}$ km
   (d) You multiply by 1 000. (e) You divide by 1 000.
3. (a) 20 mm (b) 230 mm
   (c) 5 mm (d) 200 mm
4. (a) 5 cm (b) 230 cm
   (c) 700 cm (d) 7 200 cm
   (e) $\frac{1}{2}$ cm (f) 1 000 cm

Teaching guidelines

In our base-10 place value system, each unit of a higher power is ten times the value of an adjacent unit of a lower power: 10 units make 1 ten; 10 tens make 1 hundred; 10 hundreds make 1 thousand, etc. We can show this using numbers only – see next page.

The metric system also works on groupings or powers of tens. This is why, from the 1790s, it was called the decimal metric system.

4.4 Writing lengths in different units

You already know the following:
- There are 10 millimetres in 1 centimetre.
- There are 100 centimetres in 1 metre.
- There are 1 000 metres in 1 kilometre.

1. (a) How many millimetres are there in 5 cm?
   (b) How many centimetres are there in 20 mm?
   (c) How many centimetres are there in 25 mm?
   (d) What do you do to write a length which is given in centimetres in millimetres?
   (e) What do you do to write a length which is given in millimetres in centimetres?
   (f) Take a ruler and check your answers. Discuss this with a classmate.
2. (a) How many metres are there in 6 km?
   (b) How many kilometres are there in 3 000 m?
   (c) How many kilometres are there in 7 500 m?
   (d) What do you do to write a length which is given in kilometres in metres?
   (e) What do you do to write a length which is given in metres in kilometres?
3. Write the following lengths in millimetres:
   (a) 2 cm (b) 23 cm
   (c) one half of a centimetre (d) one fifth of a metre
4. Write the following lengths in centimetres:
   (a) 50 mm (b) 2 300 mm
   (c) 7 m (d) 72 m
   (e) 5 mm (f) 10 m

Teaching guidelines

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	tenths	hundredths	thousandths
$1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000$	$1 \times 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$	$1 \times 10 \times 10 \times 10 \times 10 = 10\,000$	$1 \times 10 \times 10 \times 10 = 1\,000$	$1 \times 10 \times 10 = 100$	$1 \times 10 = 10$	1	$\frac{1}{10}$	$\frac{1}{10 \times 10} = \frac{1}{100}$	$\frac{1}{10 \times 10 \times 10} = \frac{1}{1\,000}$

Page 143 of the Grade 5 Learner Book (Term 2 Unit 4 Section 4.1) shows a table of standard metric units for measuring length. Each unit of a higher power is ten times the value of an adjacent unit of a lower power. Learners can use a table like the one below to do conversions.

Kilometre (km)	Hectometre (hm)	Decametre (dam)	Metre (m)	Decimetre (dm)	Centimetre (cm)	Millimetre (mm)

Learners simply follow these steps:

1. They write the number under the correct unit of measurement.
2. They mark which unit they are converting to.
3. If converting from a unit of a higher power to a unit of a lower power, they multiply by 10 each time they move to a unit of a lower power.
   Example: 6 km = 6 × 10 × 10 × 10 m = 6 000 m
4. If converting from a unit of a lower power to a unit of a higher power, they divide by 10 each time they move to a unit of a higher power.
   Example: 7 500 m = 7 500 ÷ 10 ÷ 10 ÷ 10 km = 7 km and 500 m. 500 m ÷10 ÷10 ÷ 10 km = $\frac{5}{10}$ km = $\frac{1}{2}$ km

If you want learners to be able to work with all the metric units between milli- and kilo-, then teach them a mnemonic to help them remember the units (names and order). The Department of Basic Education* provides the following mnemonic for learners to remember the order of the units of measurement: "Kids Have Dreams Making Dad Chocolate Muffins". However, you can make any sentence you like with words that start with the letters k, h, d, m, d, c, m. You will find many examples on the internet.

* Annual National Assessment of 2014. *Diagnostic report. Intermediate and Senior Phases. Mathematics*. DBE (2015). Government Printers. Pretoria. Page 37.

Answers

5. (a) 4 m (b) 4 m
   (c) $10\frac{1}{2}$ m or 10,5 m (d) 100 m
6. 2 m 7 cm and 3 mm
7. (a) 25 000 m (b) 14 000 m (c) 3 500 m (d) 1 250 m
8. (a) 7 km (b) 26 km
   (c) $1\frac{1}{2}$ km or $1\frac{500}{1\,000}$ km (d) 10 km

4.5 Measuring distances accurately

Mathematical notes

Learners often measure with rulers. However, it is inconvenient to use a ruler for measuring distances longer than 30 cm.

When measuring distances longer than 1 m you need to think about more than the number that you read off the measuring tape, builder's tape measure, etc. at the end of the distance. You also need to consider how many whole metres have preceded it.

Teaching guidelines

Remind learners to always estimate the distance before measuring (this is different to rounding off the measurement after measuring) and to always state the units that they are using (not just to give the number).

Possible misconceptions

When using measuring tapes or builder's tape measures, learners sometimes assume that the distance is the number they read off the tape at the end of the distance. For example, if the width of the classroom is 8 m and 85 cm, they may just state or write 85. They may forget to take into account how many full metres there are in addition to the 85 cm.

Answers

1. (a) Learners' answers will differ. Learners can measure up to the end of their ruler or measuring tape, mark the point on the floor, write down the distance and then start measuring at the mark from zero again. At the end, add up all the distances.
   (b) Learners' estimates will differ.
   (c) Comparisons will also differ from class to class.
   (d) Learners' measurements will differ. Comparisons will differ from class to class.

5. Write the following lengths in metres:
   (a) 400 cm (b) 4 000 mm
   (c) 1 050 cm (d) 10 000 cm
6. 1 342 mm can be written as 1 m 34 cm and 2 mm.
   Write 2 073 mm as m, cm and mm.
7. Write the following lengths in metres:
   (a) 25 km (b) 14 km
   (c) $3\frac{1}{2}$ km (d) $1\frac{1}{4}$ km
8. Write the following lengths in kilometres:
   (a) 7 000 m (b) 26 000 m
   (c) 1 500 m (d) 10 000 m

4.5 Measuring distances accurately

For distances longer than 2 m, a **measuring tape** or a **builder's tape measure** works better than a metre stick.

1. (a) You can use a builder's tape measure to measure the distance from the door of the classroom to the teacher's desk. Write the distance in metres and centimetres. What plan can you make if you do not have a builder's tape measure?
   (b) Estimate how far it is from the back of the class to the teacher's desk. Use your measurement in (a) to help you.
   (c) Compare your estimate with the estimates of other learners. Discuss how much your estimates differ.

   Measure the distance from the back of the class to the teacher's desk. Whose estimate was closest?
   (d) Work in pairs. Measure the distance around the class. Compare your measurements. Discuss how much your measurement differs from other learners' measurements.

Mathematical notes

Here the main focus is on using a trundle wheel. Some trundle wheels are called metre wheels.

The second focus is on choosing suitable units of length for measuring different distances and lengths.

Resources

Trundle wheel. If you don't have a trundle wheel, try to get builder's or surveyor's tape measures or 1 m lengths of string and some measuring tapes.

Teaching guidelines

If you have a trundle wheel, let learners first check how far it is when the wheel makes a full turn, for example one full turn may be 1 metre. Explain to learners that they need to count the number of clicks as they measure, and multiply this by the distance of one full turn. For example, if they count 27 clicks this would represent 27 metres. If the wheel does not click at the end of the distance that they measure, they will need to check what part of a metre was covered after the last click. This will be shown by the pointer on the wheel, for example 6 centimetres.

If you can't access either a trundle wheel or builder's tape measure, cut metre long lengths of string for each learner. Let them use these lengths of string to measure long distances. Use a ruler or tape measure to measure the final bit of the distance, which may be less than a whole metre. For example, the total distance may be 27 m 6 cm.

Answers

2. Learners' estimates will differ. The distances measured will also differ.
3. (a) Builder's tape measure
   (b) Trundle wheel
   (c) Ruler
   (d) Ruler
   (e) Measuring tape

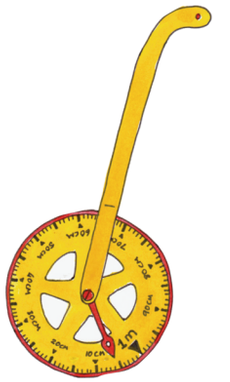
4.6 Rounding off

Teaching guidelines

You can use the tinted passage to motivate the need for rounding off.

A **trundle wheel** is a simple device for measuring distances that are too long for a tape measure, or that are not always in a straight line. It consists of a wheel with a scale around its edge. The wheel is attached to a long handle. You can push it along the path that you want to measure. Every time it completes a full turn, it makes a click.

To use a trundle wheel to measure distances, you need to know what distance is covered if the wheel turns around once.



A trundle wheel

2. If your school has a trundle wheel, use it to measure some distances. First estimate the distance and then measure it.

3. Which instrument will you use to measure the following lengths or distances?
   (a) the length of the longest wall of your school building
   (b) the distance around a soccer field
   (c) the length of your pencil
   (d) the thickness of a toffee
   (e) the height of the door in your classroom

4.6 Rounding off

In mathematics, we often round off numbers to numbers that are easy to work with. **Rounding** is a way of estimating. Numbers that are rounded off are approximations.

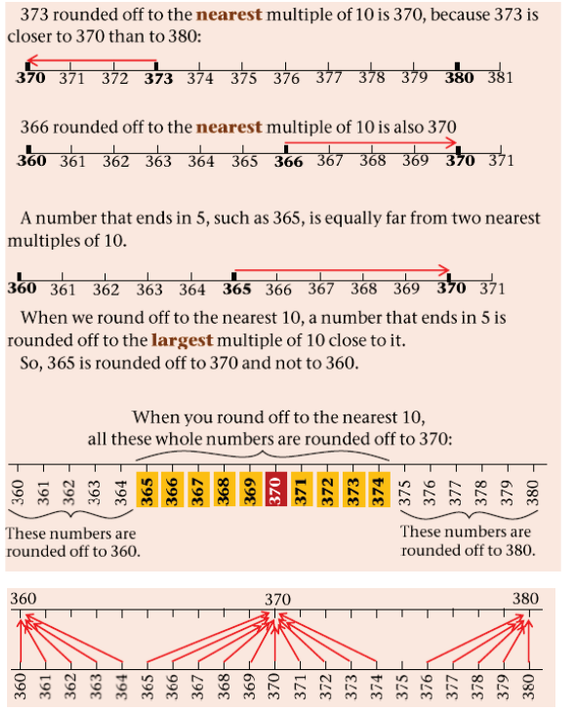
An approximation is close to the exact number, a little more or a little less.

Mathematical notes

Measurement provides a useful context for learners to understand estimation and rounding off. Questions that ask: "Is it closer to ... or ... " help learners to understand rounding off.

Teaching guidelines

Pages 14 to 16 of the Grade 6 Learner Book illustrate how to use a number line to show rounding off (an extract is shown alongside). You can use this, together with the tinted passage on pages 43 to 44 in the Grade 4 Learner Book, to explain rounding to the nearest 10 to learners.



You can show the numbers on a number line or measuring tape so that learners can actually see which ten the number is closer too. Learners can use their rulers to see the nearest 10 in questions 3(a) and (b).

Some teachers tell learners to underline the power they are rounding to, and circle the digit that follows. For example, if rounding 142 to the nearest 10, they would say underline the digit 4 and circle the digit 2, i.e. 1$\underline{4}$(2). This is *not* recommended. It does *not* help learners to understand the meaning of rounding off. It is also confusing from Grade 5 onwards, when learners need to start rounding off to the nearest 5.

Answers

1. (a) Possible answers include: the number of people in a crowd, such as at a sports match, or the hours spent studying for a test of exam.
   (b) Possible answers include: when baking or when a pharmacist mixes medicines.
2. No, it was probably an estimate.
3. (a) 30 cm (b) 140 mm (c) 1 230 mm
   (d) 50 340 mm (e) 10 km (f) 10 km

Sometimes it is not necessary to give the exact number or measurement, even if it is possible. For example: The annual school fun walk has an exact distance of 5 km 346 m. If someone asked you the distance of the fun walk, you will probably answer (about) 5 km.

1. Discuss these questions with one or two classmates.
   (a) When do you think it would be a good thing to round off a number or a measurement?
   (b) When do you think it is necessary to give the exact number or quantity?
2. The newspaper said there were 30 000 people at the soccer game. Do you think that is the exact number of people that attended the game?

To round a number usually means to find a multiple of 10, 100 or 1 000 that is close to the number. We say that we round up or down to the nearest 10 or 100 or 1 000.

Examples:

142 is closer to 140 than to 150, so 142 rounded to the nearest 10 is 140.

147 is closer to 150 than to 140, so 147 rounded to the nearest 10 is 150.

145 is the same distance from 140 and 150. "Halfway" numbers are usually rounded up to the larger of the possibilities. So, 145 rounded to the nearest 10 is 150.

3. Round off to the nearest 10:
   (a) 26 cm (b) 144 mm
   (c) 1 231 m (d) 50 335 mm
   (e) 6 km (f) 5 km

Teaching guidelines

Learners can use a measuring tape to see the nearest 100 in questions 4(a) and (b).

Answers

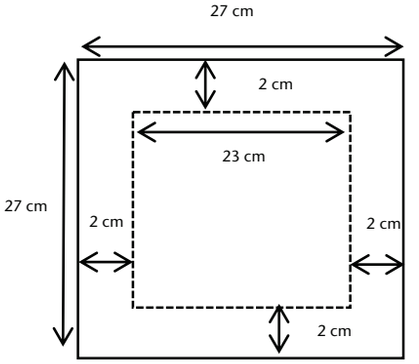
4. (a) 100 cm (b) 100 cm (c) 1 000 mm
   (d) 56 100 km (e) 300 mm (f) 23 500 km
5. (a) 2 000 mm (b) 6 000 km (c) 4 000 cm
   (d) 3 000 m (e) 1 000 mm (f) 2 000 m
6. 500 km
7. Total distance = 87 720 m + 887 m = 88 597 m
   Rounded to the nearest 100 m = 88 600 m

4.7 Apply your skills

Teaching guidelines

In question 1, learners first need to convert the measurements to a common unit before ordering the lengths.

In question 2, let learners discuss and motivate the calculation plans. The serviettes may need a 2 cm seam on each side. This means cutting squares with sides 23 cm + 4 cm = 27 cm.



Answers

1. First convert all measurements to millimetres, then arrange.
   (a) 900 cm; 2 m; 1 000 mm (b) one quarter of a km; 39 m; 100 cm; 395 mm
   (c) 2 km; 500 m; 125 cm; 248 mm (d) 449 m; 944 cm; 4 944 mm
2. You can cut 7 squares with side length 27 cm along the 2 m width (7 × 27 cm = 189 cm). You can cut 18 squares with side length 27 cm along the 5 m length (18 × 27 cm = 486 cm). So you can make 7 × 18 = 126 serviettes (there will be a little material left over).
3. There are several solutions (but only one answer). Learners could work out the height of 100 floors: $100 \times 3\frac{1}{2}\text{ m} = 350\text{ m}$. The height of 10 floors: $10 \times 3\frac{1}{2}\text{ m} = 35\text{ m}$. So the height of 90 floors = 350 m – 35 m = 315 m.

4. Round off to the nearest 100:
   (a) 78 cm (b) 145 cm (c) 991 mm
   (d) 56 072 km (e) 301 mm (f) 23 450 km

5. Round off to the nearest 1 000:
   (a) 1 991 mm (b) 6 072 km (c) 4 490 cm
   (d) 2 500 m (e) 690 mm (f) 1 932 m

6. Mr Bengu is driving to Cofimvaba this weekend. The distance is 543 km. How far is the distance to the nearest 100 km?

7. The course for the Comrades marathon race varies from year to year. In 2015 the official distance was 87 720 m. But, there were road works on the course. So, the organisers were forced to add another 877 m to the course. What was the total distance to the nearest 100 m?

4.7 Apply your skills

1. Arrange from the longest to the shortest:
   (a) 1 000 mm; 900 cm; 2 m
   (b) 39 m; 395 mm; one quarter of a km; 100 cm
   (c) 125 cm; 248 mm; 2 km; 500 m
   (d) 449 m; 944 cm; 4 944 mm

2. Mrs Tailor bought 5 m of material with a width of 2 m. She wants to make serviettes. The serviettes are squares with side length 23 cm. How many serviettes will she be able to make? Allow 2 cm for seams around each serviette.

3. A skyscraper has 90 floors. Each floor is 3 m 50 cm high. About how high is the building?

Teaching guidelines

For questions 4(c), (d) and (e), learners can measure the distance with a ruler and then find a similar length on the map for which the distance is provided alongside the map. For example, the distance on the map from Johannesburg to Kimberley is about the same as the distance from Musina to Pretoria, i.e. about 460 km.

Answers

4. (a) 1 400 km
   (b) 700 km
   (c) About the same as the distance between Musina and Pretoria: 460 km
   (d) About three-quarters of the distance between Port Elizabeth and Cape Town: 560 km
   (e) About the same as distance between East London and Beaufort West: 575 km

4. Study the map and answer the questions:



Distances

Johannesburg to Cape Town: 1 398 km

East London to Beaufort West: 575 km

Port Elizabeth to Cape Town: 749 km

Musina to Pretoria: 460 km

(a) It is 1 398 km from Johannesburg to Cape Town. Round off the distance to the nearest 100 km.

(b) It is 749 km from Cape Town to Port Elizabeth. Round off that distance to the nearest 100 km.

(c) *About* how far is it from Johannesburg to Kimberley? Estimate the distance.

(d) *About* how far is it from East London to Bloemfontein?

(e) *About* how far is it from Upington to Bloemfontein?

Hint: Look at the distances given next to the map. Use your ruler to compare and measure distances on the map.

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 Revision	Basic multiplication facts (tables)	161 to 163
5.2 Learn more multiplication facts	Multiples of 10 × multiples of 10	164 to 165
5.3 Multiplying with bigger numbers	Two-step distribution	166 to 167
5.4 Apply your knowledge	Word problems	168
5.5 Difference, ratio and rate	Ratio and rate	169 to 171
5.6 Breaking down in a different way to multiply	Multiplication by factoring	172

CAPS time allocation	6 hours
CAPS page references	13 to 15 and 76 to 77

Mathematical background

Multiplication with two 2-digit numbers, for example 26 × 43, can be done as follows:

1. Break down one number into place value parts, for example 43 = 40 + 3.
2. Distribute multiplication over addition. This means that both parts (or all, if there are more than two parts) of the number are multiplied. In this example 40 is multiplied by 26 and 3 is multiplied by 26.
3. Break down the other number into place value parts, for example 26 = 20 + 6.
4. Distribute multiplication over addition again. In this example 40 is multiplied by both 20 and 6, and 3 is multiplied by both 20 and 6.
5. Apply known multiplication facts.
6. Build up the answer.

$$
\begin{aligned}
26 \times 43 &= 26 \times (40 + 3) \\
&= 26 \times 40 + 26 \times 3 \\
&= (20 + 6) \times 40 + (20 + 6) \times 3 \\
&= 20 \times 40 + 6 \times 40 + 20 \times 3 + 6 \times 3 \\
&= 800 + 240 + 60 + 18 \\
&= 1\,118
\end{aligned}
$$

This method can only be used effectively if learners have sound knowledge of basic multiplication facts. Sections 5.1 and 5.2 are devoted to such knowledge (mental mathematics).

Multiplication can be distributed over addition as shown above: 26 × (40 + 3) = 26 × 40 + 26 × 3. This means that all the parts, or terms in the brackets, are multiplied by the number.

Multiplication can also be distributed over subtraction, for example 51 × 17 = 51 × (20 – 3) = 51 × 20 – 51 × 3: all the parts, or terms in the brackets, are multiplied by the number.

5.1 Revision

Mathematical notes

This section revises what was done in multiplication in Term 1. The focus is on multiplying by units and by multiples of 10. It includes some examples of multiplying multiples of 10 by multiples of 10, for example 20 × 30.

Notes on questions

Flow diagrams are employed as a vehicle to develop knowledge of basic multiplication facts.

The input numbers in the flow diagram and function table are deliberately not in counting sequences. This makes it more difficult for learners to get to the answer by skip counting. The aim is to encourage learners to multiply and not to skip count.

Teaching guidelines

Try to move quickly through Section 5.1: aim to cover this section in 45 minutes. One possibility is to use

- question 4 for mental mathematics,
- questions 1, 2, 3(a) and 5 for concept development,
- questions 3(b), 6 and 7 for classwork, and
- questions 3(c) and (d), 8 and 9 for additional practice.

Continue to encourage learners to think: "*What have I done before that can help me here?*"

You can start by asking learners to complete the flow diagrams in question 1. To save time you can copy the templates of the flow diagrams and function table in the Addendum (page 448). Encourage learners to use the answers in 1(a) to complete 1(b) (where possible), for example *what* × 7 = 70 is shown in the flow diagram. Explain to learners the link between the flow diagrams and the function tables.

Answers

1. (a) Output numbers from top to bottom: 49; 63; 70; 42; 21
   (b) Input numbers from top to bottom: 10; 5; 4; 9; 6

2.

Number	7	9	10	6	3	10	5	4	9	6
Number × 7	49	63	70	42	21	70	35	28	63	42

UNIT 5

WHOLE NUMBERS: MULTIPLICATION

5.1 Revision

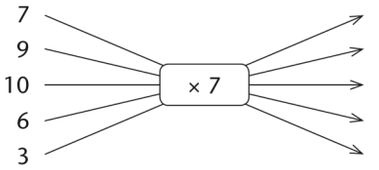
This term you will learn to do multiplication with two two-digit numbers, such as 34 × 47.

To be able to do this, you need to know your multiplication facts very well and learn some more facts, such as 30 × 60 = 1 800 and 40 × 70 = 2 800.

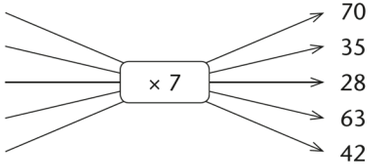
Before we start with the bigger numbers, you need to revise the multiplication facts you learnt in Term 1.

1. Complete the following flow diagrams.

(a)



(b)



2. Copy this table and write your answers for question 1 in it.

Number	7	9	10	6	3					
Number × 7						70	35	28	63	42

Division is called the **inverse** of multiplication, and multiplication is called the **inverse** of division.

Teaching guidelines

In Term 1 learners multiplied by multiples of 10, so this continues to be revision. Ask them what happens when you multiply a number by 10. Ask them what is an easy way to multiply by a multiple of 10. (In Term 1 they saw that you break the number down into two factors, one of which is 10, for example 50 × 15 = 5 × 10 × 15 = 5 × 150.) If learners cannot do this calculation, then work through questions 6(a) and (b) with them.

Note that learners do not have to copy the flow diagrams in question 3. If they do not know what 20 × 7 is, ask them which multiplication fact from questions 1 and 2 can help them. If they double 10 × 7, they will get 20 × 7. Ask them what the relationship is between 20 and 40. This will enable them to double the answer to 20 × 7 to get the answer to 40 × 7. They can repeat this process with 80 × 7. Ask learners which multiplication facts from questions 1 and 2 will help them to fill in the other answers in question 3(a): 7 × 7; 3 × 7; 10 × 7; 5 × 7.

Templates for the tables in question 3 are provided in the Addendum (page 449).

Critical knowledge

Multiplying a number by 10.

Multiplying a number by a multiple of 10.

Answers

3. (a)

Number	20	40	80	70	30	100	50
Number × 7	140	280	560	490	210	700	350

(b)

Number	30	60	90	80	50	40	70
Number × 6	180	360	540	480	300	240	420

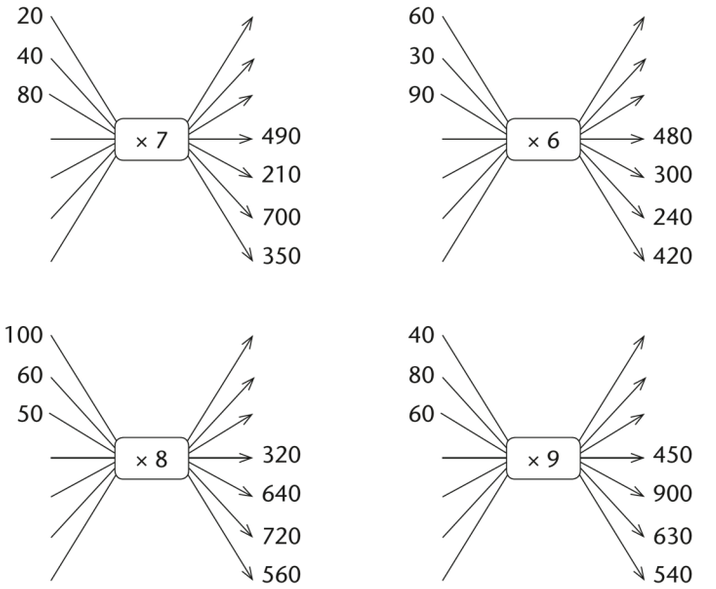
(c)

Number	100	60	50	40	80	90	70
Number × 8	800	480	400	320	640	720	560

(d)

Number	40	80	60	50	100	70	60
Number × 9	360	720	540	450	900	630	540

3. Do not copy these flow diagrams. Copy the tables below and write the missing input and output numbers in the tables.



(a)

Number	20	40	80				
Number × 7				490	210	700	350

(b)

Number	30	60	90				
Number × 6				480	300	240	420

(c)

Number	100	60	50				
Number × 8				320	640	720	560

(d)

Number	40	80	60				
Number × 9				450	900	630	540

Teaching guidelines

Encourage learners to count in 30s up to 360. They can use this to complete question 4. It is important that learners do not just skip count from 30 each time in 4(b) and (c). They can rather either count on 30 from 4(a) to get 4(b), and count on 30 from 4(b) to get 4(c), or use multiplication.

Learners should use the answer in 4(a) to write down the answer to 5(a). Show learners how they can use the answer in 5(a) to get the answer in 5(b); the answer in 5(b) to get the answer in 5(c); the answer in 5(c) to get the answer in 5(d). Remind learners that when multiplication occurs in the same expression as addition and/or subtraction, you always do the multiplication first. Ask learners which answers in 5(a) to (d) they can use to get the answer in 5(e), and how 5(e) relates to 5(f). Repeat this for 5(g) and 5(h).

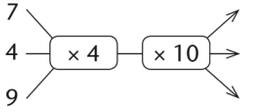
Once learners have completed question 6, ask them what they noticed about 6(a) and 6(b). How were they the same and how did they differ? What does this tell you about multiplying by a multiple of 10? Does it always work?

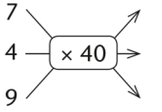
Answers

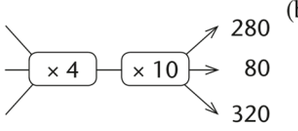
4. (a) 300 (b) 330 (c) 360
5. (a) 300 (b) 600 (c) 1 200 (d) 2 400
   (e) 1 800 (f) 1 800 (g) 2 700 (h) 2 700
6. (a) 280; 160; 360 (b) 280; 160; 360
7. (a) 7; 2; 8 (b) 7; 2; 8
8. (a) 350; 300; 450 (b) 350; 300; 450
9. (a) 4; 8; 16 (b) 4; 8; 16

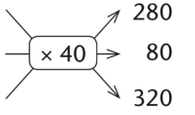
4. How much is each of the following?
   (a) 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30
   (b) 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30
   (c) 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30

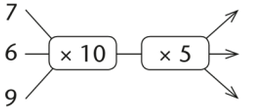
5. Calculate.
   (a) 10 × 30 (b) 20 × 30
   (c) 40 × 30 (d) 80 × 30
   (e) 20 × 30 + 40 × 30 (f) 60 × 30
   (g) 10 × 30 + 80 × 30 (h) 90 × 30

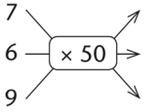
6. Write the missing output numbers in each case.
   (a)
   

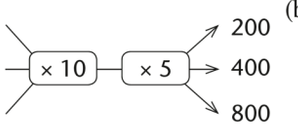
   (b)
   

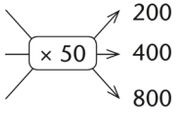
7. Write the missing input numbers in each case.
   (a)
   

   (b)
   

8. Write the missing output numbers in each case.
   (a)
   

   (b)
   

9. Write the missing input numbers in each case.
   (a)
   

   (b)
   

5.2 Learn more multiplication facts

Mathematical notes

This section continues to prepare learners to multiply 2-digit numbers by 2-digit numbers, by focusing on multiplying multiples of ten by multiples of ten.

Teaching guidelines

Try to move quickly through Section 5.2: aim to cover this section in 1 hour. One possibility is to use

- questions 2 and 3 for mental mathematics,
- questions 1, 4, 5 and 6 for concept development,
- questions 7, 8 and 9(a) to (h) for classwork, and
- questions 9(i) to (r) for additional practice.

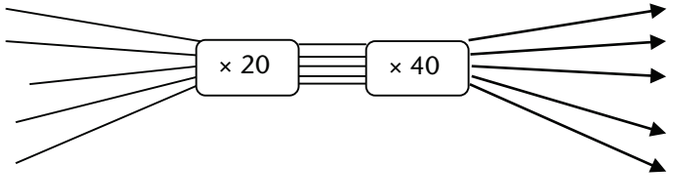
Continue to encourage learners to think: "W*hat have I done before that can help me here?",* by modelling how you can draw on one question to answer another, and draw on work done in Section 5.1 and in Term 1 to do calculations here.

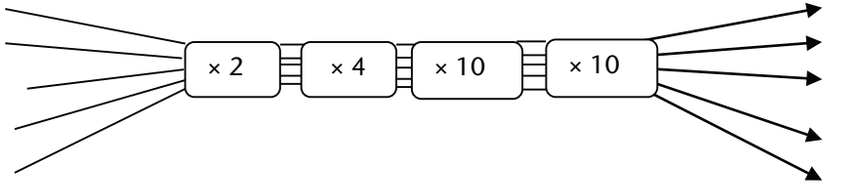
To save time you can photocopy the template of the multiplication grid on page 164. It is provided in the Addendum (page 450).

To keep learners working at an appropriate pace, you can read out the multiplication "tables" in question 2 at a steady pace (2 × 20, 3 × 20, 4 × 20, etc.) while learners write down the answers in their exercise books. Ask learners what they notice about all answers (products) where they had multiplied units by multiples of 10. They should be able to see that all the answers end in a zero.

You can read out the sequence in question 3(a) and ask learners to write down the next five numbers in their exercise books. Ask learners which row in the multiplication grid is the same as their answers to question 3(a). Then you can ask them what is the same about questions 3(a) and 3(b), what is the same about 3(b) and 3(c). Learners can use the patterns in 3(a) to help them to complete 3(b) and 3(c). You can read question 4 to them. Remind them that they have already used doubling to get from ×1 to ×2, to ×4, to ×8. Learners can write down the answers to questions 5 and 6 as you read these to them.

Ask learners what patterns they notice in the multiplication grid. Ask them what they notice about the answers (products) when they multiply a multiple of 10 by a multiple of 10. You can use the examples of breaking down numbers into factors shown under the heading "Critical knowledge" below to explain to learners why this works. You can show this using flow diagrams:





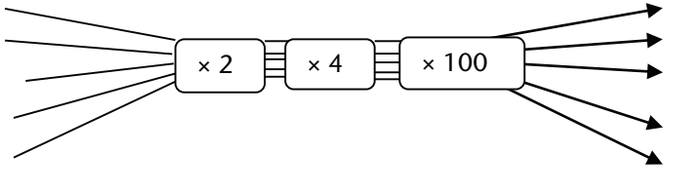
Critical knowledge

Multiplying multiples of 10 by multiples of 10.

If you change the order in which you multiply two or more numbers, the answer remains the same.

You can break numbers down into factors to make multiplication easier, for example:

20 × 10 = 2 × 10 × 10 = 2 × 100 = 200
20 × 40 = 2 × 10 × 4 × 10 = 2 × 4 × 10 × 10 = 2 × 4 × 100 = 8 × 100 = 800



Answers to questions 1, 2, 4, 5, 6 and 7

×	10	20	30	40	50	60	70	80	90	100
2	20	40	60	80	100	120	140	160	180	200
20	200	400	600	800	1 000	1 200	1 400	1 600	1 800	2 000
3	30	60	90	120	150	180	210	240	270	300
30	300	600	900	1 200	1 500	1 800	2 100	2 400	2 700	3 000
4	40	80	120	160	200	240	280	320	360	400
40	400	800	1 200	1 600	2 000	2 400	2 800	3 200	3 600	4 000
5	50	100	150	200	250	300	350	400	450	500
50	500	1 000	1 500	2 000	2 500	3 000	3 500	4 000	4 500	5 000
6	60	120	180	240	300	360	420	480	540	600
60	600	1 200	1 800	2 400	3 000	3 600	4 200	4 800	5 400	6 000
7	70	140	210	280	350	420	490	560	630	700
70	700	1 400	2 100	2 800	3 500	4 200	4 900	5 600	6 300	7 000
8	80	160	240	320	400	480	560	640	720	800
80	800	1 600	2 400	3 200	4 000	4 800	5 600	6 400	7 200	8 000
9	90	180	270	360	450	540	630	720	810	900
90	900	1 800	2 700	3 600	4 500	5 400	6 300	7 200	8 100	9 000
10	100	200	300	400	500	600	700	800	900	1 000
100	1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000	10 000

5.2 Learn more multiplication facts

1. Copy this table.

×	10	20	30	40	50	60	70	80	90	100
2										
20										
3										
30					1 500					
4										
40										
5										
50										
6										
60					3 000					
7										
70										
8										
80										
9										
90										
10										
100										

2. Complete the rows for 2, 3, 4, 5, 6, 7, 8, 9 and 10. If you know the answers for some other cells, you can fill them in too.

Notes on questions

Ask learners to compare the answers in the ×2 row with the answers in the ×20 row; the ×3 row with the ×30 row, the ×4 row with the ×40 row. They can use the patterns they see to complete other rows.

Ask learners what is the same and what is different about questions 8(a) and 9(b); 8(b) and 9(c); 8(c) and 9(d); 8(d) and 9(e); 8(e) and 9(f); 8(f) and 9(h). Ask how this can help them to get the answers to 9(b), (c), (d), (e), (f) and (h).

Answers

3. (a) 140; 160; 180; 200; 220
   (b) 280; 300; 320; 340; 360
   (c) 380; 400; 420; 460; 480

4. (a) 200; 400
   (b) Learners fill in the cells for 20 × 10 and 20 × 20.
   (c) 800 = cell 20 × 40. Learners might realise that they can fill in 40 × 20 from this.

5. (a) 300 (b) 400 (c) 600 (d) 900

6. (a) Learners fill in more of the cells in the table.
   (b) Doubled: 600; 800; 1 200; 1 800
   Doubled again:1 200; 1 600; 2 400; 3 600
   (c) 600 = 20 × 30; 30 × 20
   800 = 20 × 40; 40 × 20
   1 200 = 40 × 30; 30 × 40; 20 × 60; 60 × 20
   1 800 = 20 × 90; 90 × 20; 60 × 30; 30 × 60

7. Learners complete the table.

8. (a) 3 × 3 = 9 (b) 4 × 4 = 16 (c) 5 × 5 = 25
   (d) 6 × 6 = 36 (e) 7 × 7 = 49 (f) 9 × 9 = 81

9. (a) 400 (b) 900 (c) 1 600
   (d) 2 500 (e) 3 600 (f) 4 900
   (g) 6 400 (h) 8 100 (i) 3 000
   (j) 1 200 (k) 4 200 (l) 4 800
   (m) 6 300 (n) 5 400 (o) 7 200
   (p) 5 600 (q) 1 800 (r) 2 000

3. Write the next five numbers in each pattern.
   (a) 20 40 60 80 100 120
   (b) 200 220 240 260
   (c) 300 320 340 360

4. (a) How much is 20 × 10 and how much is 20 × 20?
   (b) Write your answers in the correct places in the table you made in question 1.
   (c) Double your answer for 20 × 20. Can you use this to fill in another cell in your table?

5. How much is each of the following?
   (a) 10 × 30 (b) 10 × 40
   (c) 10 × 60 (d) 10 × 90

6. (a) Enter your answers for question 5 in your table.
   (b) Double each of your answers, and double them again.
   (c) Can you use this to fill in more cells in your table?

7. Complete your table. You may see some patterns that can help you. (This is an investigation.)

8. How much is each of the following?
   (a) 3 × 3 (b) 4 × 4 (c) 5 × 5 (d) 6 × 6 (e) 7 × 7 (f) 9 × 9
   Write your answers as number sentences, such as 8 × 8 = 64.

9. How much is each of the following?
   (a) 20 × 20 (b) 30 × 30 (c) 40 × 40
   (d) 50 × 50 (e) 60 × 60 (f) 70 × 70
   (g) 80 × 80 (h) 90 × 90 (i) 60 × 50
   (j) 40 × 30 (k) 60 × 70 (l) 60 × 80
   (m) 70 × 90 (n) 60 × 90 (o) 80 × 90
   (p) 70 × 80 (q) 60 × 30 (r) 40 × 50

5.3 Multiplying with bigger numbers

Notes on questions

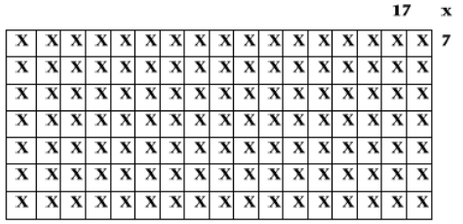
In questions 1 and 2 learners first apply knowledge of basic multiplication facts and then combine the answers (products) to multiply with 2-digit numbers.

In question 3 the order is reversed: learners now have to break down 2-digit numbers to convert the task into smaller tasks that can be done by applying knowledge of basic facts.

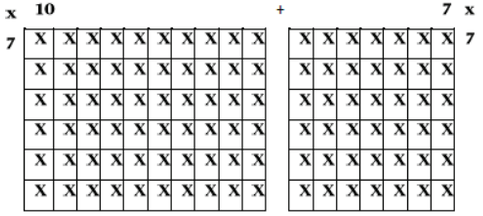
The aim is to develop sound understanding of the logic of the breaking down and building up method of multiplication.

Teaching guidelines

You can visually illustrate this to help learners further to understand the logic of breaking down numbers into place value parts when multiplying. This can be done before question 3. Give learners square grid paper. Ask them to shade or mark with crosses 7 rows of 17. Ask them to repeat this on another grid.



They can paste one of the grids into their exercise books and show that it represents 17 × 7. This is not easy to multiply mentally.



They can then cut or tear the second grid to form two grids. One grid with 7 rows of 10, representing 7 × 10, and the other grid with 7 rows of 7, representing 7 × 7. These are easier to multiply and learners should be able to see that this gives them 70 and 49, so 119 in total.

Answers

1. (a) 1 161; 1 161 (b) 840; 840 (c) 1 058; 1 058 (d) 966; 966
2. (a) 918 (b) 1 118 (c) 924 (d) 783
3. (a) 7 × 30 + 7 × 6 = 210 + 42 = 252
   (b) 30 × 30 + 30 × 6 = 900 + 180 = 1 080
   (c) 1 080 + 252 = 1 332
   (d) Yes, add 1 080 from (b) and 252 from (a) to get 1 332.

5.3 Multiplying with bigger numbers

1. (a) If 20 × 43 = 860 and 7 × 43 = 301,
   how much is 27 × 43,
   and how much is 43 × 27?
   (b) If 30 × 24 = 720 and 5 × 24 = 120,
   how much is 35 × 24,
   and how much is 24 × 35?
   (c) If 40 × 23 = 920 and 6 × 23 = 138,
   how much is 46 × 23,
   and how much is 23 × 46?
   (d) If 60 × 14 = 840 and 9 × 14 = 126,
   how much is 69 × 14,
   and how much is 14 × 69?
2. (a) If 30 × 20 = 600 and 30 × 7 = 210
   and 4 × 20 = 80 and 4 × 7 = 28,
   how much is 34 × 27?
   (b) If 40 × 20 = 800 and 40 × 6 = 240
   and 3 × 20 = 60 and 3 × 6 = 18,
   how much is 43 × 26?
   (c) If 30 × 20 = 600 and 30 × 8 = 240
   and 3 × 20 = 60 and 3 × 8 = 24,
   how much is 33 × 28?
   (d) If 20 × 20 = 400 and 20 × 9 = 180
   and 7 × 20 = 140 and 7 × 9 = 63,
   how much is 27 × 29?
3. (a) Calculate 7 × 36 by breaking 36 down into parts.
   (b) Calculate 30 × 36 by breaking 36 down into parts.
   (c) How much is 30 × 36 + 7 × 36?
   (d) Can you now just add some of your answers to find out how much 37 × 36 is?

Teaching guidelines

You can remind learners that they have already seen, in Section 5.1, an example of what it looks like when you represent 2-digit by 2-digit multiplication as an array broken down into place value parts. You can copy the array on page 60 and ask learners to draw lines breaking the array down into (10 + 6) × (10 + 8). Then calculate each of the parts.

You can show learners how to imagine multiplying 2-digit numbers by 2-digit numbers using a simpler grid. For example, 36 × 37 can be shown as alongside.

×	30	+ 6
30	30 × 30	30 × 6
+ 7	7 × 30	7 × 6

Learners can learn to draw these simple grids to show how to multiply each part of each number separately.

Notes on questions

In questions 5(a) to (j), pairs of questions have the same numbers multiplied; only the order of the numbers is swapped around. Ask learners what is the same about questions 5(a) and 5(b); 5(c) and 5(d); 5(e) and 5(f); 5(g) and 5(h); 5(i) and 5(j). Learners only have to calculate once for each pair.

Learners should look for the patterns of halving and doubling in the numbers multiplied in question 6. In question 6(a), 60 is double 30, so 60 × 53 is double 30 × 53.

Mathematical notes

Although it need not be mentioned explicitly, this work amounts to application of the distributive property.

Answers

4. (a) 40 × 30 = 1 200; 40 × 4 = 160; 6 × 30 = 180; 6 × 4 = 24
   (b) 70 × 60 = 4 200; 70 × 8 = 560; 3 × 60 = 180; 3 × 8 = 24
   (c) 50 × 90 = 4 500; 50 × 3 = 150; 4 × 90 = 360; 4 × 3 = 12
   (d) 80 × 20 = 1 600; 80 × 9 = 720; 2 × 20 = 40; 2 × 9 = 18

5. (a) 1 472 (b) 1 472 (c) 884 (d) 884
   (e) 2 666 (f) 2 666 (g) 1 472 (h) 1 472
   (i) 574 (j) 574 (k) 240 (l) 810
   (m) 420 (n) 1 375

6. (a) Double 1 590 = 3 180 (b) Half of 1 700 = 850
   (c) Half of 1 840 = 920 (d) Double 1 720 = 3 440
   (e) Add one more 67 to 2 412 = 2 479

4. Write the four multiplication facts that will make it easy to calculate each of the following.
   (a) 46 × 34 (b) 73 × 68
   (c) 54 × 93 (d) 82 × 29

43 × 57 can be calculated as follows:
43 = 40 + 3 hence 43 × 57 = 40 × 57 + 3 × 57
40 × 57 and 3 × 57 are now calculated separately:
40 × 57 = 40 × 50 + 40 × 7 = **2 000** + **280**
3 × 57 = 3 × 50 + 3 × 7 = **150** + **21**

To get the answer for 43 × 57 all the parts now have to be added up:
43 × 57 = 2 000 + 280 + 150 + 21
= 2 280 + 150 + 21
= 2 430 + 21 = 2 451

When you do your own calculations you need not write it in so much detail.

5. Calculate each of the following.
   (a) 23 × 64 (b) 64 × 23
   (c) 26 × 34 (d) 34 × 26
   (e) 62 × 43 (f) 43 × 62
   (g) 32 × 46 (h) 46 × 32
   (i) 41 × 14 (j) 14 × 41
   (k) 15 × 16 (l) 45 × 18
   (m) 12 × 35 (n) 25 × 55

6. (a) If 30 × 53 = 1 590, how much is 53 × 60?
   (b) If 50 × 34 = 1 700, how much is 17 × 50?
   (c) If 80 × 23 = 1 840, how much is 23 × 40?
   (d) If 40 × 43 = 1 720, how much is 43 × 80?
   (e) If 36 × 67 = 2 412, how much is 37 × 67?

5.4 Apply your knowledge

Notes on questions

In all these questions learners practise multiplying 2-digit numbers by 2-digit numbers. In questions 9(b), (c) and (d) learners first have to calculate the time Lebogang walks for, before multiplying this by her walking speed.

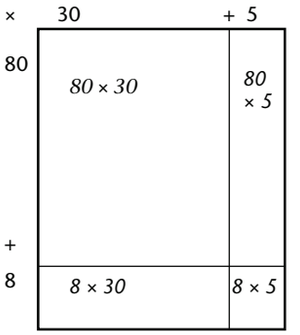
Teaching guidelines

Aim to cover this section in 45 minutes. One possibility is to use

- questions 1 and 9(d) for concept development,
- questions 2, 4(c) and (d), 5, 7 and 9(a) and (b) for classwork, and
- questions 3, 4(a) and (b), 6, 8 and 9(c) for additional practice.

You can use question 1 to model for learners how they can use a grid to see the parts of the numbers to multiply.

In question 9(d) you can model for learners how to calculate the time elapsed from 11:48 a.m. to 1:18 p.m.



Answers

1. 3 080 oranges (88 × 35 oranges)
2. R2 774 (R38 × 73)
3. 2 814 seats (67 × 42 seats)
4. (a) R925 (b) R1 702 (c) R2 738 (d) R3 293
5. 1 260 cm (45 × 28 cm)
6. 874 beads (38 × 23 beads)
7. 952 turns (56 × 17 turns)
8. 2 025 bricks (45 × 45 bricks)
9. (a) 1 092 m (26 × 42 m)
   (b) 1 554 m (From 10:17 to 10:54 is 37 minutes. 37 × 42 m)
   (c) 1 848 m (From 10:54 to 11:38 is 38 minutes + 6 minutes. 44 × 42 m)
   (d) 3 780 m (From 11:48 to 12:48 is 60 minutes. From 12:48 to 13:00 is 12 minutes. From 13:00 to 13:18 is 18 minutes. [60 + 12 + 18] minutes = 90 minutes. 90 × 42 m)

5.4 Apply your knowledge

1. There are 88 bags with 35 oranges each on a truck. How many oranges are there in total?
2. Joe buys 73 chickens at R38 each. How much does he pay in total?
3. There are 42 seats on a bus. How many seats are there on 67 such buses?
4. One packet of sugar costs R37. Calculate the total price of each of the following:
   (a) 25 packets (b) 46 packets
   (c) 74 packets (d) 89 packets
5. Jason must cut 45 pieces of string, each 28 cm long. How much string does he need?
6. A certain bead design consists of 38 rows of beads, each with 23 beads. How many beads are there in total?
7. A wheel on a certain machine makes 17 turns each minute. How many turns does the wheel make in 56 minutes?
8. A certain brick wall has 45 bricks in each row. How many bricks are there in 45 rows?
9. Lebogang walks approximately 42 m each minute.
   (a) Approximately how far will she walk in 26 minutes?
   (b) Approximately how far will Lebogang walk from 10:17 a.m. until 10:54 a.m.?
   (c) Approximately how far will Lebogang walk from 10:54 a.m. until 11:38 a.m.?
   (d) Approximately how far will Lebogang walk from 11:48 a.m. until 1:18 p.m.?

5.5 Difference, ratio and rate

Mathematical notes

Quantities can be compared in two ways:

Example: Susan earns R36 000 per month and William earns R12 000 per month.

- We can state the difference: how much more the one quantity is than the other, for example: "Susan earns R24 000 more than William each month."
- We can state the **ratio**: by what the one quantity must be multiplied to get the other quantity, for example: "Susan earns 3 times as much as William each month."

Both difference and ratio are used to compare two quantities of the same kind.

Rate is something entirely different. Rate is used to compare two quantities of different kinds. A rate describes how much of one quantity (e.g. money) corresponds to one unit (e.g. volume of petrol) of another quantity: R10,40 may correspond to 1 litre of petrol.

In many situations, for example in question 2, two rates are compared by stating the ratio between the rates.

Notes on questions

Question 2 provides learners with an opportunity to learn about and distinguish between difference, ratio and rate. The various parts of question 2 are intended to induce learners to take note of different aspects of the given information, with a view to empower them for the introduction of the idea of rate on the next page.

Teaching guidelines

To help learners who may be challenged by question 2, you may suggest that they start by making a more complete table, with entries for 4, 6, 7 and 9 seconds too. This will help them to answer questions 2(a), (d) and (f). Note that in 2(a) learners are not asked to say how long it will take each machine to produce 100 m, just which machine will produce 100 m first.

Answers

1. 212 oranges
2. (a) Machine B, it produces more rope per second.
   (b) 6 metres (16 m – 10 m)
   (c) 9 metres (24 m – 15 m)
   (d) A: 60 m (From column 1, Machine A makes 10 m every 2 s. 50 m + 10 m = 60 m)
   B: 96 m (From column 1, Machine B makes 16 m every 2 s. 80 m + 16 m = 96 m)
   (e) 16 m
   (f) 80 m

5.5 Difference, ratio and rate

We can compare two quantities by stating what the **difference** is.

If Musi eats 8 slices of bread and Mary eats 5 slices, we can say that Musi eats **3 slices more** than Mary.

You will soon learn about a different way of comparing two quantities.

1. Jansen picked 643 oranges and Gertrude picked 855 oranges. How many more oranges did Gertrude pick than Jansen?
2. Two machines are used to make rope. The lengths of rope made by the two machines in different periods after they were switched on, are given in the table.

	after 2 seconds	after 3 seconds	after 5 seconds	after 8 seconds	after 10 seconds
Machine A	10 m	15 m	25 m	40 m	50 m
Machine B	16 m	24 m	40 m	64 m	80 m

   (a) Which machine do you think will first complete 100 m of rope? Why do you think so?

   (b) What is the difference between the lengths of rope made after 2 seconds?

   (c) What is the difference between the lengths of rope made after 3 seconds?

   (d) What lengths of rope do you think will each of the two machines make in a period of 12 seconds? Give reasons for your answer.

   (e) How much rope does Machine B make while Machine A makes 10 m of rope?

   (f) How much rope does Machine B make while Machine A makes 50 m of rope?

Mathematical notes

The concept of ratio is introduced at the top of the page, although the word "ratio" can still be withheld for a while. (It is better to withhold the word till the concept has been experienced in at least one other context.) Ensure that learners relate the statement to what they have done in question 2.

Notes on questions

Question 3 is critical. You may extend it by asking learners to check for other periods of time too (multiples of 5 seconds).

Questions 4 and 5 provide learners with experiences of ratio in different contexts.

Teaching guidelines

Try to complete this section in $1\frac{1}{2}$ hours. One possibility is to use

- questions 2, 3, 7, 8 and 9 for concept development,
- questions 1, 4, 6 and 10 for classwork, and
- questions 5 and 11 for additional practice.

At this stage you do not need to show learners procedures for calculating ratios. You just need to encourage them to work sensibly with the information.

Learners could use a range of different methods to find solutions. One approach in question 4 is to produce a table similar to the one in question 2.

Cups of concentrate	3	6	9	12	15
Cups of water	5	10	15	20	25

They can then count in 3s to complete the top row, and in 5s to complete the bottom row.

Another approach is to say: for every 3 cups of concentrate she uses 5 cups of water. 6 cups concentrate is 2 × 3 cups, so you need 2 × 5 cups water; 12 is 4 × 3 cups, so you need 4 × 5 cups of water; 20 cups of water is 4 × 5 cups, so you need 4 × 3 cups of concentrate.

Learners can use skip counting to produce tables for questions 5, 6 and 7, but this is not efficient as they are not asked to provide lots of examples. It is less time-consuming for learners to think about the isolated multiples required.

See the next page for notes on questions 5 and 6.

Answers

3. The statement is true.
4. (a) 10 cups (b) 20 cups (c) 25 cups (d) 12 cups
5. 36 red beads
6. (a) 16 cups of white flour (b) 36 cups of wholewheat flour

The speed of rope making by Machines A and B can be compared by saying: *For every 5 m of rope made by Machine A, 8 m of rope is made by Machine B.*

3. Check whether this statement is true for the lengths of rope made by the two machines in a period of 5 seconds, and in a period of 10 seconds.
4. Sally makes juice by mixing juice concentrate with water. For every 3 cups of concentrate, she uses 5 cups of water.
   (a) How many cups of water will she use with 6 cups of concentrate?
   (b) How many cups of water will she use with 12 cups of concentrate?
   (c) How many cups of water will she use with 15 cups of concentrate?
   (d) How many cups of concentrate will she use with 20 cups of water?

The word **ratio** can be used to describe situations like the above.

In the juice that Sally makes, concentrate and water are mixed **in the ratio 3 to 5**.

This means that for every 3 parts of concentrate, there are 5 parts of water.

5. Betty has 3 red beads for every 2 blue beads that she has. How many red beads does she have, if she has 24 blue beads?
6. Jackson is baking bread. He mixes white flour and wholewheat flour in the ratio 2 to 3.
   (a) How many cups of white flour will he mix with 24 cups of wholewheat flour?
   (b) How many cups of wholewheat flour will he mix with 24 cups of white flour?

Notes on questions

In question 5 learners can calculate that 24 blue beads are 12 groups of 2 blue beads, so they need 12 groups of 3 red beads = 36 red beads.

In question 6(a), 24 cups of wholewheat flour are 8 times 3 cups, so he needs 8 × 2 = 16 cups of white flour. In question 6(b), 24 cups of white flour are 12 times 2 cups, so he needs 12 × 3 = 36 cups of wholewheat flour.

Teaching guidelines

Question 8 alerts learners to the possibility that two ratios may be equivalent. It is not necessary to dwell on this in Grade 4.

The concept of rate is introduced by means of question 10.

Answers

7. (a) 16 cups of white flour
   (b) 36 cups of wholewheat flour

8. The answers in questions 6 and 7 are the same.
   All questions use 24 cups of the one kind of flour.
   The ratios are equivalent, i.e. the ratio 2 to 3 is the same as the ratio 4 to 6.
   This makes sense because 2 × 2 = 4 and 2 × 3 = 6.

9. (a) The ratio between the distance run by Athlete A and the distance run by Athlete B is 3 to 4. However, learners may look to the first column of the table and give the ratio as 300 to 400. This answer is also acceptable at this stage. However, it is easier for learners to work with 3 to 4 to check the other ratios.
   (b) Learners check their answer in (a) against the distances given in the table.

   Learners can check their answers by dividing each distance for Athlete A by 3, and for Athlete B by 4. This will give them the same distance for each athlete, for example 100 m after 5 minutes; 160 m after 8 minutes; 200 m after 10 minutes; 280 m after 14 minutes. They could also divide each column by the number of minutes to get a ratio of 60 to 80 each time.

10. (a) Yes, because 300 m in 5 minutes means 300 m ÷ 5 = 60 m per minute.
    (b) 80 m (400 ÷ 5 = 80 m per minute)

11. 1 489 cm (1 387 cm + 34 cm + 34 cm + 34 cm = 1 387 cm + 102 cm)

7. For the bread she bakes, Melanie mixes white flour and wholewheat flour in the ratio 4 to 6.
   (a) How many cups of white flour will she mix with 24 cups of wholewheat flour?
   (b) How many cups of wholewheat flour will she mix with 24 cups of white flour?

8. Compare your answers for questions 6 and 7.
   What can you say about the ratios 4 to 6 and 2 to 3?

9. The distances covered by two long-distance runners after different periods of time are given in the table.

	after 5 minutes	after 8 minutes	after 10 minutes	after 14 minutes
Athlete A	300 m	480 m	600 m	840 m
Athlete B	400 m	640 m	800 m	1 120 m

   (a) Use ratio to compare the running of the two athletes.
   (b) Make sure that your comparison is correct for all the time periods in the table.

10. (a) Is it correct to say that Athlete A runs about 60 m in each minute?
    (b) Approximately what distance does Athlete B run in one minute?

We can say that Athlete A runs at a **rate** of approximately **60 m per minute**.

Note that **rate** is a different word than **ratio**.

We say "approximately" because he may sometimes run a bit faster, and sometimes a bit slower.

11. A certain tree was 1 387 cm tall at the end of 2015.
    If it grows at a rate of 34 cm per year, how tall can you expect it to be at the end of 2018?

5.6 Breaking down in a different way to multiply

Mathematical notes

In previous sections, learners broke down numbers into place value parts and used the distributive property to convert multiplication tasks into smaller tasks. Multiplication can also be done by breaking down numbers into factors. This method is useful where numbers have lots of simple factors, for example 27 × 36. The aim is to break at least one of the numbers down into factors which are easy to multiple by, for example 2, 3, 5, 10. This method is not useful where numbers have few factors, for example 37 × 34, because 37 × 2 × 17 does not make the calculation easier.

Learners have already encountered breaking down numbers into factors to multiply in Section 5.1 questions 6, 7, 8 and 9 (page 163) and in Term 1 in Section 5.6 (pages 71 and 72).

Learners will sometimes use this method of breaking down numbers when multiplying to divide: see Term 2 Unit 10 Section 10.1.

Teaching guidelines

Try to complete this section in 1 hour.

Once learners have completed question 1, ask them what is the same about questions 1(a), (b) and (c). Learners may express this differently, but all the answers are equal to 20 × 2 × 2 × 2. Ask them what it is about the numbers multiplied that allow the answers to be the same. Repeat this for questions 1(d), (e) and (f). Learners may express this differently, but all the answers are equal to 10 × 3 × 3 × 2.

Use the tinted passage to explain to learners how to do multiplication by breaking numbers down into factors.

Answers

1. (a) 160 (b) 160 (c) 160
   (d) 180 (e) 180 (f) 180
2. Learners' answers might differ. (a), (c), (e) and (f) have the same answer.
3. (a) 960 (b) 1 020 (c) 960
   (d) 880 (e) 960 (f) 960
4. (a) 4 (b) 12 (c) 36
   (d) 24 (e) 18 (f) 54
5. (a) Question 4(c) (b) Question 4(f)
6. Learners follow the prescribed method. The answers are:
   (a) 972 (b) 516 (c) 888
   (d) 1 242 (e) 1 809 (f) 2 784

5.6 Breaking down in a different way to multiply

1. Calculate the following.
   (a) 80 × 2 (b) 40 × 4 (c) 20 × 8
   (d) 90 × 2 (e) 30 × 6 (f) 10 × 18
2. Do not do these calculations now. Just state which of the calculations you think will have the same answer.
   (a) 24 × 40 (b) 34 × 30 (c) 48 × 20
   (d) 44 × 20 (e) 96 × 10 (f) 12 × 80
3. Do the calculations in question 2 now and check your answer for question 2.
4. Calculate.
   (a) 2 × 2 (b) 2 × 2 × 3
   (c) 2 × 2 × 3 × 3 (d) 2 × 2 × 2 × 3
   (e) 2 × 3 × 3 (f) 2 × 3 × 3 × 3
5. (a) Which calculation in question 4 has the answer 36?
   (b) Which calculation in question 4 has the answer 54?

32 × 18 can be calculated like this:

32 × 18 = 32 × 2 × 9
= 32 × 2 × 3 × 3
= 64 × 3 × 3
= 192 × 3
= (100 + 90 + 2) × 3
= 300 + 270 + 6
= 576

6. Calculate each of the following in the way 32 × 18 was calculated above.
   (a) 27 × 36 (b) 43 × 12 (c) 37 × 24
   (d) 23 × 54 (e) 67 × 27 (f) 58 × 48

Grade 4 Term 2 Unit 6 Properties of three-dimensional objects

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
6.1 Objects with different shapes	Examining the surfaces of three-dimensional objects Identifying spheres, cylinders, pyramids and rectangular prisms	173 to 175
6.2 Make prisms, pyramids and cones	Making paper models of prisms, pyramids and cones in order to focus on their properties	176 to 178
6.3 Put faces together	Focusing on the faces of objects by building up objects from two-dimensional faces	179
6.4 Differences between shapes	Identifying properties to compare different three-dimensional objects	180

CAPS time allocation	5 hours
CAPS page references	22 and 78 to 79

Mathematical background

All real objects are three-dimensional. Spheres, cylinders, prisms and pyramids are solid geometrical objects. In order to focus on the properties of geometrical objects we can build models of the objects. In this unit we build models from paper and card. Such models and drawings become our references as we develop an understanding of the properties and relationships between different three-dimensional objects. Sometimes we also use real-life, everyday objects to represent geometrical objects. These everyday objects, and models of the objects, may have some, but not necessarily all the properties of geometrical objects. For example, an open box may represent a rectangular prism, but you will need to ask learners to imagine that it is a closed box.

We can also represent the objects in a drawing. We follow mathematical conventions when we try to draw three-dimensional objects, for example using solid lines to show edges of faces that we can see (these are on the front of the object as we see it) and broken lines to show the edges of the faces at the back of the object (edges that we cannot see as we view the object).

Three-dimensional objects can be grouped according to their properties (once the properties are grasped). This is called classification. Classification systems are all based on sameness and difference. Questions such as: "*Compare these objects. What is the same about them?*", "*Which properties of these two objects differ?*", "*Is this object a special form of that object?*", etc. help learners to focus on the properties of the object, and can be used as the basis for grouping like objects (classification).

Grade 4 learners first focus on the surfaces of objects. These surfaces can be flat (polygons, circles, etc.) or curved (cones, cylinders or spheres). The surfaces of objects are one way to identify and compare different objects. Learners then look at the shape of the flat surfaces (or faces) of the objects. In Grade 4 learners look for circular faces, triangular faces and rectangular faces (including squares).

Resources

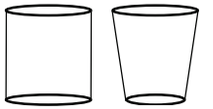
This topic involves a number of practical activities for which various resources are needed. These resources are listed in the various sections.

6.1 Objects with different shapes

Mathematical notes

A flat surface of a 3-D object is called a face. On page 174 a rectangular prism is described as an object with flat surfaces that are all in the shape of rectangles. In the units on two-dimensional shapes it was explained that a square is a special kind of rectangle. This means that faces of rectangular prisms can be square. So a cube is a special kind of rectangular prism.

A cylinder has one curved surface and two flat, circular surfaces of the same size. A paper cup or a plant pot is not a cylinder.



Cylinder Not a cylinder

Some everyday objects, for example cylindrical tubes, are open at one or both ends. The face(s) need(s) to be imagined. Technically these are not cylinders, but models of cylinders.

Similarly, an open box is shaped like a rectangular prism; it is a model of a rectangular prism. Learners will need to imagine that the open face is closed (or they can take a sheet of paper and close it).

Resources

Objects with curved surfaces including, but not limited to, spheres, for example balls, oranges, marbles, etc.

Objects with flat surfaces including, but not limited to, rectangular prisms, for example boxes, bricks, etc.

Objects with flat and curved surfaces including, but not limited to, cylinders, for example cardboard tubes, cylindrical piping, cans, caps of some spray bottles, glue sticks, shoe polish tins, snuff or mentholated ointment tins, etc.

A sheet of paper (can be scrap paper) for each learner; sticky tape; scissors.

Encourage learners to bring objects with as many different shapes as possible to class.

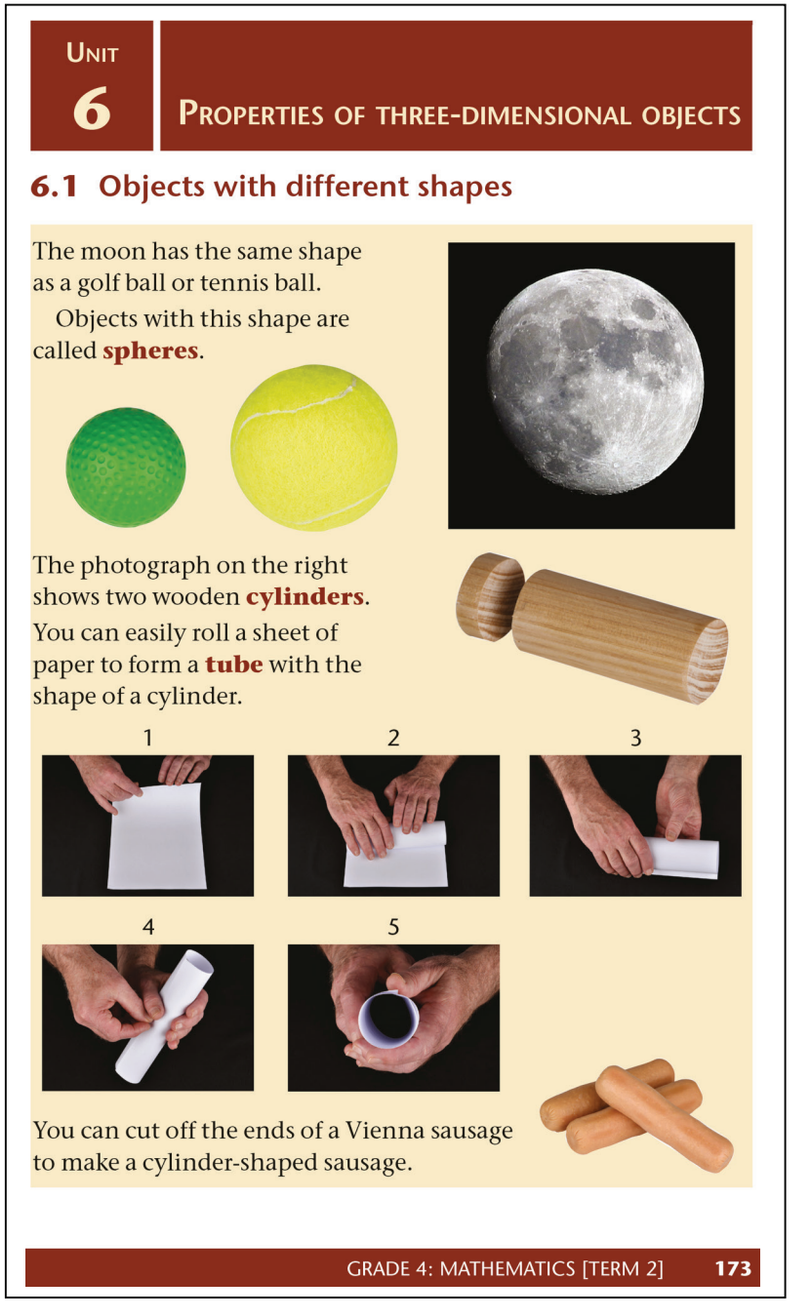
Learners can also make spheres and cylinders from clay, or roll cylinders from paper. Explain to learners that a cylinder must be the same width all along.

You can also make square-based pyramids and rectangular prisms from the templates provided in the Addendum (pages 451 and 452).

Teaching guidelines

Aim to spend about 1 hour on this section.

It is much easier to learn about three-dimensional objects when working with real objects than when working with pictures and descriptions of the objects.



UNIT 6 PROPERTIES OF THREE-DIMENSIONAL OBJECTS

6.1 Objects with different shapes

The moon has the same shape as a golf ball or tennis ball.

Objects with this shape are called **spheres**.

The photograph on the right shows two wooden **cylinders**.

You can easily roll a sheet of paper to form a **tube** with the shape of a cylinder.

1 2 3

4 5

You can cut off the ends of a Vienna sausage to make a cylinder-shaped sausage.

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If you have sufficient objects, let learners work in groups to sort the objects into three groups: objects with flat surfaces only; objects with curved surfaces only; objects with flat and curved surfaces. If you have too few objects, find an open space where you can place the objects on the ground. Label areas for objects with flat surfaces only; objects with curved surfaces only; objects with flat and curved surfaces. Let learners each choose an object and place it in the correct group.

Explain to learners that they need to imagine that all objects are closed. So they need to imagine surfaces on, for example, the open ends of tubes.

Allow time for discussion about whether everyone agrees with the groupings.

Ask learners whether any of the objects are shaped like three-dimensional geometric objects. If so, let them hold these up and give their names. In Grade 3 learners will have called *spheres* ball-shapes and *rectangular prisms* box-shapes. You will need to introduce the correct terminology. Check that learners can identify rectangular prisms, spheres and cylinders. You can introduce them to triangular prisms, but they don't need to be assessed on their knowledge thereof.

You can read questions 1 and 2 to learners while they look at the pictures on page 175. Ask them to write down the answers.

Give each learner a piece of scrap paper and some sticky tape, and ask them to roll a cylinder. Keep these cylinders in a safe place. Learners will work with them over the next few days.

Possible misconceptions

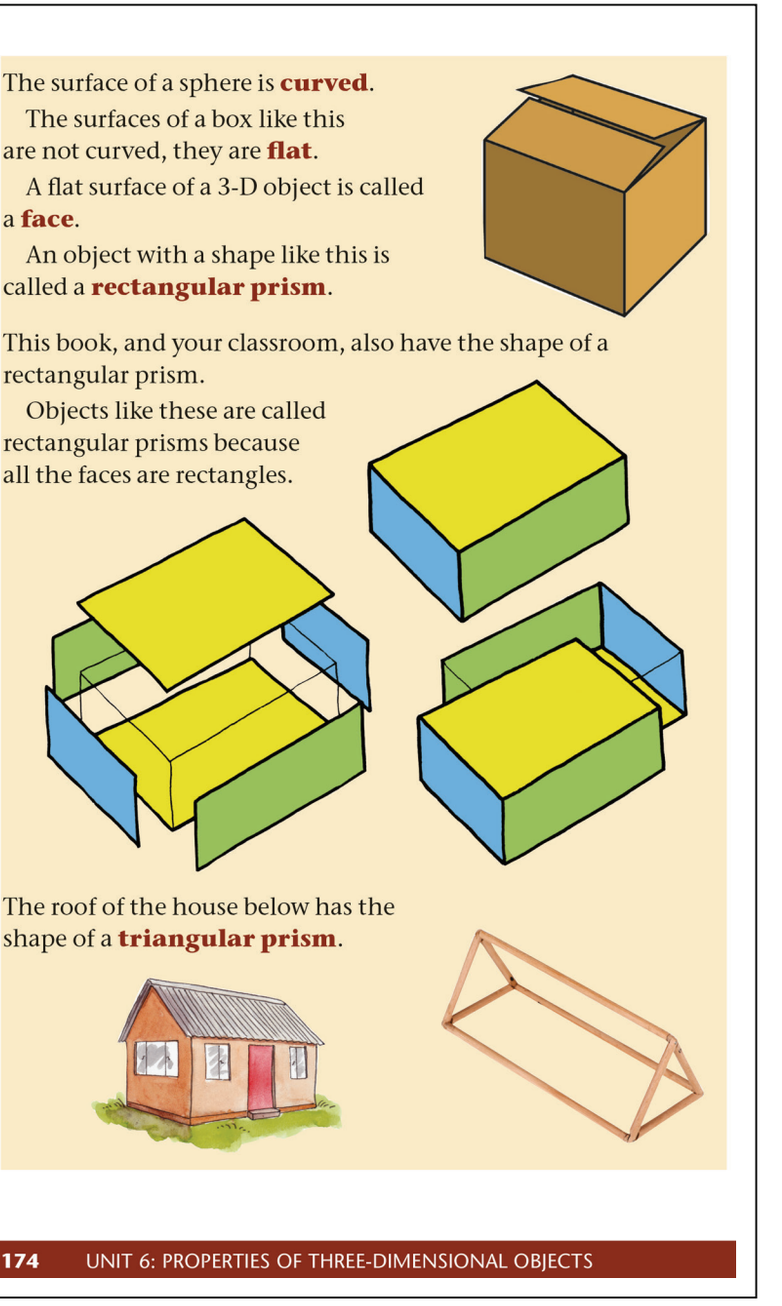
Some learners may not distinguish clearly between three-dimensional objects and the two-dimensional figures that form their faces. This can only be properly addressed with a stock of objects and a focused discussion. In Term 4 learners will make objects from cardboard cut-outs – this will further help to clarify this.

Sometimes learners think that a cylinder must be taller than it is wide. This is not true. The photograph of the two wooden cylinders on page 173 clearly shows that the smaller cylinder is wider than it is tall. Try to collect other examples, such as shoe polish tins, snuff tins, ointment tins, and coins.

Critical knowledge

Learners must be able to identify and name rectangular prisms, spheres and cylinders. They should know that:

- rectangular prisms have flat surfaces (faces)
- spheres have curved surfaces
- cylinders have two circular faces and a curved surface.


The surface of a sphere is **curved**.

The surfaces of a box like this are not curved, they are **flat**.

A flat surface of a 3-D object is called a **face**.

An object with a shape like this is called a **rectangular prism**.

This book, and your classroom, also have the shape of a rectangular prism.

Objects like these are called rectangular prisms because all the faces are rectangles.

The roof of the house below has the shape of a **triangular prism**.

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Mathematical notes

The roof of the house and the skeleton of the triangular prism on page 174 are shaped like triangular prisms, but because they do not have five faces, they are not actually triangular prisms. They are models of triangular prisms.

Notes on questions

Question 3 is more difficult than questions 1 and 2 because learners are working from abstract language and have to imagine the geometrical objects. Give learners examples of spheres, cylinders and rectangular prisms. Read the question while they try to find the object described. They should write the answers in their exercise books.

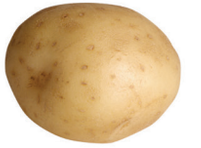
Answers

1. (a) Potato
   (b) Cooking pot, battery
   (c) Brick
   (d) Brick, pyramid
   (e) Potato
   (f) Cooking pot, clay pot, battery
2. Circle
3. (a) Sphere
   (b) Rectangular prism
   (c) Cylinder

1. Look at the objects in the photographs.
   (a) Which of the objects have the shape of a sphere?
   (b) Which of the objects have the shape of a cylinder?
   (c) Which of the objects have the shape of a rectangular prism?
   (d) Which of the objects have flat surfaces only?
   (e) Which of the objects have curved surfaces only?
   (f) Which of the objects have curved and flat surfaces?



A brick



A potato



A pyramid



A cooking pot



A clay pot



A battery

2. What is the shape of the face of the glass that touches the table?



3. In each case state what kind of 3-D object it can be.
   (a) It has only one surface, and the surface is curved.
   (b) It has six surfaces, and the surfaces are all flat.
   (c) It has two flat surfaces and one curved surface.

6.2 Make prisms, pyramids and cones

Mathematical notes

In this section learners fold rectangular prisms, pyramids and cones. The aim is for learners to have three-dimensional objects that they can work with and use to further explore their properties.

In Term 1 learners used drawings of two-dimensional shapes to explore their properties by talking about their similarities and differences. Similarly learners will use the prisms, pyramids and cones to look at what is the same about them and what is different. This helps learners to learn the properties of these objects.

Critical knowledge

Identify and name rectangular prisms, cones and square-based pyramids from real objects and drawings.

Prisms and pyramids have flat faces.

Cones have a flat, circular face and a curved surface.

Resources

The paper cylinders folded in the previous lesson.

Three sheets of paper (can be scrap paper) for each learner.

Sticky tape.

Scissors.

Teaching guidelines

Aim to spend about 2 hours on this section.

One of each pair of learners can use their cylinders to fold a rectangular prism. When learners fold the second set of creases, it is important that the first set of creases line up exactly. Check that all the corners of the prism are right angles; the imagined faces should be square. Remind learners that they should imagine that the prism is closed. Focus learners' attention on key features of the prism by asking questions like: "*Are all faces of my prism the same?*"

Help learners to compare prisms with cylinders by asking questions like: *"How do the surfaces of cylinders and prisms differ?", "What is the same about prisms and cylinders?"* (Answers: Cylinders' surfaces are rounded, prisms' surfaces are flat. They both have a constant width.)

Answers

1. Learners fold their own rectangular prism.

6.2 Make prisms, pyramids and cones

1. Follow these instructions to make a tube in the shape of a rectangular prism:

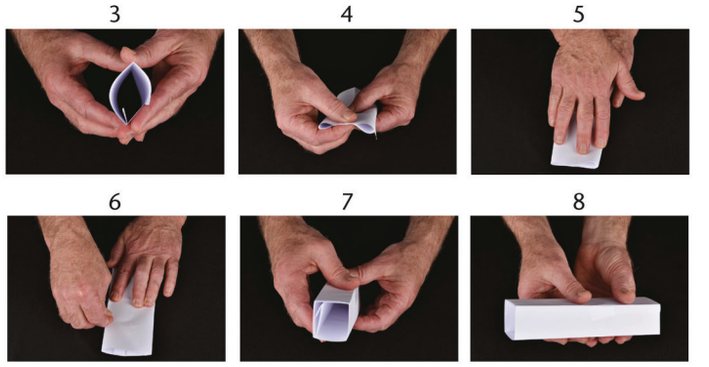
   Use a sheet of paper and roll it into a tube. You can use sticky tape to keep the tube together.

   Press it flat and make the creases sharp.

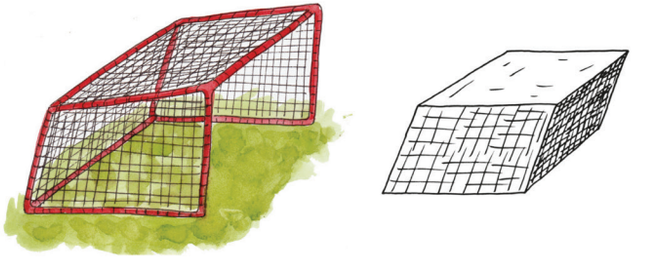
   Continue as shown in the photographs to make two more edges.







These objects have a prism shape, but they are not rectangular.



Possible misconceptions

Some learners may struggle with cutting and folding. These are important skills. Use peer support as far as possible and step in to assist where necessary. A good strategy is to show a struggling learner a few of the steps they need to follow and then ask them to redo these steps, and also the remaining steps.

Teaching guidelines

Give each learner a piece of scrap paper and some sticky tape, and ask them to roll a cone. You can demonstrate how to do it.

Remind learners that they should imagine the cone is closed. Focus learners' attention on key features of cones by asking questions like: "*Are the surfaces of cylinders and cones the same or different?*", "*What is the same about cones and cylinders?*", "*What is different about cones and cylinders?*" (Answers: A cylinder has two flat surfaces and one curved (rounded) surface. A cone has only one flat surface and a curved surface. Cones and cylinders both have one curved surface. A cone has a vertex and one circular edge, while a cylinder has no vertices but two identical circular edges.)

Mathematical notes

The paper model of the cone is open at its base: it does not have a circular base. We can say that it is shaped like a cone. Technically it is not a cone, only a model of a cone. The template of the cone provided in the Addendum (page 466) does make a closed cone.

Answers

2. Learners fold their own cones.

A cone has a curved surface and one flat face called the **base**. The base of a cone is in the form of a circular disk.



2. To make a cone, roll a sheet of paper as shown in these photographs.

1



2



3



4



5



6



7



You can use sticky tape to keep the cone together. You can cut off some of the paper to give it a proper cone shape.

Teaching guidelines

One of each pair of learners can use their cones to fold a pyramid. When learners fold the second set of creases, it is important that the first set of creases line up exactly. Check that all the corners of the pyramid are right angles: the imagined face should be square.

Remind learners that they should imagine that the pyramid is closed. Focus learners' attention on key features of the pyramid by asking questions like: "*Are all faces of my pyramid the same?", "Do the surfaces of the cone and pyramid differ?", "How do they differ?", "What is the same about the pyramid and cone?"* (Answers: A pyramid has triangular and square faces. A cone has flat and curved surfaces. A pyramid has only flat faces. They are both wide at one end and come to a point at the other end.)

Help learners to compare:

- the cone and the cylinder: *How are they the same? How do they differ?*
- the rectangular prism and the pyramid: *How are they the same? How do they differ?*

Remind learners to imagine that the pyramid and prism are closed: *"What shape are most of the faces of the square-based pyramid?", "What shape are most of the faces of the rectangular prism?"* (Answers: Most of the faces of any pyramid are triangles. All the faces of a rectangular prism are rectangles.)

Mathematical notes

All squares are rectangles: a square is a special kind of rectangle with all the sides the same length.

There are many different kinds of prisms and pyramids. Grade 4 learners only need to be able to identify and name rectangular prisms and square-based pyramids.

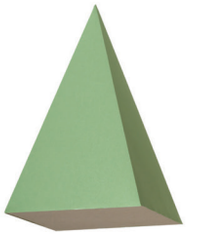
A prism has flat faces, most of which are rectangles. Two faces are identical and opposite each other, and they could be triangles, pentagons, hexagons, etc.

A pyramid has flat faces, most of which are triangles. It has one face that can be any polygon: triangle, square, pentagon, hexagon, etc.

Answers

3. (a) Cones and pyramids are wide at one end and come to a point at the other end. Cylinders and rectangular prisms have a constant width.
   (b) Pyramids and rectangular prisms have only flat faces. Cones and cylinders have one curved surface and one or two flat surfaces.
   Rectangular prisms and pyramids have rectangular bases. Cones and cylinders have circular bases.
4. Learners fold their own pyramids.

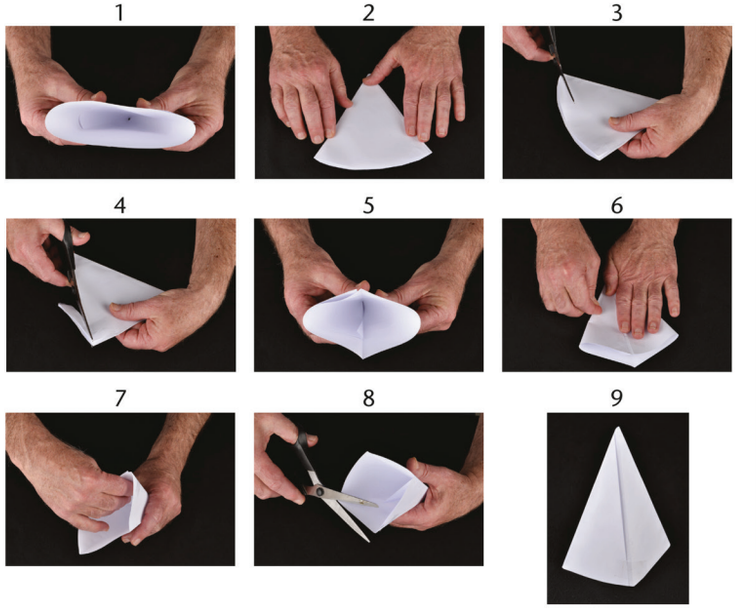
A 3-D object with flat triangular faces that all meet at one point (apex) is called a **pyramid**.



*This pyramid has a square base, hence it is called a **square-based pyramid**.*

3. (a) How do cones and pyramids differ from cylinders and rectangular prisms?
   (b) How do pyramids and rectangular prisms differ from cones and cylinders?

4. To make a square-based pyramid, first make an open cone and then do what the person in the photographs is doing. Use sticky tape to keep it together, and cut off waste paper to make a neat pyramid.



6.3 Put faces together

Mathematical notes

Question 2 of this section asks learners to do a freehand drawing of a square-based pyramid, a cone, a cylinder and a rectangular prism. In primary school learners are shown drawings of geometrical and everyday objects from many angles, but these drawings are always shown using the same rules of perspective. There are, however, different kinds of perspectives, each with their own rules of construction. These different perspectives are called projections. Since learners have not yet been shown how to draw objects according to the standard rules of perspective, you cannot expect their drawings to follow these rules. Question 2 simply prepares learners for the drawings that they will see in Section 6.4.

Resources

Learners should have the prisms, pyramids, cones and cylinders that they built earlier on their desks.

Teaching guidelines

The questions ask learners to work "in their heads" with the given shapes. This is a challenging activity. Let learners work with the 3-D objects and try to match the faces of the objects with the shapes in question 1. Aim to spend about 1 hour on this section.

Possible misconceptions

Learners who are not successful in meeting the challenge posed by the questions will *have to be given paper cut-outs* to work with to support their spatial reasoning: use the templates provided for Term 4 Unit 4 Section 4.1 in the Addendum (pages 463 to 466). Let learners work in pairs to make the objects from the faces.

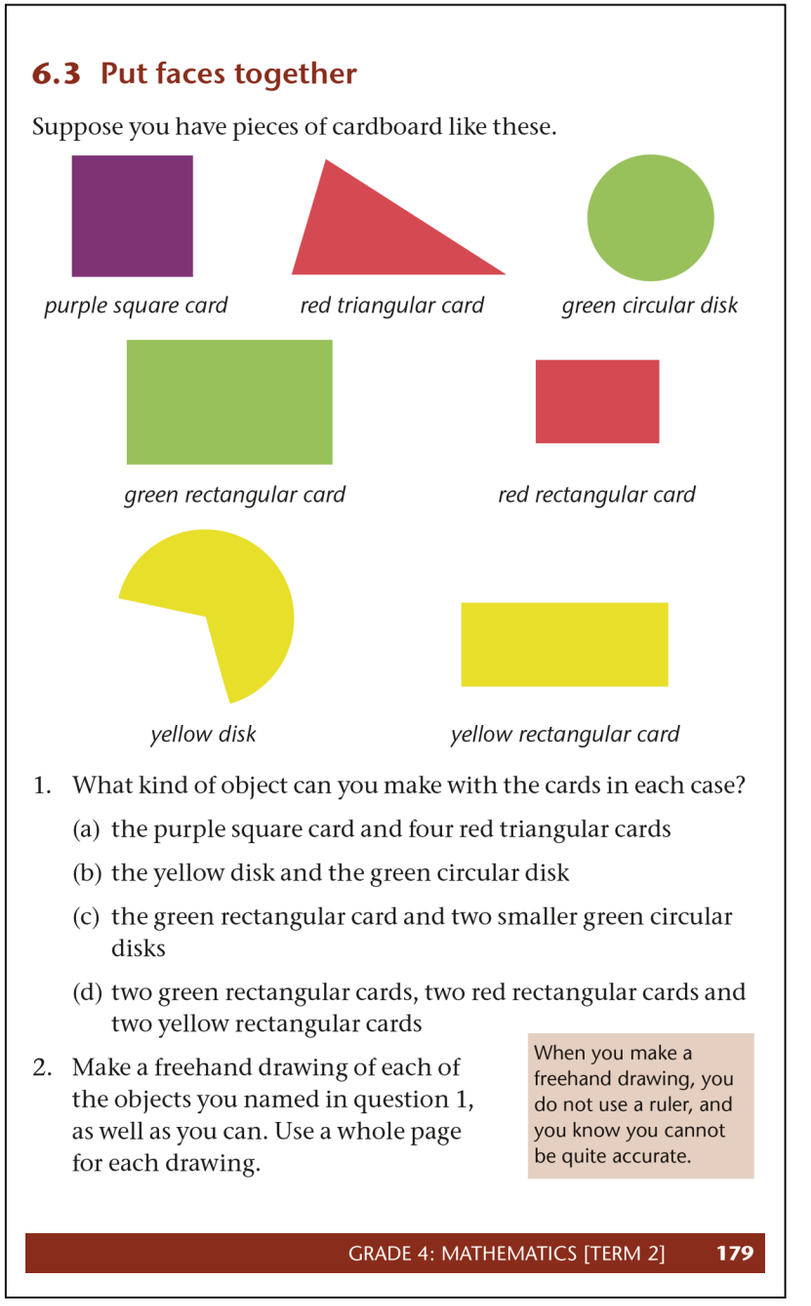
Some learners may struggle to represent three-dimensional objects as flat drawings. Allow them to come back to this (i.e. question 2) after they have completed Section 6.4.

Notes on questions

Question 2 requires some support. It will be useful to introduce the convention of showing edges that are out of the line of sight as broken lines.

Answers

1. (a) Square-based pyramid (b) Cone
   (c) Cylinder (d) Rectangular prism
2. Learners' own drawings. Do not expect learners' drawings to follow the standard conventions of perspective.



6.3 Put faces together

Suppose you have pieces of cardboard like these.

1. What kind of object can you make with the cards in each case?
   (a) the purple square card and four red triangular cards
   (b) the yellow disk and the green circular disk
   (c) the green rectangular card and two smaller green circular disks
   (d) two green rectangular cards, two red rectangular cards and two yellow rectangular cards
2. Make a freehand drawing of each of the objects you named in question 1, as well as you can. Use a whole page for each drawing.

When you make a freehand drawing, you do not use a ruler, and you know you cannot be quite accurate.

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6.4 Differences between shapes

Mathematical notes

This section consolidates the kinds of reasoning learners have developed in the previous three sections. It also shows the conventional way we *represent* three-dimensional objects as two-dimensional drawings (because it is impossible to *draw* a 3-D object on a flat piece of paper).

Resources

Learners should have the prisms, pyramids, cones and cylinders that they built earlier on their desks. They should also have a sphere.

Teaching guidelines

Let learners have access to all the objects listed in question 1. Remind learners that they must think of all the objects as closed.

Once they have completed question 1, let learners take turns to explain their answers to the class, using the models of the objects. This can also be done after question 4.

Let learners first attempt questions 2 and 3, then show them how the lines in the drawings represent the edges of the shapes that they have built. Show learners how to draw a rectangular prism. Aim to spend about 1 hour on this section.

Answers

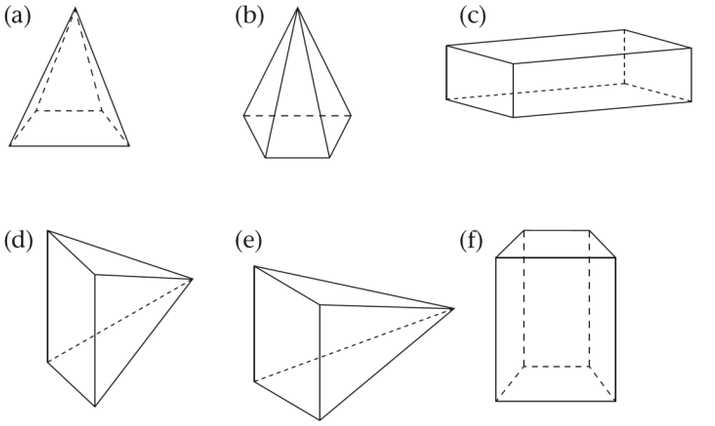
1. (a) Rectangular prisms and square-based pyramids
   (b) Spheres
   (c) Cylinders, cones
   (d) Rectangular prism
   (e) Square-based pyramids
2. Rectangular prisms: (c), (f)
3. Square-based pyramids: (a), (e)
4.

	Number of faces	Shapes of faces	Number of curved surfaces
Rectangular prism	6	rectangles	0
Sphere	0		1
Cylinder	2	circles	1
Cone	1	circle	1
Square-based pyramid	5	square triangles	0

6.4 Differences between shapes

1. You know about spheres, cylinders, rectangular prisms, cones and square-based pyramids. Which of these objects have
   (a) only flat surfaces? (b) only curved surfaces?
   (c) flat and curved surfaces? (d) only rectangular surfaces?
   (e) triangular and rectangular surfaces?

2. Which drawings below show rectangular prisms?



3. Which of the above drawings show square-based pyramids?

4. Complete this table.

	Number of faces	Shapes of faces	Number of curved surfaces
Rectangular prism			
Sphere			
Cylinder			
Cone			
Square-based pyramid			

Grade 4 Term 2 Unit 7 Geometric patterns

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 Our geometric art heritage	Introduction	181
7.2 Clever counting	Visual recognition of repeating units	182 to 183
7.3 Do not count – *calculate!*	Writing calculation plans	183 to 184
7.4 Growing patterns	Introduction to continuing geometric patterns	184 to 185
7.5 From pictures to tables	Representing geometric patterns as numeric patterns in tables	186
7.6 Writing our plans as flow diagrams	Expressing patterns as flow diagrams	187 to 188

CAPS time allocation	4 hours
CAPS page references	19 and 80 to 82

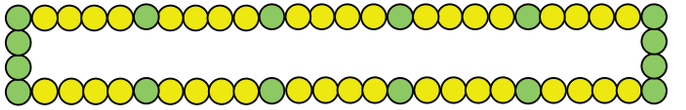
"The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test." GH Hardy

Mathematical background

Below we describe the principles and guidelines underlying our work on geometric patterns.

Given a geometric situation in which some repetition occurs, people often immediately make a table, and then handle it exactly as they handle a numeric pattern (using common differences, etc. to find a rule). However, the approach here is to use the *visual* aspects of geometric patterns to find rules based on the *structure* of the situation.

To enable learners to use the *structure* of the situation, it is vital that learners progress from counting the objects in ones to counting in larger units – what we call *clever counting*. Clever counting should reinforce learners' number concepts and number knowledge. For example, instead of counting these beads one by one, we can identify units of 4 and rather count in fours: 4, 8, 12 ... while keeping track of how many fours we have counted.

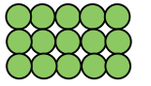


However, in this unit we want to encourage learners not to count at all, but rather to write a numerical expression (calculation plan) and start to use *calculation*, for example:

10×4 (the yellows) + $2\times4 + 8$ (the greens) or $12\times4 + 8$ or 14×4

This process of using the *structure* forms the basis of developing algebra – the counting unit will later become the *variable* in the situation; the counting *method* will become a numerical and later an *algebraic expression*.

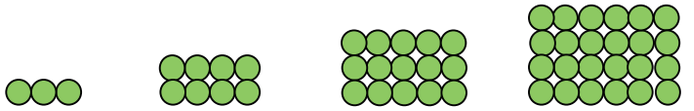
A second example: given this rectangular array of beads, learners should not *count* 1, 2, 3, ... but should see larger units and *calculate* the number of objects as, for example, $3 + 3 + 3 + 3 + 3$ or $5 + 5 + 5$ or 3×5 or 5×3. This type of work will then also reinforce the properties of operations, for example the commutativity of multiplication.



7.1 Our geometric art heritage

Mathematical background *(continued)*

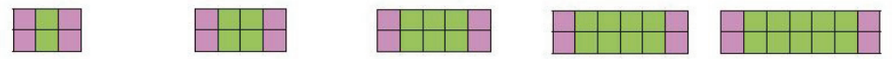
In growing patterns such as the one below, such a structural approach using appropriate counting units means that we do not try to generalise the *number sequence* (the numbers) 3, 8, 15, 24, ... but it helps us to generalise the numeric expressions (our clever counting *method,* the *structure*): 3×1, 4×2, 5×3, 6×4, ...



Picture 1	Picture 2	Picture 3	Picture 4	Picture 100
3×1	4×2	5×3	6×4	102×100

We will not introduce any symbols like *x* and *y* for the formulation of the rules, but we use the words *Number* and *Picture Number* as our variables. We will use flow diagrams and number expressions to formulate our rules – these are both calculation procedures (we call them *plans*) that help learners to find input and output values.

The emphasis in this work should not be on getting a *numerical answer,* but on *seeing*, *writing* and *using* the *structure* in the situation, as in this example:



Picture 1	Picture 2	Picture 3	Picture 4	Picture 5	Picture 87
1 + 1 + 4	2 + 2 + 4	3 + 3 + 4	4 + 4 + 4	5 + 5 + 4	87 + 87 + 4
2×1 + 4	2×2 + 4	2×3 + 4	2×4 + 4	2×5 + 4	2× 87 + 4
			or		
3 + 3	4 + 4	5 + 5	6 + 6	7 + 7	89 + 89
2 × 3	2 × 4	2 × 5	2 × 6	2 × 7	2 × 89

UNIT 7

GEOMETRIC PATTERNS

7.1 Our geometric art heritage

In this unit we study geometric patterns, such as the decorations on the walls of the homes of the Ndebele people in Mpumalanga.



The Ndebele like to decorate the outside of their homes with colourful designs.



The Ndebele also make and wear beautiful bead necklaces and bracelets.

They love geometric patterns! Do you?

A **geometric pattern** is a repeated decorative design.

In this unit we will not focus on the types of figures (such as triangles, rectangles, and so on) but on the *number* of figures or beads in such repeating patterns.

7.2 Clever counting

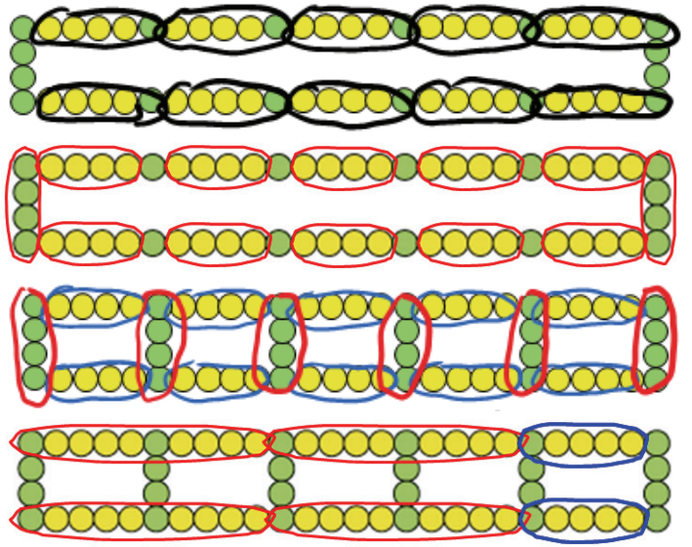
Mathematical notes

The objective in this section is to get learners away from counting in ones or twos towards identifying convenient larger units, so that they can count more efficiently.

Teaching guidelines

We suggest that instead of letting learners read question 1(b), you rather teach question 1 live in class through questions and answers (with different content if needed). This involves that learners will attempt to count the number of beads and that you will identify the different strategies used by learners and discuss these strategies, focusing on their advantages and disadvantages.

The advantage of "clever counting" should be clear – it is faster and less error-prone. However, you should allow learners enough time to engage with the questions. It will be good if you encourage them to not jump into counting, but to first analyse the *structure* of the drawings (to *think* before they jump) and to mark their units on a copy of the bracelets (see Addendum page 453) to make counting easier. Here are some examples of learners' work on question 2:



2(a): *Count in fives:*
5, 10, 15, ..., 45, 50
And then 54, 56

Count in fours:
4, 8, 12, ..., 48
And then 50, 52, 54, 56

2(b): *Count in fours:*
4, 8, 12, 16, ..., 60, 64

Count in tens and fives:
10, 20, 30, 40, 45, 50
Then 52, 54, 56, 58, 60, 64

Answers

1. (a) 32
   (b) Mia and Thea. Both use larger units; it is faster than counting in ones.
2. (a) 56 (b) 64

7.2 Clever counting

1. (a) How many beads are there in this string? Explain your method.



(b) Look at these methods that other learners used. Who is correct? Which method do you think is the best?

Simon points with his finger and counts the beads one by one: "One, two, three ... 28, 29."

Amir says: "There are five yellows in each group and three greens. So I count 5, 8, 13, 16, 21, 24, 30, 33."

Mia says: "There are five yellows in each group and three greens. So I count the yellows first: 5, 10, 15, 20. Then I continue to count the greens: 23, 26, 29, 32."

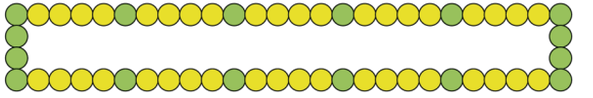
Thea says: "I group the yellows and greens together like this and then count in eights: 8, 16, 24, 32."



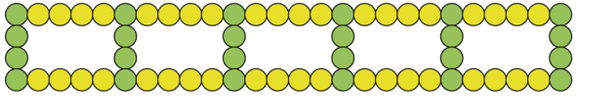
2. Use clever counting to easily find the number of beads in each of the four bracelets below. Explain how you do it. Compare your method with some of your classmates' methods.

Simon counted the beads one by one. The other learners counted *bigger units* like 3 and 5 and 8. They used **clever counting**!

(a)

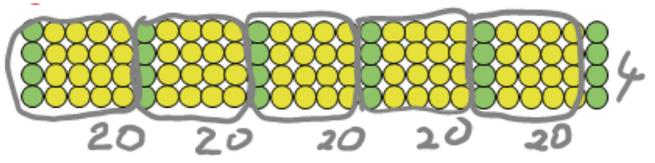


(b)

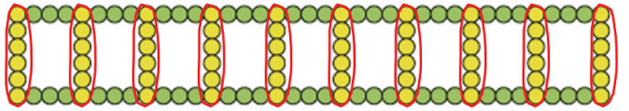


Answers

Here are some examples of learners' work on questions 2(c) and (d):



2(c): *Count in twenties:*
20, 40, 60, 80, 100
and then 104



2(d): *Count in sixes and threes:*
6, 12, 18, ..., 54, 60
63, 66, 69, 72, ..., 111, 114

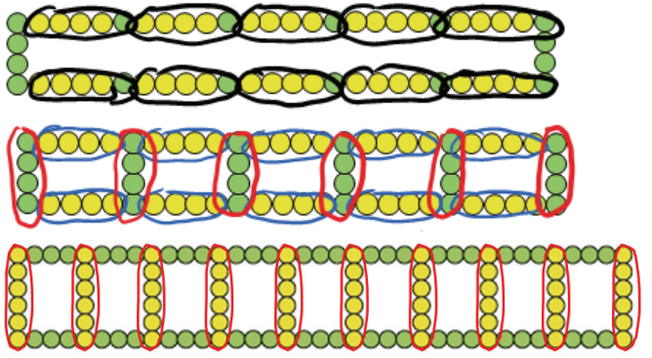
7.3 Do not count – *calculate!*

Mathematical notes

In this section we use the same drawings as in the previous section, but it is very important that learners now progress to *not* actually counting the units, but to write down the **calculation plan** as a **numeric expression**.

Note that we are not calculating the number of beads, but just saying what our plan is – how we would calculate it if required. It is important that learners learn to withhold calculation and make the expression (plan) the object of reflection. This uncalculated plan will become the **rule** or **formula** in our work on numeric patterns, and the **algebraic expression** in algebra in high school. *Structure*, and not *number*, is the basis of algebra!

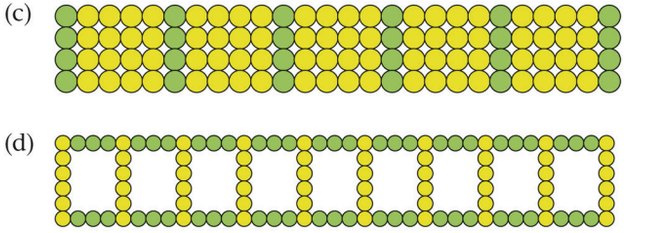
Here are some examples of learners' calculation plans for the total number of beads (see Diagrams 1, 2 and 3 on page 184).



1.
Calculation plan: 10×5 + 4 + 2

2.
Calculation plan: 16×4

3.
Calculation plan: 10×6 + 18×3



7.3 Do not count – *calculate*!

When we count many objects, it can take a very long time and we may make mistakes!

A better way is to first write down your thinking as a **calculation plan**. Write down what you are going to do.

Then you can *calculate* the answer instead of counting. For example:

A **plan** describes your **method**. You can write a plan in **words**, as a **calculation plan** or as a **flow diagram**.



Amir: Number of beads = 5 + 3 + 5 + 3 + 5 + 3 + 5 + 3

Mia: Number of beads = 5 + 5 + 5 + 5 + 3 + 3 + 3 + 3 = 4 × 5 + 4 × 3

Thea: Number of beads = 8 + 8 + 8 + 8 = 4 × 8

For each bracelet on the next page, write down your calculation plan to find out:

(a) how many yellow beads there are

(b) how many green beads there are

(c) how many beads there are altogether.

Note: *Do not calculate now.* The method (plan) is the important thing! You can calculate later or even use a calculator once you have written down your plans.

4 × 5 + 4 × 3 and 4 × 8 and 5 + 3 etc. are calculation plans. A **calculation plan** says what calculations we are going to do before we actually do them.

Answers

The following are examples of many possibilities. You should encourage different plans.

1. (a) 10×4 (b) $2 \times 4 + 8$ (c) $12 \times 4 + 8 = 14 \times 4$
2. (a) 10×4 (b) 6×4 (c) $10 \times 4 + 6 \times 4 = 16 \times 4$
3. (a) 10×6 (b) 18×3 (c) $10 \times 6 + 18 \times 3 = 19 \times 6$

7.4 Growing patterns

Mathematical notes

In this section we build on the ideas developed in the previous sections, i.e. not counting in ones and identifying appropriate larger units, and we continue to develop the know-how to articulate and write our calculation plans as arithmetic expressions (rules or formulas), and then analysing the expression, instead of jumping to calculation.

We very briefly illustrate the process of using structure to deduce a rule for the geometric sequence in question 1. We represent the number of squares numerically in the table below, with colours clearly showing the *structure* of beads in each picture.

Picture no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**60**	**70**
Top row	1+1	2+1	3+1	4+1	5+1	6+1	7+1	60+1	70+1
Bottom row	1	2	3	4	5	6	7	60	70
Total	2×1+1	2×2+1	2×3+1	2×4+1	2×5+1	2×6+1	2×7+1	2×60+1	2×70+1

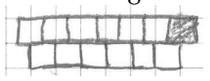
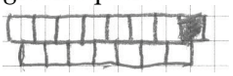
Whether the process is recorded in a table or not, the process and structure should definitely be part of discussing and formulating in class. Here are two different approaches:

No. of squares in Picture 6 = 2 rows with 6 green squares each + 1 pink square = $2 \times 6 + 1 = 13$

No. of squares in Picture 6 = 7 in top row + 6 in bottom row = $7 + 6 = 13$

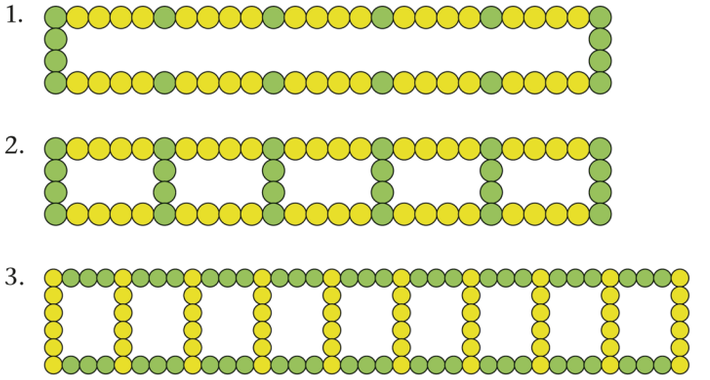
It is clear that we should understand the importance of the *relationship between numbers* in order to *see structure* and *generalise the rule*, e.g. in Picture 6, 7+6 are not random numbers, but 7 *is one more than 6* (the Picture no.), and this *relationship* applies to all the pictures.

Answers (questions 1(a) and (b))

Picture 6: A row of 6 green squares with a pink square at the end, on top of a row of 6 green squares.  $2 \times 6 + 1 = 13$ $7 + 6 = 13$	**Picture 7:** A row of 7 green squares with a pink square at the end, on top of a row of 7 green squares.  $2 \times 7 + 1 = 15$ $8 + 7 = 15$

Picture 60: 60 green squares and one pink square, on top of 60 green squares.
The total number of squares is $2 \times 60 + 1 = 121$

Picture 70: 70 green squares and one pink square, on top of 70 green squares.
The total number of squares is $2 \times 70 + 1 = 141$



7.4 Growing patterns

1. Lindiwe is making this growing pattern of pictures with squares:



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Lindiwe plans to continue the pattern.

(a) Describe Picture 6 and Picture 7 in words.
Now draw Picture 6 and Picture 7.
How many squares are there in Picture 6 and how many in Picture 7?

(b) Describe Picture 60 and Picture 70 in words. *Do not draw them!* Imagine them; "see" them in your head!
Calculate the number of squares in Picture 60 and in Picture 70.

Teaching guidelines

Alert learners to question 2(b) before they start working on question 2(a). In question 2(b) they will have to determine the numbers of squares in pictures 60 and 70 of each pattern without drawing the pictures, and without counting. The way in which they do question 2(a) will enable them to find a way of doing question 2(b). Impress on learners that they should not determine the number of squares in pictures 6 and 7 by counting, since this will leave them helpless when they come to question 2(b).

You should encourage different learners to use different methods, and give them the opportunity to explain their methods to their classmates, so that the classmates can decide if it is correct or not. If different methods are correct (give the same answers), the different calculation plans are equivalent expressions. For example in Pattern A:

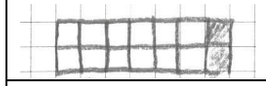
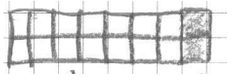
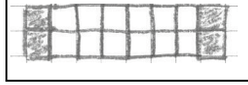
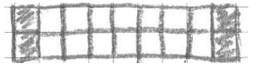
Picture 60: 2 rows with 60 green squares each + 2 pink squares = $2\times60 + 2 = 122$

Picture 60 = 2 rows with 61 squares each = $2\times61 = 122$

This demonstrates that $2\times60 + 2 = 2\times61$.

Explaining *why* these two plans give the same answer can lead to more interesting mathematical discussions based on the properties of operations, in this case adding multiples of 2 (60 twos + 1 two = 61 twos): $2\times60 + 2\times1 = 2\times(60+1) = 2\times61$

Answers (questions 2(a) and (b))

Pattern A, Picture 6: 2 rows, each with 6 green squares and one pink square at the end.  $2\times6 + 2\times1 = 14$ $2\times7 = 14$	**Picture 7**: 2 rows, each with 7 green squares and one pink square at the end.  $2\times7 + 2\times1 = 16$ $2\times8 = 16$
Pattern B, Picture 6: 2 rows, each with a pink square followed by 6 green squares and ending with another pink square.  $2\times1 + 2\times6 + 2\times1 = 16$ $2\times8 = 16$	**Picture 7**: 2 rows, each with a pink square followed by 7 green squares and ending with another pink square.  $2\times1 + 2\times7 + 2\times1 = 18$ $2\times9 = 18$

Pattern A, Picture 60: 2 rows, each with 60 green squares and a pink square at the end. The total number of squares is $2\times60 + 2\times1 = 122$

Pattern A, Picture 70: 2 rows, each with 70 green squares and a pink square at the end. The total number of squares is $2\times70 + 2\times1 = 142$

Pattern B, Picture 60: 2 rows, each with a pink square followed by 60 green squares and ending with another pink square. The total number of squares is $2\times1 + 2\times60 + 2\times1 = 124$

Pattern B, Picture 70: 2 rows, each with a pink square followed by 70 green squares and ending with another pink square. The total number of squares is $2\times1 + 2\times70 + 2\times1 = 144$

The answers for patterns C and D are on the next page.

2. Simphiwe is making these growing patterns of pictures with squares.

Pattern A



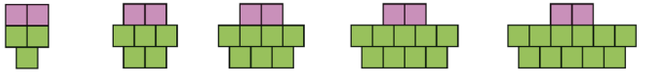
Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Pattern B



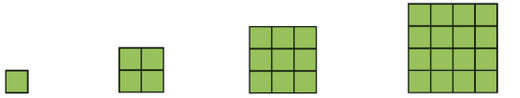
Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Pattern C



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Pattern D



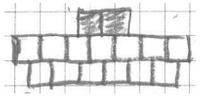
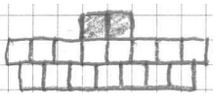
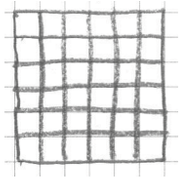
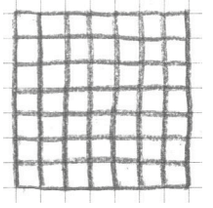
Picture 1 Picture 2 Picture 3 Picture 4

Answer questions (a) and (b) for each of Patterns A, B, C and D.

(a) Describe Picture 6 and Picture 7 in words.
Then draw Picture 6 and Picture 7.
How many squares are there in Picture 6 and how many in Picture 7?

(b) Describe Picture 60 and Picture 70 in words. *Do not draw them!* Imagine them; "see" them in your head!
Calculate the number of squares in Picture 60 and in Picture 70.

Answers (*continued*)

Pattern C, Picture 6: 2 pink squares in the middle on top of row of 7 green squares, and a row with 6 green squares in the middle below.  2 + 7 + 6 = 15	**Picture 7**: 2 pink squares in the middle on top of a row of 8 green squares, and a row with 7 green squares in the middle below.  2 + 8 + 7 = 17
Pattern D, Picture 6: 6 rows of green squares with 6 squares in each row.  6×6 = 36	**Picture 7:** 7 rows of green squares with 7 squares in each row.  7×7 = 49

Pattern C, Picture 60: 2 pink squares in the middle on top of row of 61 green squares, and a row with 60 green squares in the middle below.
The total number of squares is 2 + 61 + 60 = 123

Pattern C, Picture 70: 2 pink squares in the middle on top of row of 71 green squares, and a row with 70 green squares in the middle below.
The total number of squares is 2 + 71 + 70 = 143

Pattern D, Picture 60: 60 rows of green squares with 60 squares in each row
The total number of squares is 60×60 = 3 600

Pattern D, Picture 70: 70 rows of green squares with 70 squares in each row
The total number of squares is 70×70 = 4 900

2. Simphiwe is making these growing patterns of pictures with squares.

Pattern A



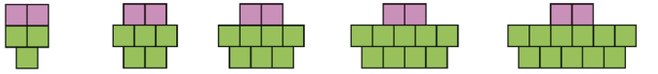
Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Pattern B



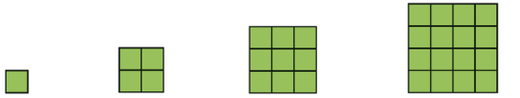
Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Pattern C



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Pattern D



Picture 1 Picture 2 Picture 3 Picture 4

Answer questions (a) and (b) for each of Patterns A, B, C and D.

(a) Describe Picture 6 and Picture 7 in words.
Then draw Picture 6 and Picture 7.
How many squares are there in Picture 6 and how many in Picture 7?

(b) Describe Picture 60 and Picture 70 in words. *Do not draw them!* Imagine them; "see" them in your head!
Calculate the number of squares in Picture 60 and in Picture 70.

7.5 From pictures to tables

Mathematical notes

Going from pictures to tables here is not a goal in itself, but a vehicle to help learners to deduce general formulae (rules) that will enable them to *easily* calculate the number of triangles in *any* figure.

What should therefore be recorded is not the *number* of triangles in the given figures, but the *structure* of the figures. Here is an example for the number of green triangles:

Green 1	Green 2	Green 3	Green 4	Green 5	Green 6	Green 30
2+1	3+2	4+3	5+4	6+5	7+6	31+30

Generalising this structure requires that we formulate the *relationships* between numbers in words, for example:

"*The number of green triangles in the bottom row is equal to the Figure number, and there is one more green triangle in the top row than in the bottom row.*"

This formulation then enables learners to say that in Figure 100 there will be 100 green triangles in the bottom row and 101 in the top row, for a total of 100 + 101 green triangles.

Answers

1.

Figure number	1	2	3	4	5	6	10
No. of pink triangles	1	2	3	4	5	6	10
No. of green triangles	3	5	7	9	11	13	21
Total no. of triangles	4	7	10	13	16	19	31

2. Pink sequence: consecutive numbers
   Green sequence: the odd numbers, starting from 3.
   Total sequence: start with 4 and count on in threes.

3. 30 pink triangles, 30 green triangles in the bottom row and 31 green triangles in the top row
   (a) 30 (b) $2 \times 30 + 1 = 61$ (c) $3 \times 30 + 1 = 91$

4. $31 + 32 = 2 \times 31 + 1 = 63$

5. $(31 - 1) \div 2 = 15$

7.5 From pictures to tables

Elspeth is making this growing pattern of pictures with pink and green triangles.

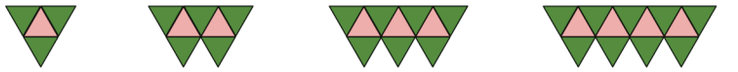


Figure 1 Figure 2 Figure 3 Figure 4

1. Complete this table and describe your methods.

Figure number	1	2	3	4	5	6	10
No. of pink triangles	1	2	3				
No. of green triangles	3	5	7				
Total no. of triangles	4	7					

2. Describe the number patterns you see in the table.

3. Describe Figure 30 in words.
   (a) How many pink triangles are there in Figure 30?
   (b) How many green triangles are there in Figure 30?
   (c) How many triangles are there in Figure 30?

4. If Elspeth makes a figure with 31 pink triangles, how many green triangles does the figure have?

5. If Elspeth makes a figure with 31 green triangles, how many pink triangles does the figure have?

7.6 Writing our plans as flow diagrams

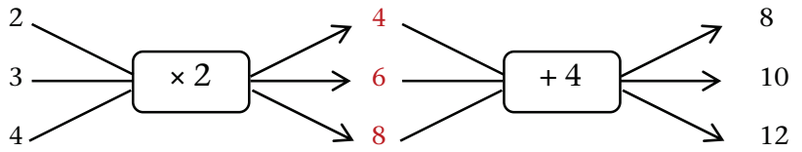
Mathematical notes

Learners should have adequate background knowledge to engage with the work in this section. But instead of learners reading the page, you may present it as a lesson, and interactively with learners deduce the calculation plan and its representation as a flow diagram. Learners can then practise what they have learnt in the rest of the questions.

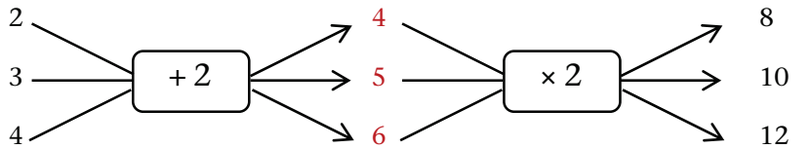
Note on flow diagrams and BODMAS

You should make sure that learners understand the flow diagram notation.

The flow diagram representation carries an intuitive left-to-right procedure: the first input produces the first output, the second input produces the second output. If you find that it is not so intuitively clear to your learners, you may wish to show them the flow diagram with the intermediate input-output values filled in, like this:



The left-to-right convention means that there is no need to learn rules like BODMAS for the order of operations (e.g. first multiply before you add). BODMAS does not apply in these diagrams. For example, the following flow diagram is equivalent to the above.



The flow diagram left-to-right procedure plays the same role as brackets. To calculate the output value for the input 3, the first diagram uses the arithmetic expression (3×2) + 4, and the second diagram uses (3+2) × 2, and of course (3×2) + 4 = (3+2) × 2 = 10.

Note that due to space limitations we will in some answers use this alternative **one-line notation** for the above two flow diagrams, intended *only* for teachers, *not* for learners:

2, 3, 4 → [× 2] → [+ 4] → 8, 10, 12

2, 3, 4 → [+ 2] → [× 2] → 8, 10, 12

You may think of the input numbers moving from left to right into the operator one at a time.

Answers

1. Learners should check if it is true that 1×2 + 4 = 6, 2×2 + 4 = 8, 3×2 + 4 = 10 ...

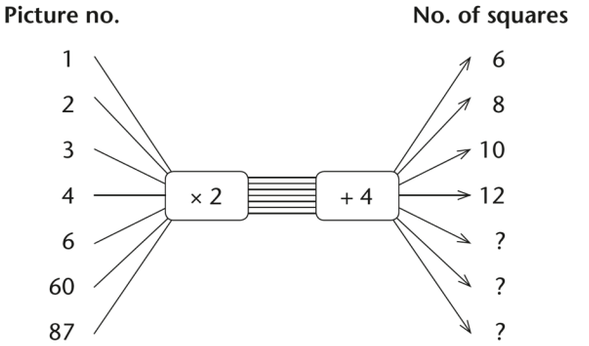
7.6 Writing our plans as flow diagrams

You have already seen this growing pattern in Section 7.4. We will now look at it again, but in a different way. How will you calculate the number of squares in Picture 6, Picture 60 and Picture 87? What is *your* plan?



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Mia writes her plan as a flow diagram:



A **flow diagram** describes a plan (method) using **input → rule → output**.

This flow diagram shows that you must first multiply any **input number** by 2 and then add 4 to get the **output number**.

3 × 2 → 6; 6 + 4 → 10

4 × 2 → 8; 8 + 4 → 12

1. First check if Mia's flow diagram (plan) is correct for the input and output numbers that we already have. Discuss with your classmates how you can check her plan.

Teaching guidelines

Again, describing the structure in words is very important for the development of understanding. Indeed, the language carries the underlying relationships and allows us to generalise away from the specific example to *any* Picture number. Note that with "any" we do not mean that we should introduce algebraic notation for variables. With "any" we here mean any specific number, for example the 100th, or that we may use *words* as names for variables, for example in *Picture no.* = 2 × *No. of squares* + 1.

Here are examples of how to describe the pictures (the number of squares) in question 5(a):

Picture 4: 2 rows of 4 squares plus 1 pink or *4 squares on top and 5 at the bottom*
Picture 5: 2 rows of 5 squares plus 1 pink or *5 squares on top and 6 at the bottom*

It is this relationship (between the *Picture number* and the *number of squares*, and the relationship between the two sentences) that must now be extended and generalised. There is no need to draw:

Picture 6: 2 rows of 6 squares plus 1 pink or *6 squares on top and 7 at the bottom*
Picture 60: 2 rows of 60 squares plus 1 pink or *60 squares on top and 61 at the bottom*

It should be clear that it is vital to distinguish between the **constants** (2 rows, 1 pink) and the **variables** (the *Picture no.* and the *No. of squares* per row).

This **generalisation** then serves as a **calculation plan** that enables us to calculate the number of squares for any Picture number in the sequence, for example:

Picture 60: 2×60 + 1 = 121 squares or 60 + 61 = 121.

Answers

2. 6×2 + 4 = 16 60×2 + 4 = 124 87×2 + 4 = 178
3. Yes, we can calculate instead of count.
4. One alternative is –[+ 2]–[× 2]→
5. (a) *Picture no.* –[× 2]–[+ 1]→ *No. of squares*
   Picture 6: 6×2 + 1 = 13
   Picture 60: 60×2 + 1 = 121
   Picture 87: 87×2 + 1 = 175
   (b) *Picture no.* –[× 2]–[+ 2]→ *No. of squares*
   Picture 6: 6×2 + 2 = 14
   Picture 60: 60×2 + 2 = 122
   Picture 87: 87×2 + 2 = 176
6. Learners' own work

2. If Mia's plan is correct, use her plan to calculate the missing output numbers.
3. Mia says, with the flow diagram it is easy to calculate the number of squares in Picture 60 or in Picture 87 or in *any* Picture number. Do you agree?
4. If you or your classmates have a different plan than Mia, write it as a flow diagram.
5. Write your plans for each of these patterns as flow diagrams, and calculate the number of squares in Picture 6, Picture 60 and Picture 87. Then compare your methods with some classmates' methods.

(a)



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

(b)



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

6. Make your own growing geometric pattern with squares.
   (a) Draw the first four pictures.
   (b) Calculate the number of squares in Picture 5, Picture 6, Picture 50 and Picture 60.
   (c) Now let some classmates solve your problem in (b) and check if they are correct.

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
8.1 Identical parts that fit onto each other	Introducing symmetry and lines of symmetry	189
8.2 Lines of symmetry	Focusing on lines of symmetry, seen as "fold lines"	190 to 192

CAPS time allocation	2 hours
CAPS page references	23 and 82

Mathematical background

Symmetry occurs when a two-dimensional figure can be divided into two halves that are "mirror images" of each other. One cannot talk about symmetry without referring to a line of symmetry. A line of symmetry is an imaginary line through a two-dimensional figure that can act as a "fold line". If we imagine folding the two-dimensional figure along the line of symmetry we will find that the two halves fit perfectly on top of each other. If we imagine that our figures are drawn on a transparent sheet and cut along the line of symmetry, we will see that when we place the two halves on top of each other, they are exactly the same (both the overall shape and any features drawn inside the outlines of the two figures).

Some figures have two or more lines of symmetry (e.g. a rectangle has two, a square has four and a circle has more lines of symmetry than can be counted). If a figure does not have at least one line of symmetry we say that it is not symmetrical (or asymmetrical).

Symmetry, or lack of symmetry, are very important properties of two-dimensional figures. They provide another way to compare different figures (instead of just saying which sides or corners are the same, or not). Symmetry is touched on again in Term 3 Unit 6, question 1, page 255.

Resources

Photocopies of templates mentioned in "Teaching guidelines" on the next page.

Sheets of paper and scissors.

8.1 Identical parts that fit onto each other

Mathematical notes

A two-dimensional figure may not be symmetrical as a whole. However, certain *parts* of the figure may be symmetrical. It is important to be able to see what is symmetrical in a figure even though the figure may not be symmetrical as a whole. Identifying symmetry is equivalent to identifying a line of symmetry. The two go hand in hand. Symmetry is a very important concept and needs to be given as much attention as is practically possible.

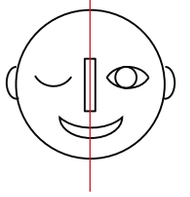
Teaching guidelines

Aim to spend about 1 hour on this section.

You can start with some paper folding activities to help learners to begin to see lines of symmetry in figures. Learners can trace and cut out any of the figures on pages 306, 307, 308 and 338 (or you could photocopy pages 463 to 466 or pages 469 to 474 in the Addendum). Once learners have cut out the figures, they should fold them so that the top and bottom half fit exactly on top of each other. When learners open out the figure, the fold line will be the line of symmetry.

You can use the tinted passage on page 189 to explain that as a whole, a two-dimensional figure may not be symmetrical. However, certain *parts* of the figure may be symmetrical. You may, time permitting, provide additional examples of diagrams that have, or do not have, symmetry "hidden" in them.

In the face alongside the mouth, nose, each ear and each eye are symmetrical, but the whole face is not because the one eye is open and the other closed.



You will have to provide lines that may or may not be lines of symmetry.

Possible misconceptions

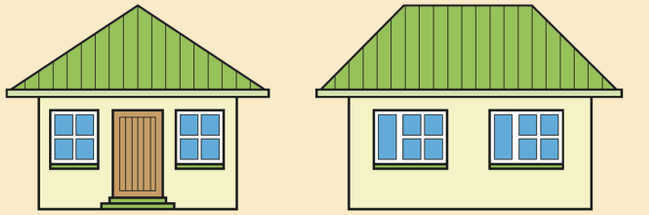
Learners may see symmetry only as a "total" property of a figure or object. Such learners will need support to realise that they can *choose* to ignore parts that are not symmetrical in order to be able to "see" the parts of the figure or object that are symmetrical. They need to be able to describe what they are focusing on in such cases.

Answers

1. No
2. Yes
3. Because each set of windows is not symmetrical and the two sets of windows are not symmetrical to each other.

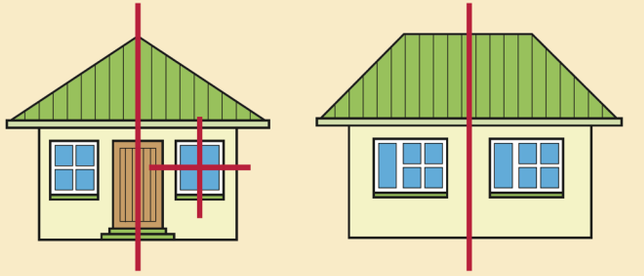
UNIT 8 SYMMETRY

8.1 Identical parts that fit onto each other



Front view *Side view*

If you fold the picture of the front view of the house on the red line, the part that is on the left of the line will fold exactly onto the part that is on the right of the line.



The front view of this house is **symmetrical**. It has one **line of symmetry**. The side view is not symmetrical.

Each front window of the house is also symmetrical, and has two lines of symmetry.

1. Are the windows on the side of the house symmetrical?
2. Is the roof as seen from the side symmetrical?
3. Why can we say that the side view of the house is not symmetrical?

GRADE 4: MATHEMATICS [TERM 2] 189

8.2 Lines of symmetry

Mathematical notes

The key to this section is being able to identify all lines of symmetry in each of the figures.

Teaching guidelines

Aim to spend about 1 hour on this section.

Encourage learners to always identify the line of symmetry when they deal with symmetrical objects. Ask questions that justify opinions, ideas, etc. For example: "*You say this object is symmetrical – show me the line of symmetry.*", "*Is everything in this figure symmetrical or are only some parts symmetrical?*", "*How many lines of symmetry does that object have?*", etc.

If time permits, allow for some group discussion. Moderate the discussions to ensure that the focus remains on the important issues: justifying symmetry or the lack thereof, using the idea of line of symmetry ("fold line"), and individual parts that are symmetrical around the line.

Possible misconceptions

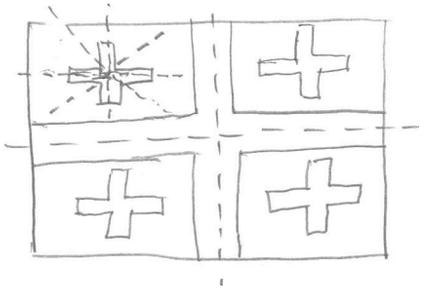
Some learners may struggle to see that two parts are not symmetrical. Question 2 is meant to help address this matter. To assist such learners, you may have to provide additional cut-outs of figures with a fold line and have them fold the halves together to see whether parts overlap exactly or not.

Notes on questions

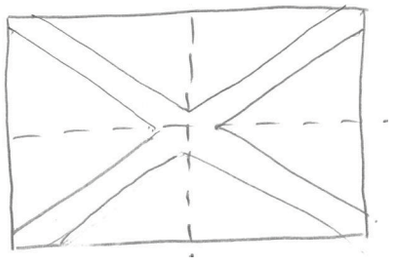
If learners find any part of question 4 difficult or get it wrong, ask them to trace the figures (including the broken lines) and cut them out. Then they should fold the figures on the lines to see which one(s) is/are the line(s) of symmetry.

Answers

1. (a) Flag of Georgia



(b) Flag of Jamaica



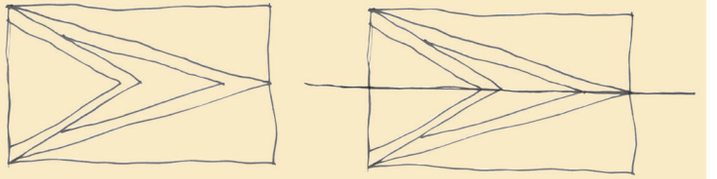
8.2 Lines of symmetry

This is a picture of the flag of Guyana.



The different colours and the different lines and shapes make it interesting.

You can easily make a rough drawing of the lines and shapes in this flag, as shown on the left below.



The same sketch is shown on the right, with a line of symmetry drawn on the sketch.

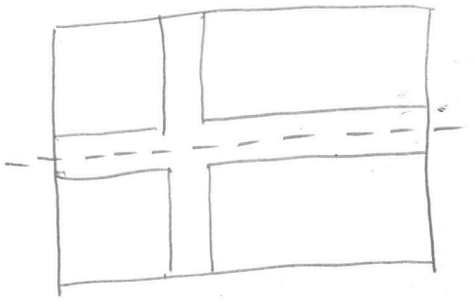
You can think of the line of symmetry as a "fold line". If you fold a picture on the line of symmetry, the parts of the picture will fit exactly on top of each other. Imagine you fold the flag in half so that the parts fit exactly onto each other.

1. The flags of some countries are shown on the next page. Some flags have only one line of symmetry, and others have two or more lines of symmetry.

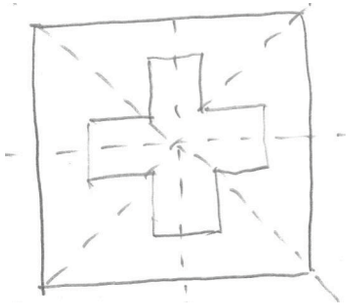
   For each flag, make a rough drawing to show the shapes and lines on the flag. Draw the line or lines of symmetry on the drawing.

Answers *(continued)*

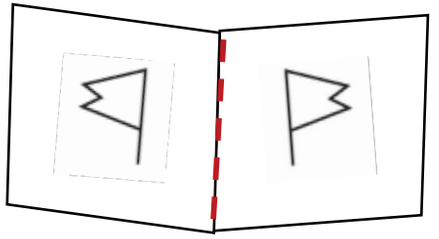
1. (c) Flag of Sweden



(d) Flag of Switzerland

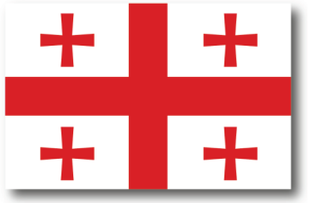


2. Learners' drawings should look something like the illustration below (or the one shown on page 191). Note that the fold line is the line of symmetry.



3. Learners' own drawings. There are too many possibilities to illustrate. Check that the two drawings make a symmetrical image on the page.

(a) The flag of Georgia



(b) The flag of Jamaica



(c) The flag of Sweden

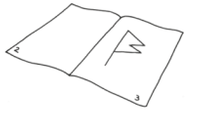


(d) The flag of Switzerland



2. Fold a clean sheet of paper in the middle and number the pages 1, 2, 3 and 4.
Draw a large figure like the one on the right on page 3.
Use a blunt pencil to make a thick dark line.

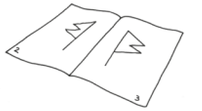




Fold half of the sheet over. On page 1, trace a copy of the sketch you have made on page 3.



Open the sheet again.
On page 2, trace a copy of what you have drawn on page 1.



Did you make a symmetrical drawing?
Draw the line of symmetry.

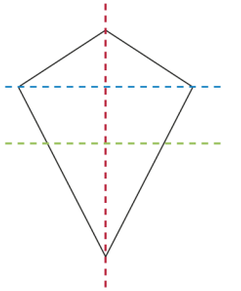
3. Make two drawings on two halves of a clean page, so that you have symmetry.

Answers

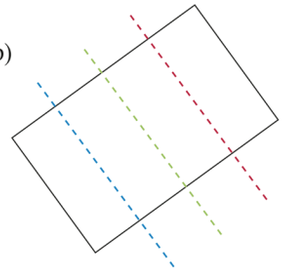
4. (a) The red line
   (b) The green line
   (c) The red and blue lines
   (d) The red, blue and green lines
   (e) The red, blue and green lines
   (f) None
   (g) None
   (h) The red line

4. For each figure below, state which of the coloured broken lines are lines of symmetry.

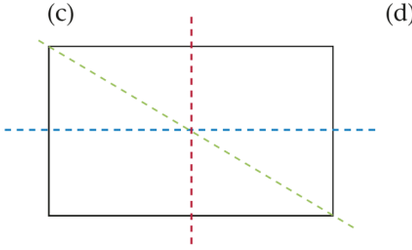
(a)



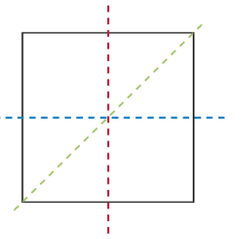
(b)



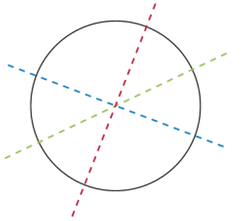
(c)



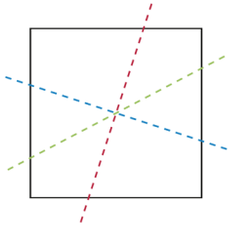
(d)



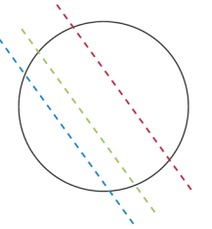
(e)



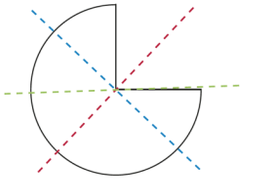
(f)



(g)



(h)



Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
9.1 Another way to add	Adding by breaking down one number, and by breaking down both numbers	193 to 194
9.2 Another way to subtract	Subtraction with the breaking-down and building-up method	195 to 197
9.3 Solve problems	Word problems	198 to 199

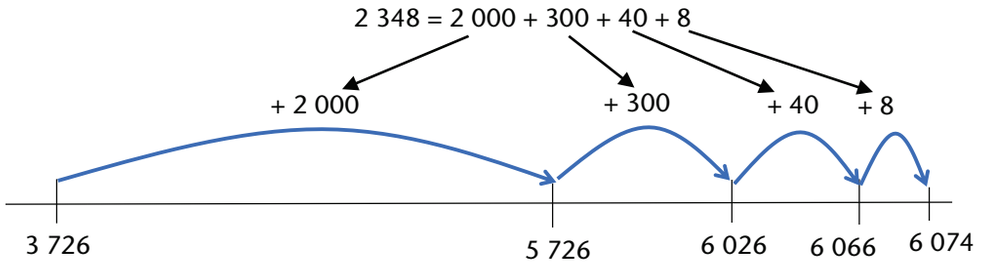
CAPS time allocation	4 hours
CAPS page references	14 to 15, 69 to 71 and 83

Mathematical background

Addition can be performed by breaking down one number and adding its place value parts one by one to the other number, for example:

3 726 + 2 348 = 3 726 + 2 000 + 300 + 40 + 8
= 5 726 + 300 + 40 + 8
= 6 026 + 40 + 8
= 6 066 + 8
= 6 074

As an alternative, adding on may proceed from the smaller parts to the bigger parts.



Addition can also be performed by breaking down both numbers into place value parts, rearranging the parts (commutative and associative properties), and building up the answer:

3 726 + 2 348 = 3 000 + 700 + 20 + 6 + 2 000 + 300 + 40 + 8
= 3 000 + 2 000 + 700 + 300 + 20 + 40 + 6 + 8
= 5 000 + 1 000 + 60 + 14
= 6 000 + 70 + 4
= 6 074

9.1 Another way to add

Mathematical notes

In Unit 2 of this term learners used a range of methods to add and subtract. Amongst these were breaking down only the number that is added, and breaking down both numbers.

In this section learners revise adding by breaking down numbers into place value parts, rearranging the parts, and building up the answer. The work on this page serves as revision and builds up towards addition by breaking down both numbers on the next page.

Teaching guidelines

Aim to cover this section in 1 hour. One possibility is to use

- the tinted passage on page 193 and question 1(a), and the tinted passage on page 194 and question 4(a) for concept development,
- questions 1(b), (d) and (f), 2, 3, 4(b), 5 and 6 for classwork, and
- questions 1(c) and (e), and 4(c) and (d) for additional practice.

You can start by counting on in multiples of tens, hundreds and thousands for mental mathematics. Start with numbers that are not multiples of what you are counting in, for example, count on in multiples of 4 000 from 1 234.

Learners have used rounding off for calculating. In this unit and subsequent units you can help learners to use rounding off and estimating as a way of checking whether their answers are reasonable. At first it is useful to give learners a range of estimated answers to choose from, ask them to choose the best one and then to *explain* their choice. For example, for question 1 they can choose the best estimate from the following:

Question 1(a):	(i) about 9 000	(ii) about 8 000	(iii) about 10 000	(iv) about 2 000
Question 1(b):	(i) about 7 000	(ii) about 5 000	(iii) about 8 000	(iv) about 9 000
Question 1(c):	(i) about 4 000	(ii) about 8 000	(iii) about 9 000	(iv) about 6 000
Question 1(d):	(i) about 7 000	(ii) about 2 000	(iii) about 8 000	(iv) about 9 000
Question 1(e):	(i) about 5 000	(ii) about 8 000	(iii) about 10 000	(iv) about 9 000
Question 1(f):	(i) about 6 000	(ii) about 7 000	(iii) about 2 000	(iv) about 2 500

Answers

1. (a) 9 900 (b) 8 820 (c) 9 072
   (d) 9 041 (e) 9 263 (f) 2 455
2. (a) 5 878 (b) Yes (c) 5 878
3. 5 000 + 3 000 + 800 + 500 + 60 + 70 + 4 + 3

9.1 Another way to add

One way to calculate 3 785 + 4 667 is to break the 4 667 down into its place value parts and add them one by one to 3 785:

4 667 = 4 000 + 600 + 60 + 7
3 785 + 4 000 = 7 785
7 785 + 600 = 8 385
8 385 + 60 = 8 445
8 445 + 7 = 8 452

The smaller parts can be added first if you prefer to do so:

3 785 + 7 = 3 792
3 792 + 60 = 3 852
3 852 + 600 = 4 452
4 452 + 4 000 = 8 452

1. Calculate.
   (a) 1 212 + 8 688 (b) 5 421 + 3 399
   (c) 5 583 + 3 489 (d) 7 354 + 1 687
   (e) 4 675 + 4 588 (f) 4 789 – 2 334
2. (a) Calculate 2 000 + 600 + 40 + 5 + 3 000 + 200 + 30 + 3 in the order in which the numbers are given.
   (b) Will you get the same answer if you rearrange the numbers as done in the line below, and do the calculations in the order in which the numbers are now written?
   2 000 + 3 000 + 600 + 200 + 40 + 30 + 5 + 3
   (c) Do the calculations to check your answer.
3. Rearrange the numbers in
   5 000 + 800 + 60 + 4 + 3 000 + 500 + 70 + 3
   so that the thousands parts are together, also the hundreds parts, the tens parts and the units parts.

Mathematical notes

The rearrangement of the place value parts of the two numbers is possible because of the commutative and associative properties of addition.

Most of this section focuses on addition. However, it is important that learners don't close their minds to the possibility of other operations. That is why question 1(f) on page 193 is a subtraction calculation. It is also important that learners continue to keep the relationships between operations in mind. This is one reason why question 5 uses repeated addition and question 6 uses multiplication and division related to the repeated addition in question 5.

Notes on questions

Question 5 uses repeated addition of 387. Once learners have calculated the answer to question 5(a), they can use this answer (1 935) and add another 387 to get the answer to 5(b). They do not need to add up all the 387s again. Similarly, in question 5(c) learners can just add 387 to the answer to question 5(b).

Learners do not need to calculate any answers in question 6. They should rather use their knowledge of multiplication as repeated addition and the answer in 5(a) for question 6(a), and the answer in 5(c) for question 6(b). Learners can use question 6(a) to answer questions 6(c) and (d), because multiplication and division are inverse operations.

Teaching guidelines

While doing the examples on the board, explain that the task of adding the two numbers is replaced with the task of doing several easy additions separately and then combining the answers by adding them up.

Continue to help learners to estimate answers before calculating. This will help them to judge whether their answers are reasonable. For example, you can ask learners to choose the best estimated answers from:

Question 4(a):	(i) about 9 000	(ii) about 8 000	(iii) about 10 000	(iv) about 2 000
Question 4(b):	(i) about 500	(ii) about 5 000	(iii) about 8 000	(iv) about 9 000
Question 4(c):	(i) about 400	(ii) about 4 000	(iii) about 3 000	(iv) about 2 500
Question 4(d):	(i) about 2 000	(ii) about 3 000	(iii) about 3 500	(iv) about 4 000

It is very important that learners explain why they had chosen one of the estimated answers.

Answers

4.	(a) 8 305	(b) 8 166	(c) 2 511	(d) 3 722
5.	(a) 1 935	(b) 2 322	(c) 2 709	
6.	(a) 1 935	(b) 2 709	(c) 5	(d) 387

To calculate 3 465 + 4 574 you can break *both* numbers down, work with the parts of the same kind, and then build the answer up.

Break down:

3 465 = 3 000 + 400 + 60 + 5
4 574 = 4 000 + 500 + 70 + 4

Writing the expanded notations in separate lines makes it easier to see which parts belong together.

Work with the parts:

3 000 + 4 000 = 7 000
400 + 500 = 900
60 + 70 = 130
5 + 4 = 9

Build up the answer:

3 465 + 4 574
= 7 000 + 900 + 130 + 9
Transfer 100 from 130 to 900
= 7 000 + 1 000 + 30 + 9
Transfer 1 000 to 7 000
= 8 000 + 0 + 30 + 9
= 8 039

4. Break all numbers down to calculate the following:
   (a) 4 478 + 3 827 (b) 6 289 + 1 877
   (c) 866 + 967 + 678 (d) 1 287 + 991 + 658 + 786
5. Calculate.
   (a) 387 + 387 + 387 + 387 + 387
   (b) 387 + 387 + 387 + 387 + 387 + 387
   (c) 387 + 387 + 387 + 387 + 387 + 387 + 387
6. Use your answers for question 5 to state how much each of the following is.
   (a) 387 × 5 (b) 387 × 7
   (c) 1 935 ÷ 387 (d) 1 935 ÷ 5

9.2 Another way to subtract

Teaching guidelines

Aim to cover this section in 1 hour. One possibility is to use

- the tinted passage on page 196 and questions 8(c) and (d), and the tinted passage on page 197 and question 10 for concept development,
- questions 1, 2, 3, 4(a), (c) and (e), 5, 6, 7 and 12(a) to (e) for classwork, and
- questions 4(b), (d) and (f), 8(a) and (b), 9, 11 and 12(f) for additional practice.

Allow learners to engage with the questions on page 195 on their own. This provides them with opportunities to activate their own ideas. These questions are designed to develop understanding of subtraction situations (in this case taking away), as well as to develop skills for subtraction. You could, however, ask learners a few questions before they start, for example:

- Will Vusi have more or less money in his purse after he has bought the shoes?
- Will Tebogo have more or less than R3 787 after she has bought the washing machine?
- About how much money will she have:
  (i) about R5 000 (ii) about R6 000 (iii) about R2 000 (iv) about R1 000?
- Will Sarah have more or less than R4 958 after she has bought the refrigerator?
- About how much money will she have:
  (i) about R5 000 (ii) about R8 000 (iii) about R2 000 (iv) about R3 000?

The breaking-down and building-up method of subtraction is introduced on the next page.

Answers

1. (a) R3 000 + R700 + R80 + R7 = R3 787
   (b) R 2 428
2. R1 242
3. R2 622

9.2 Another way to subtract

1. (a) Vusi has this money in his purse. How much is it?



(b) Vusi takes R1 359 from his purse to pay for a pair of shoes. How much money is left in his purse?

2. Tebogo also has R3 787 in her purse. She pays R2 545 for a washing machine. How much money does she have left?
3. Sarah has R4 958. She buys a refrigerator for R2 336. How much money does she have left?

Notes on questions

Questions 4 and 5 serve to develop the idea of subtraction as the inverse of addition. Question 5 revises a method of subtraction that learners have used previously.

Teaching guidelines

Once learners have completed question 7, ask them why the answers to questions 5 and 6(b) are the same. If they cannot answer you, use simpler examples such as:

Why will the answers to the following be the same?

$3 + \square = 10$ and $10 - 3 = \square$

$411 + \square = 800$ and $800 - 411 = \square$ etc.

Do one or two examples of the breaking-down and building-up method of subtraction on the board. While doing this, explain that the given subtraction task is replaced with the task of doing several easy subtractions separately and then combining the answers by adding them up.

Answers

4. (a) $2\,137 + 63 = 2\,200$ (b) $3\,437 + 63 = 3\,500$
   (c) $2\,364 + 36 = 2\,400$ (d) $4\,917 + 83 = 5\,000$
   (e) $4\,286 + 14 = 4\,300$ (f) $5\,324 + 76 = 5\,400$

5. 2 644. Some learners may use some or all of the following stages:
   $3\,324 + \underline{6} \rightarrow 3\,330 + \underline{70} \rightarrow 3\,400 + \underline{600} \rightarrow 4\,000 + \underline{1\,000} \rightarrow 5\,000 + \underline{968} \rightarrow 5\,968$. However, in this example it is not necessary and learners can write down the missing number directly.
   $3\,324 + \underline{2\,644} \rightarrow \underline{5\,968}$

6. (a) 2 000; 600; 40; 4
   (b) 2 644

7. Learners check and make corrections if necessary.

4. Find the missing number in each case.
   (a) $2\,137 + \ldots = 2\,200$ (b) $3\,437 + \ldots = 3\,500$
   (c) $2\,364 + \ldots = 2\,400$ (d) $4\,917 + \ldots = 5\,000$
   (e) $4\,286 + \ldots = 4\,300$ (f) $5\,324 + \ldots = 5\,400$

5. Calculate 5 968 – 3 324 by adding on.
   $3\,324 + \ldots\ldots \rightarrow \ldots\ldots\ldots\ldots$

6. (a) How much is each of the following?
   $5\,000 - 3\,000$ $900 - 300$ $60 - 20$ $8 - 4$
   (b) Calculate $(5\,000 - 3\,000) + (900 - 300) + (60 - 20) + (8 - 4)$.

7. If your answers for questions 5 and 6(b) are not the same, you have made a mistake. Correct it if this is the case.

To calculate 6 878 – 4 465 you can break *both* numbers down into place value parts, work with the parts of the same kind, and then build the answer up.

Break down:

$6\,878 = 6\,000 + 800 + 70 + 8$

$4\,465 = 4\,000 + 400 + 60 + 5$

Work with the parts:

$6\,000 - 4\,000 = 2\,000$

$800 - 400 = 400$

$70 - 60 = 10$

$8 - 5 = 3$

Build up the answer:

$6\,878 - 4\,465 = 2\,000 + 400 + 10 + 3 = 2\,413$

The above actions can also be described by using brackets:

$$\begin{aligned} 6\,878 - 4\,465 &= (6\,000 + 800 + 70 + 8) - (4\,000 + 400 + 60 + 5) \\ &= (6\,000 - 4\,000) + (800 - 400) + (70 - 60) + (8 - 5) \\ &= 2\,000 + 400 + 10 + 3 \\ &= 2\,413 \end{aligned}$$

Teaching guidelines

Once you have worked through question 8(a) on the board with learners using the breaking-down method, you can ask them: *"What is the same about questions 8(c) and 8(d)?"* Learners should realise that the same amount (3 325) is subtracted from both numbers, and that the starting numbers are nearly the same. The difference between 6 559 and 6 552 is only 7. This means that learners do not need to calculate 6 559 – 3 325; they only need to calculate 3 234 (the answer to 8(c)) – 7. Remind learners that they should develop the habit of thinking: *"What do I already know that can help me here?"*

Do examples of subtraction that require transfer on the board. Emphasise the idea of **replacing** the expanded notation of the bigger number with a different way of breaking it down, to make the subtraction by parts possible.

You can ask learners: *"What is the same about the numbers in question 12?"* They should see that the starting numbers in 12(a) to (e) are the same, but that the numbers subtracted are different. Ask learners questions like: *"Which answer will be the smallest?", "Which answer will be the biggest?"* Ask them to arrange the answers to 12(a) to (e) from the biggest to the smallest *before* they do the calculations. If learners cannot do this, then start by giving them easier calculations, for example 75 – 9; 75 – 18; 75 – 27; 75 – 36.

Notes on questions

Questions 10 and 11 are more easily done using "transfers"; they motivate the need for doing transfers. You may like to first show learners how to do transfers before they do questions 10 and 11. In questions 12(a) to (e) the initial number is the same (5 346). Learners can use this to estimate the sizes of answers relative to each other.

Possible misconceptions

If learners do not understand the logic of the subtraction method properly, they may make the mistake of subtracting the smaller of the two place value parts.

Answers

8. (a) 5 344 (b) 2 424 (c) 3 234 (d) 3 227
9. $7\,698 - 2\,354 = (7\,000 - 2\,000) + (600 - 300) + (90 - 50) + (8 - 4)$
   $= 5\,000 + 300 + 40 + 4$
   $= 5\,344$
10. R1 557
11. 3 378
12. (a) 2 218 (b) 3 072 (c) 3 521
    (d) 2 169 (e) 2 787 (f) 3 478

8. Calculate each of the following by breaking both numbers down into place value parts, working with the parts and building the answers up.
   (a) 7 698 – 2 354 (b) 6 567 – 4 143
   (c) 6 559 – 3 325 (d) 6 552 – 3 325
9. Describe the work that you did in question 8(a) by using brackets, as demonstrated at the bottom of the previous page.
10. Ben has R4 325 and he has to pay R2 768 rent for his house. How much money will he have left?
11. Calculate 6 254 – 2 876.

In a case like 6 254 – 2 876, some place value parts of the bigger number are smaller than the parts you have to subtract.

In a case like this, you can make **transfers** (shifts) between the place value parts of the bigger number to make it easier to do the subtractions with the place value parts:

$6\,254 = 6\,000 + 200 + 50 + 4$
(1 000, 100, 10)
$= 5\,000 + 1\,100 + 140 + 14$
$2\,876 = 2\,000 + 800 + 70 + 6$

$5\,000 - 2\,000 = 3\,000$
$1\,100 - 800 = 300$
$140 - 70 = 70$
$14 - 6 = 8$

$6\,254 - 2\,876 = 3\,000 + 300 + 70 + 8$
$= 3\,378$

12. Calculate by breaking both numbers down into place value parts. Check each answer by doing addition.
   (a) 5 346 – 3 128 (b) 5 346 – 2 274
   (c) 5 346 – 1 825 (d) 5 346 – 3 177
   (e) 5 346 – 2 559 (f) 7 132 – 3 654

9.3 Solve problems

Mathematical notes

In this section learners solve problems that involve addition and subtraction. The section also focuses on addition and subtraction as inverse operations, i.e. if the numbers used are the same then what the one operation does the other undoes. Inverse operations can be used to check calculations: see questions 1 to 4; 5 to 7; 8 and 9.

Teaching guidelines

Aim to cover this section in 2 hours. One possibility is to use

- questions 1, 2, 3, 4 and 10 for concept development,
- questions 5, 6, 7, 8, 9 and 12 for classwork, and
- questions 11, 13, 14 and 15 for additional practice.

Remind learners to always ask themselves: *"What have I learnt or done before that can help me here?"* Using information, number facts and strategies that they have used before will not only boost learners' confidence and independence, but will also save them a lot of time.

Using questions for concept development does *not* mean writing out the calculations on the board as worked examples. Rather read the questions to learners and ask them: *"What will you do now?"* Once you have completed question 1, ask learners where they have seen the numbers in questions 2 and 3 before. Ask learners to match the number sentences in questions 2 and 3 with questions 1(a), (b) and (c).

Learners can use the fact that addition and subtraction are inverse operations, for example 248 – 128 = ☐; ☐ + 128 = 248. Learners should do this for all calculations in question 1.

Notes on questions

The number sentences in questions 6 and 7 are the same as the calculations learners do in question 5. Learners can use the answers from question 5 for questions 6 and 7.

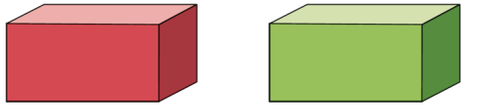
Answers

1. (a) 120 (b) 90 (c) 158
2. (a) 120 (b) 90 (c) 158
3. 128 + 120 = 248
4. The answers to question 1 should be the same as the answers to question 2.
5. (a) 3 132 (b) 1 113 (c) 4 366
6. (a) 3 132 (b) 1 113 (c) 4 366

9.3 Solve problems

1. There are 128 lemons in the red box.

   There are 248 lemons in the two boxes together.

   (a) How many lemons are there in the green box?

   (b) 38 lemons are moved from the red box to the green box. How many lemons are left in the red box?

   (c) How many lemons are there in the green box now?

2. Calculate.

   (a) 248 – 128 (b) 128 – 38 (c) 120 + 38

3. Find the number that makes the number sentence true:

   128 + ... = 248

4. Check to make sure that your answers for question 1 are correct.

5. Farmer Cele has two farms. He calls them Farm A and Farm B. He has 2 347 goats on Farm A.
   Altogether, there are 5 479 goats on the two farms.

   (a) How many goats are there on Farm B?

   (b) Farmer Cele takes 1 234 goats from Farm A to Farm B. How many goats are left on Farm A?

   (c) How many goats are there on Farm B now?

6. Calculate.

   (a) 5 479 – 2 347

   (b) 2 347 – 1 234

   (c) 3 132 + 1 234

Notes on questions

If learners realise that the calculations in question 9 are different versions of the calculation in question 8, it will save them time. They can use the answer for question 8 to get the answers in question 9 (without doing calculations).

In question 11 it is much easier to find the answer by adding on than by breaking up the numbers, i.e. R5 775 + R200 → R5 975 + R20 → R5 995 + R4 → R5 999

Answers

7. 2 347 + 3 132 = 5 479
8. 4 397
9. (a) 4 397 + 2 176 = 6 573
   (b) 2 176 + 4 397 = 6 573
   (c) 6 573 – 2 176 = 4 397
10. 4 397 people
11. R224
12. (a) 922 learners
    (b) 8 654 learners
13. 2 908 chickens
14. 2 899 toilets
15. R7 928

7. Find the number that makes this number sentence true:
   2 347 + ☐ = 5 479
8. Calculate 6 573 – 2 176.
9. Find the numbers that are missing from these number sentences.
   (a) ☐ + 2 176 = 6 573
   (b) 2 176 + ☐ = 6 573
   (c) 6 573 – 2 176 = ☐
10. Between 10 o'clock and 11 o'clock 2 176 people enter a soccer stadium. At 11 o'clock there are 6 573 people in the stadium. How many people were there at 10 o'clock?
11. A refrigerator costs R5 775 at a shop. The same model costs R5 999 at another shop. What is the difference between the two prices?
12. There are 4 788 Grade 4 learners in School District A and 3 866 learners in School District B.
    (a) How many more Grade 4 learners are there in District A than in District B?
    (b) How many Grade 4 learners are there in the two districts together?
13. On five consecutive days a supermarket sold 657, 358, 724, 547 and 622 chickens. How many chickens were sold altogether?
14. A contractor has to build 8 276 flush toilets in a township. He has completed 5 377. How many are still outstanding?
15. An airline charges a fare of R4 480 for a return flight from Johannesburg to Nairobi. There are also additional fees and taxes of R3 448. What is the total cost of the air ticket?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
10.1 What is division?	Division as equal sharing and as grouping. Multiply and divide as inverses	200 to 202
10.2 Remainders	Dividing and multiplying with remainders. Order of operations	203 to 204
10.3 Dividing bigger numbers into equal parts	Multiplying in steps to solve sharing and grouping problems	205 to 206
10.4 Use multiplication facts to solve problems	More multiplying in steps to solve sharing and grouping division problems	206 to 208

CAPS time allocation	4 hours
CAPS page references	14 to 15 and 84 to 85

Mathematical background

Multiplication and division both relate to two very different kinds of situations, which are briefly described below.

1. Situations in which quantities are made up of equal parts

The statement 23 × 37 = 851 can be used to describe 23 objects (e.g. planks) of 37 measurement units (e.g. cm) each, which is a total of 851 units.
The statement 23 × 37 = 851 can also be used to describe the cost of 23 objects at R37 each, which is R851.

Situations like these have the form ***number of parts × size of each part = total quantity*** or ***number of parts × rate = total quantity*** .
Any of the three quantities can be unknown:

- A situation in which the total quantity is unknown, i.e. when the situation can be described by a number sentence of the form ***number of parts × size of each part (or rate) = ?*** is called a **multiplication situation**. To find the unknown total, you have to multiply.
- A situation in which the number of parts is unknown, i.e. when the situation can be described by a number sentence of the form ***? × size of each part (or rate) = total quantity*** is called a **grouping situation**. To find the unknown number of parts, you have to divide.
- A situation in which the size of each part is unknown, i.e. when the situation can be described by a number sentence of the form ***number of parts × ? = total quantity*** is called a **sharing situation**. To find the unknown rate or part size, you have to divide.

2. Ratio situations

In a ratio situation two quantities are compared by using multiplication or division, not by stating the difference between the two quantities. For example, if you stick to a cake recipe that states 2 parts of sugar for 5 parts of cake flour, your amount of cake flour will always be $2\frac{1}{2}\times$ your amount of sugar, and your amount of sugar will always be your amount of cake flour $\div 2\frac{1}{2}$. Ratio situations were dealt with in Term 2 Unit 5. The challenge at the end of this unit (page 207) also involves a ratio situation.

10.1 What is division?

Mathematical notes

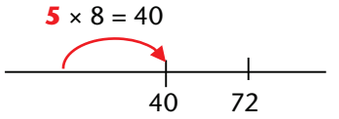
It is very empowering to understand division as the inverse of multiplication, in other words to interpret 90 ÷ 6 as the question: "*What must 6 be multiplied by to get 90?*", or 6 × ? = 90.

Somebody who doesn't know the answer immediately can find it in steps, for example 6 × 10 = 60 and 6 × 5 = 30, so 6 × 10 + 6 × 5 = 90, so 90 ÷ 6 = 15.

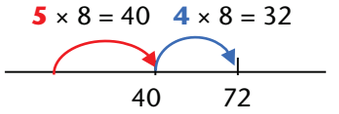
Teaching guidelines

Explain to learners that they can always rewrite a division number sentence as a multiplication number sentence. For example, in question 1 you need to know how many bags or groups of 8 apples is 72 apples. We can write this as 72 ÷ 8 = ☐ or ☐ × 8 = 72.

Ask learners what multiplication facts for 8 they know. Record these on the board. If learners do not know the answer immediately, they can build it up in steps. For example, a learner might know 5 × 8 = 40. This can be recorded on an empty number line.



Learners can then see that 72 is 32 more than 40. Ask them: *"What times 8 is 32?"* Some learners may know that it is 4. Others might try 2 × 8 and get 16. If they add 16 to 40, it will give them 56. They can try and add another 2 × 8: 56 + 16 = 72. Now ask them how many times they needed to multiply by 8 to get 72. In this example the answer is 5 + 4 times, or 5 + 2 + 2 times, in other words 9 times. So, 9 × 8 = 72. Mzwi bought 9 bags of 8 apples each.



It is important that you complete all four sections in the 4 hours allocated to this topic. Build learners' knowledge conceptually, but try to move quickly through Section 10.1. Aim to cover it in 1 hour. One possibility is to use

- questions 1, 4(a) to (c), 5(a) to (c), 6(a) and (b), and 7 for concept development,
- questions 2, 3, 4(d) to (f), 5(d) to (g), 6(c) to (f), and 7 for classwork, and
- questions 5(h) to (k), 6(g) and (h), and 7 for additional practice.

Answers

1. 72 ÷ 8 = ☐ or ☐ × 8 = 72.
   See possible solution under "Teaching guidelines" above.
   The answer is 9 bags.

UNIT 10 WHOLE NUMBERS: DIVISION

10.1 What is division?

Division and multiplication go together.

If you multiply 15 by 6 you get 90:

15 × 6 = 90

Suppose 12 was multiplied by some number and the answer was 60:

12 × . . . = 60

What you do to find the missing number 5 in this case is called **division**.

The symbol ÷ is used to indicate division.

The fact that 12 × 5 = 60 can also be stated by writing 60 ÷ 5 = 12 or 60 ÷ 12 = 5.

1. Mzwi bought 72 apples.
   The apples came in bags of 8 apples each.
   How many bags did he buy?
   (Only some of the bags are shown here.)



How did you find the answer to question 1?

One way is to find the number of bags by counting in eights:

No. of apples	8	16	24	32	40	48	56	64	72
No. of bags	1	2	3	4	5	6	7	8	9
Apples in 1 bag	8	8	8	8	8	8	8	8	8

But it is not necessary to count in eights if you know that 8 × 9 = 72. Then you can give the answer straightaway.

To find the answer to question 1, you can divide 72 by 8 and then the answer is 9. You can describe this with the number sentence 72 ÷ 8 = 9.

Mathematical notes

When you multiply numbers, you can change the way they are grouped without changing the answer, for example $4 \times 2 \times 9 = (4 \times 2) \times 9 = 4 \times (2 \times 9)$. So $8 \times 9 = 4 \times 18$. This is known as the associative property of multiplication. It is also sometimes called the grouping property of multiplication. Learners do *not* need to know the name of the property. They only need to know how to use it to make multiplication easier. This grouping property of multiplication is useful in question 4, and in a lot of other mathematics.

Critical knowledge

You can always change division into multiplication. For example, to find $63 \div 3$ you can multiply $3 \times \square$ to get 63. Here $\square = 21$.

The order in which two numbers are multiplied does not affect the answer, for example $3 \times 24 = 24 \times 3$.

Teaching guidelines

We have already mentioned the importance of linking concepts, ideas, strategies and even questions in mathematics. Remind learners to ask themselves: "*What do I already know that can help me here?*" Learners use the multiplication facts they already know to build up multiples to get the answers to the division calculations. However, learners should also see where they can use solutions or parts of solutions from one question to make the calculations in another question easier.

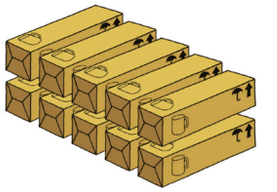
In Term 2 Unit 5 Section 5.6 learners broke numbers down into factors to make multiplication more manageable. They can use this strategy in question 4. In question 5 learners should look for answers they can take directly from question 4.

In question 6 learners can use both these strategies, i.e. factorising and seeing which answers they have already worked out.

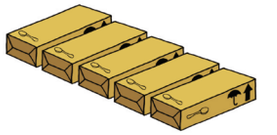
In question 4, remind learners how to break numbers down into factors. For example, in question 4(a): $8 \times 9 = 4 \times 2 \times 3 \times 3 = 8 \times 3 \times 3 = 24 \times 3 = 72$. This gives the answer to question 4(b) as well: $4 \times 18 = 4 \times 2 \times 3 \times 3$, so the answer is also 72. And question 4(c): $3 \times 24 = 24 \times 3 = (4 \times 3 \times 2) \times 3 = 72$, as before.

Show learners how to use their calculations (and answers) from question 4 to answer question 5. For example, question 5(a) uses the multiplication facts from question 4(a), so $72 \div 8$ can be calculated as $\square \times 8 = 72$. The answer is given in 4(a): $\mathbf{9} \times 8 = 72$. Question 4(a) then also gives the answer to question 5(b). The answer to question 5(c) is provided by question 4(b): $\mathbf{4} \times 18 = 72$. In fact, only questions 5(i), (j) and (k) need to be calculated. Here learners should use the answer in (i) to get (j), and the answers in (i) and (j) to get (k).

2. Cindy bought 90 plastic cups.
   The cups came in 10 small boxes.
   How many cups were in each box?



3. Cindy also bought 90 plastic teaspoons.
   The teaspoons came in 5 boxes.
   How many teaspoons were in each box?



4. Calculate each of the following.
   (a) 8×9
   (b) 4×18
   (c) 3×24
   (d) 6×6
   (e) 6×12
   (f) 18×4

5. 72 apples must be put into bags.
   (a) How many bags of 8 apples each can be made?
   (b) How many bags of 9 apples each can be made?
   (c) How many bags of 18 apples each can be made?
   (d) How many bags of 6 apples each can be made?
   (e) How many bags of 12 apples each can be made?
   (f) How many bags of 24 apples each can be made?
   (g) How many bags of 3 apples each can be made?
   (h) How many bags of 4 apples each can be made?
   (i) How many bags of 10 apples each can be made?
   (j) How many bags of 5 apples each can be made?
   (k) How many bags of 15 apples each can be made?

Teaching guidelines

While learners work on questions 1 to 6, you may advise individual learners and the whole class from time to time to use two strategies:

- Use multiplication facts you know.
- Build the answer up in steps.

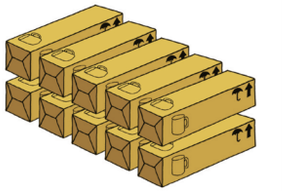
Notes on questions

Questions 1 and 5 demonstrate grouping situations. The question in each case is: "How many groups of a given size?" Questions 2 and 3 demonstrate sharing situations. The question in each case is: "How many items in each group?"

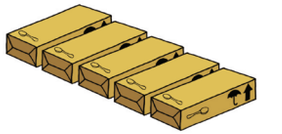
Answers

2. $90 \div 10 = \square$, so $\square \times 10 = 90$. $\boxed{9} \times 10 = 90$. So the answer is 9 cups.
3. $90 \div 5 = \square$, so $\square \times 5 = 90$. $\boxed{18} \times 5 = 90$. So the answer is 18 teaspoons.
   Learners can use the answer in question 2 to get the answer in question 3:
   If $9 \times 10 = 90$, then $9 \times 5 \times 2 = 90$ and $(9 \times 2) \times 5 = 90$, so $18 \times 5 = 90$.
4. (a) 72 (from question 1)
   (b) 72 (because $4 \times 18 = 4 \times (2 \times 3 \times 3) = (4 \times 2) \times (3 \times 3) = 8 \times 9 = 72$ from questions 1 and 4(a))
   (c) 72
   (d) 36
   (e) 72 (because $6 \times 12 = 6 \times 6 \times 2 = 36 \times 2 = 72$)
   (f) 72 (because $18 \times 4 = (6 \times 3) \times 4 = 6 \times (3 \times 4) = 6 \times 12 = 72$ from question 4(e))
5. (a) 9 bags (from questions 1 and 4(a))
   (b) 8 bags (from questions 1 and 4(a))
   (c) 4 bags (from question 4(f))
   (d) 12 bags (from question 4(e))
   (e) 6 bags (from question 4(e))
   (f) 3 bags (because $12 \times 6 = 72$, i.e. $(3 \times 4) \times (2 \times 3) = 72$, so $3 \times (4 \times 2) \times 3 = 72$, i.e. $24 \times 3 = 72$)
   (g) 24 bags (from question 5(f))
   (h) 18 bags (from question 4(f))
   (i) 7 bags with 2 apples remaining unbagged (because $7 \times 10 = 70$)
   (j) 14 bags with 2 apples remaining unbagged (because if $7 \times 10 = 70$, $7 \times (2 \times 5) = 70$, so $(7 \times 2) \times 5 = 70$)
   (k) 4 bags with 12 apples remaining unbagged (because $4 \times 15 = 60$)

2. Cindy bought 90 plastic cups.
   The cups came in 10 small boxes.
   How many cups were in each box?



3. Cindy also bought 90 plastic teaspoons.
   The teaspoons came in 5 boxes.
   How many teaspoons were in each box?



4. Calculate each of the following.
   (a) 8×9 (b) 4×18
   (c) 3×24 (d) 6×6
   (e) 6×12 (f) 18×4

5. 72 apples must be put into bags.
   (a) How many bags of 8 apples each can be made?
   (b) How many bags of 9 apples each can be made?
   (c) How many bags of 18 apples each can be made?
   (d) How many bags of 6 apples each can be made?
   (e) How many bags of 12 apples each can be made?
   (f) How many bags of 24 apples each can be made?
   (g) How many bags of 3 apples each can be made?
   (h) How many bags of 4 apples each can be made?
   (i) How many bags of 10 apples each can be made?
   (j) How many bags of 5 apples each can be made?
   (k) How many bags of 15 apples each can be made?

Teaching guidelines

Ask learners how many rows of cubes they see in Diagram B. Then ask how many columns they see.

Explain that we can write two multiplication number sentences that can describe Diagram B, i.e.

$8 \times 9 = \square$ and $9 \times 8 = \square$

Explain that we can also write two division number sentences for Diagram B, i.e.

$\square \div 8 = 9$ and $\square \div 9 = 8$

Ask learners to write two multiplication number sentences and two division number sentences for each diagram.

Answers

6. (a) 24 (24 × 3 = 72: from question 4(c))
   (b) 3 (24 × 3 = 72: from question 4(c))
   (c) 6 (12 × 6 = 72: from question 4(e))
   (d) 12 (12 × 6 = 72: from question 4(e))
   (e) 4
   (f) 18
   (g) 8
   (h) 9

7. A represents 6(a) and (b).
   B represents 6(g) and (h).
   C represents 6(c) and (d).
   D represents 6(e) and (f).

6. How much is each of the following?
   (a) 72 ÷ 3 (b) 72 ÷ 24
   (c) 72 ÷ 12 (d) 72 ÷ 6
   (e) 72 ÷ 18 (f) 72 ÷ 4
   (g) 72 ÷ 9 (h) 72 ÷ 8

7. When you have answered question 6, you can use these four diagrams to check your answers.
   For each item in question 6, find the diagram that you can use to check your answer. Write the letter of that diagram next to your answer.

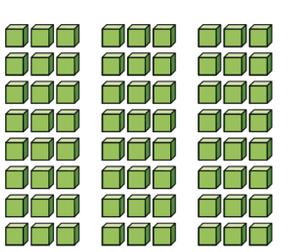


Diagram A

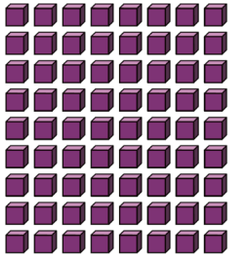


Diagram B

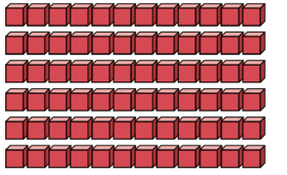


Diagram C

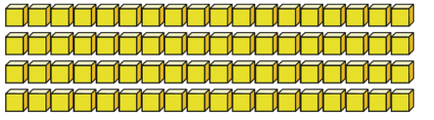


Diagram D

10.2 Remainders

Teaching guidelines

It is important to complete up to Section 10.4 in the 4 hours allocated to this topic. Build learners' knowledge conceptually, but try to move quickly through Section 10.2. Aim to cover this section in 1 hour. One possibility is to use

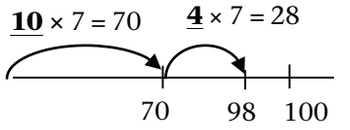
- questions 1, 4(a) and (b), 5(a), 6(a) and (b), and 7(a), as well as the tinted passage on page 203 for concept development,
- questions 2, 3, 4(c) and (d), 5(b) and (c), 6(c) to (f), 7(b) and 8 for classwork, and
- questions 7(c) and 9 for additional practice.

Explain that 72 ÷ 10 = 7 with 2 remaining, because 10 × 7 = 70. So, 10 × 7 + 2 = 72.

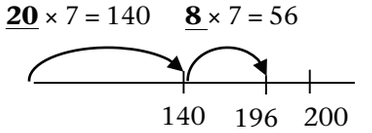
Remind learners that they already know that when an expression or number sentence contains multiplication and addition and/or subtraction, they should do the multiplication first. Use the tinted passage on page 204 to explain this again.

Answers

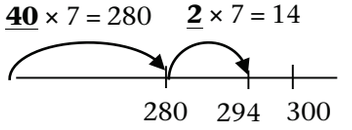
1. Yes
2. (a) 53 (because 6 × 8 = 48 and 48 + 5 = 53)
   (b) No, it is 6 remainder 5. (6 × 8 = 48 and 48 + 5 = 53)
3. By doing 5 × 7 + 3 = 38. Therefore it is not true.
4. (a) False. 44 ÷ 7 = 6 rem 2 (because 6 × 7 = 42 and 42 + 2 = 44)
   (b) False. 67 ÷ 7 = 9 rem 4 (because 9 × 7 = 63 and 63 + 4 = 67)
   (c) True (d) True
5. (a) 14 pencils with R2 remaining unspent.
   Because 10 × 7 = 70 and 4 × 7 = 28

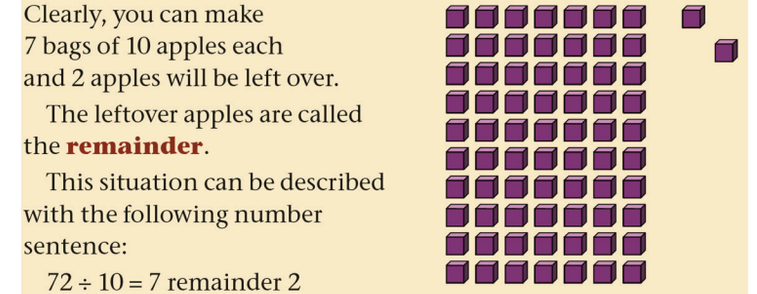


   (b) 28 with R4 remaining unspent.
   With double the money, you can buy twice as many pencils.
   28 × 7 = 196



   (c) 42 pencils with R6 remaining unspent.
   R300 = R200 + 100.
   So you can buy 14 pencils + 28 pencils = 42 pencils. Learners may use different methods to get to the answer.





10.2 Remainders

You have answered this question before:

If you have 72 apples, how many bags of 10 apples each can you make?

Clearly, you can make 7 bags of 10 apples each and 2 apples will be left over.

The leftover apples are called the **remainder**.

This situation can be described with the following number sentence:

72 ÷ 10 = 7 remainder 2

1. Is it true that 76 ÷ 10 = 7 remainder 6?
2. (a) How much is 6 × 8 + 5?
   (b) Is it true that 53 ÷ 8 = 6 remainder 3?
3. How can you check whether it is true that 40 ÷ 7 = 5 remainder 3?
4. Which of these number sentences are true, and which are false? Write true number sentences to replace the false ones.
   (a) 44 ÷ 7 = 6 remainder 3
   (b) 67 ÷ 7 = 8 remainder 8
   (c) 58 ÷ 8 = 7 remainder 2
   (d) 75 ÷ 7 = 10 remainder 5
5. (a) How many pencils at R7 each can you buy with R100?
   (b) How many pencils at R7 each can you buy with R200?
   (c) How many pencils at R7 each can you buy with R300?

Teaching guidelines

Remind learners to keep thinking: *"What have I done before that can help me here?"*

The answers to question 6 will help learners with the answers to questions 7, 8 and 9. When learners do question 8, suggest that they start with (b), then do (c), and then (a).

Answers

6. (a) 56 (b) 56 + 5 = 61 (c) 54 + 3 = 57
   (d) 63 + 5 = 68 (e) 120 + 13 = 133 (f) 132 + 5 = 137

7. (a) 9 rem 5 (from 6(d) above)
   (b) 6 rem 3 (from 6(c) above)
   (c) 7 rem 5 (from 6(b) above)

8. (a) 12 (11 full cartons and one carton with only 5 eggs)
   (b) 11 (from 6(f) above: 11 × 12 = 132. 132 + 5 = 137)
   (c) 5 eggs

9. (a) 6 pens (with R13 unspent; from 6(e) above: 120 + 13 = 133)
   (b) 6 remainder 13

People all over the world agree that 7 × 8 + 5 or 5 + 7 × 8 is a way to say that 7 × 8 must be calculated before 5 is added.

If we want to say that addition should be done first, we use brackets:

7 × (8 + 5) means 8 + 5 = 13 and 7 × 13 = 91.

But 7 × (8 + 5) can also be replaced with 7 × 8 + 7 × 5, which can be calculated like this:

7 × 8 + 7 × 5 = 56 + 35 = 91

(5 + 7) × 8 means 5 + 7 = 12 and 12 × 8 = 96

6. How much is each of the following?
   (a) 7 × 8 (b) 7 × 8 + 5 (c) 6 × 9 + 3
   (d) 7 × 9 + 5 (e) 20 × 6 + 13 (f) 12 × 11 + 5

You may use your results for question 6 when you answer questions 7, 8 and 9.

7. (a) 68 apples have to be shared equally between 7 people. How many apples will each one get and how many apples will remain?
   (b) 57 apples have to be shared equally between 9 people. How many apples will each one get and how many apples will remain?
   (c) 61 apples have to be shared equally between 8 people. How many apples will each one get and how many apples will remain?

8. 137 eggs have to be packed in cartons that can take 12 eggs each.
   (a) How many cartons are needed?
   (b) How many cartons can be filled?
   (c) How many eggs will be in the carton that is not filled?

9. (a) Isaac has R133. How many pens at R20 each can he buy?
   (b) Write the answer for 133 ÷ 20.

10.3 Dividing bigger numbers into equal parts

Critical knowledge

Learners need to know how to multiply by units, multiples of 10 and multiples of 100. In particular, learners need to be able to make multiples of 10 from multiples of units.

Teaching guidelines

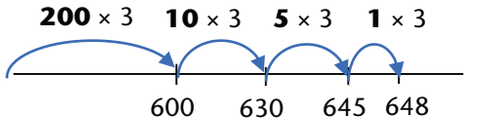
In Sections 10.3 and 10.4 learners divide numbers in the hundreds by units. They should not make dots or stripes and count in ones, or attempt to skip count in intervals of the divisor until they get the answer. Both of these approaches will take too long and learners are more likely to make mistakes. Remind learners to use known multiplication facts.

Aim to complete this section in 45 minutes. You can use question 1 for concept development, questions 2 to 5 for classwork and questions 6 to 8 for additional practice.

Learners should first count how much money there is (R600 + R30 + R18 = R648). You can show learners how to make **clue boards** of easy multiples. Ask learners what are the easiest numbers to multiply by. They will probably say 1, 2 and 10 (zero is also an easy number to multiply by, but it does not help with division). Learners can first write out 1×, and 10×. They can double both of these to get 2× and 20×. Double again to get 4× and 40×. Double again to get 8× and 80×. They can find 5× by halving 10×. They can use these values and an empty number line to calculate the answer. You can find an example of a clue board on page 85 of CAPS.

Using this method for question 1:

1 × 3 = 3	10 × 3 = 30	100 × 3 = 300
2 × 3 = 6	20 × 3 = 60	200 × 3 = 600
4 × 3 = 12	40 × 3 = 120	
8 × 3 = 24	80 × 3 = 240	
5 × 3 = 15	50 × 3 = 150	



R648 ÷ 3 = R200 + R10 + R5 + R1 = R216

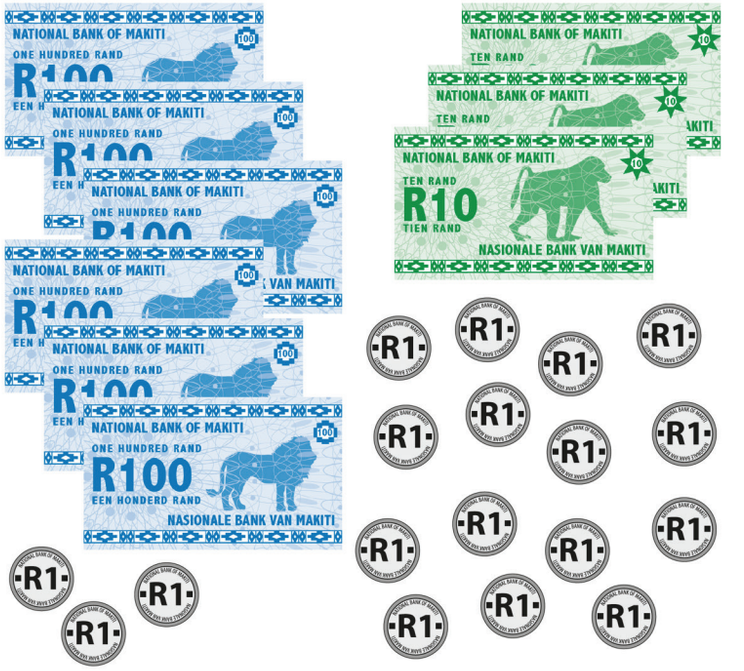
Notes on questions

Questions 1, 4 and 5 involve equal sharing, and questions 2, 3 and 6 involve grouping.

Answers

1. R216 (□ × 3 = R648)
2. 162 pears
3. 108 lemons
4. R129, or R129,60 if they have access to change.
5. R81
6. 64 pies (with R8 change)

10.3 Dividing bigger numbers into equal parts



1. How much will each person get if the money shown here is shared equally between 3 people?
2. How many pears at R4 each can be bought with the money?
3. How many lemons at R6 each can be bought with the money?
4. How much will each person get if the money is shared equally between 5 people?
5. How much will each person get if the money is shared equally between 8 people?
6. How many pies at R10 each can be bought with the money?

Answers

7. R73, or R73,40 if they have access to change.
8. 45 tins of juice

10.4 Use multiplication facts to solve problems

Teaching guidelines

Learners may have some good ideas of their own about finding out how many peaches at R3 each one can buy with R776. Put this question to them at the beginning of the lesson, before they start working on question 1, to allow them to activate their own ideas. Take feedback on their approaches to solving the problem. This will also ensure that they understand the situation and the question when they work on question 1.

Question 1 is critically important. It introduces learners to dividing by building up the total in steps, using known multiplication facts.

Use the second tinted passage on page 206 to explain how to work out how many peaches of R3 each you can buy for R776. You can use this in conjunction with the method outlined in Section 10.3. Learners can use this method for the rest of the questions in this section.

Aim to complete this section in $1\frac{1}{4}$ hour. One possibility is to use

- question 1 and the second tinted passage for concept development,
- questions 2, 3, 5, 7, 8(a), (b), (g) and (h), 10, 12 and 13 for classwork, and
- questions 4, 6, 8(c), (d), (e), (f), (i) and (j), 9(a) and (b), 15 and 16 for additional practice.

Note: Questions 11 and 14 are challenging questions; not all learners need to do them. You could decide to do question 14 in Term 4 Unit 6: *Whole numbers: Division.*

Answers

1. (a) 600; 300
   (b) Yes, because 200 × R3 is R600, which is less than R776.
   (c) No, because R600 + R300 = R900, which is more than R776.
   (d) Yes, because 50 × R3 is half of 100 × R3 = R150. R600 + R150 = R750, which is R26 less than R776.
   (e) 8 peaches. Because 8 × R3 = R24. R24 + R750 = R774. You will have R2 left.
2. 116 tins of juice
3. R116 (using the information from question 2)

7. How much can each person get if R367 is available to be shared equally between 5 people?
8. How many tins of juice at R8 each can be bought with R367?

10.4 Use multiplication facts to solve problems

You will now learn to use multiplication facts to solve division problems with bigger numbers. When you do question 1, you will see how you can find out how many peaches you can buy with R776, if each peach costs R3.

1. (a) How much is 3 × 200, and how much is 3 × 100?
   (b) If the peaches cost R3 each and you have R776, can you buy 200 peaches?
   (c) Can you buy another 100 peaches with the money that is left over?
   (d) Can you buy another 50 peaches?
   (e) How many peaches can you buy with the money that is left over from R776 after having bought 250 peaches?

The work you have done for question 1 can be written up as shown below.

To calculate 776 ÷ 3, you have to find out by what 3 must be multiplied to get 776.
3 × 200 = 600, so the answer is more than 200.
3 × 50 = 150
So 3 × **200** + 3 × **50** = 600 + 150 = 750.
But there is still 26 left over because 776 – 750 = 26.
3 × **8** = 24
So 3 × **258** = 750 + 24 = 774, and 2 is left over.
776 ÷ 3 = 258 remainder 2

2. How many tins of juice at R6 each can you buy with R700?
3. How much does one book cost if 6 books cost R696?

Notes on questions

In questions 8(d) and (i) the hundreds part, the tens part and the units part are all divisible by the divider without leaving a remainder. In such questions it may be more efficient to break numbers down into place value parts and divide each part separately.

Note: From question 4 onwards, learners should multiply out to check answers.

Answers

4. 34 bags

34 × 4	1 × 4 = 4	10 × 4 = 40
= 120 + 16	2 × 4 = 8	20 × 4 = 80
= 136	4 × 4 = 16	40 × 4 = 160
	8 × 4 = 32	80 × 4 = 320
	5 × 4 = 20	50 × 4 = 200

5. 119 loaves

1 × 8 = 8	10 × 8 = 80	100 × 8 = 800
2 × 8 = 16	20 × 8 = 160	200 × 8 = 1 600
4 × 8 = 32	40 × 8 = 320	
8 × 8 = 64	80 × 8 = 640	
5 × 8 = 40	50 × 8 = 400	

6. 16 tables Use the clue board of easy multiples in question 5 above.

7. 24 rows

1 × 7 = 7	10 × 7 = 70	100 × 7 = 700
2 × 7 = 14	20 × 7 = 140	200 × 7 = 1 400
4 × 7 = 28	40 × 7 = 280	
8 × 7 = 56	80 × 7 = 560	
5 × 7 = 35	50 × 7 = 350	

8. (a) 34 Use the clue board of easy multiples of 8 in question 5 above.

4. Kado makes up bags with 4 avocados in a bag. If he has 136 avocados, how many bags can he fill?
5. A bakery produces 952 loaves of bread and delivers the same number of loaves to 8 grocery stores. How many loaves of bread go to each store?

Always check your answers for division by doing multiplication.

6. Mrs Mbete plans a dinner for 128 guests. She wants to seat 8 guests at a table. How many tables does she need?
7. In an orchard, 168 olive trees are planted in 7 equal rows. How many trees are planted in each row?
8. Calculate.
   (a) 272 ÷ 8 (b) 714 ÷ 6
   (c) 252 ÷ 7 (d) 240 ÷ 5
   (e) 365 ÷ 5 (f) 605 ÷ 5
   (g) 714 ÷ 3 (h) 504 ÷ 7
   (i) 250 ÷ 5 (j) 750 ÷ 3
9. (a) 198 beads are arranged in 9 equal rows. How many beads are there in each row?
   (b) 238 beads are arranged in 7 equal rows. How many beads are there in each row?
10. 688 trees are planted in 8 equal rows. How many trees are there in each row?

CHALLENGE

11. For every 3 red beads in a bead pattern, there are 6 blue beads.
    (a) How many red beads are there, if there are 72 blue beads in the pattern?
    (b) How many blue beads are there, if there are 270 red beads in the pattern?

Answers *(continued)*

8. (b) 119

1 × 6 = 6	10 × 6 = 60	100 × 6 = 600
2 × 6 = 12	20 × 6 = 120	200 × 6 = 1 200
4 × 6 = 24	40 × 6 = 240	
8 × 6 = 48	80 × 6 = 480	
5 × 6 = 30	50 × 6 = 300	

(c) 36 Use the clue board of easy multiples of 7 in question 7 above.

(d) 48
Learners can divide each place value part by 5. 240 = 200 + 40. 200 ÷ 5 is the same as ☐ × 5 = 200. ☐ = **40**. 40 ÷ 5 is the same as ☐ × 5 = 40. ☐ = **8**. So 240 ÷ 5 = 48

(e) 73

1 × 5 = 5	10 × 5 = 50	100 × 5 = 500
2 × 5 = 10	20 × 5 = 100	200 × 5 = 1 000
4 × 5 = 20	40 × 5 = 200	
8 × 5 = 40	80 × 5 = 400	
5 × 5 = 25	50 × 5 = 250	

(f) 121 Use the clue board of easy multiples of 5 in (e) above.

(g) 238 Draw a clue board of easy multiples of 3.

(h) 72 Use the clue board of easy multiples of 7 in question 7 above.

(i) 50
Learners can divide each place value part by 5. 250 = 200 + 50. 200 ÷ 5 is the same as ☐ × 5 = 200. ☐ = **40**. 50 ÷ 5 is the same as ☐ × 5 = 50. ☐ = **10**. So 250 ÷ 5 = 50

(j) 250 Use the clue board of easy multiples of 3 in (g) above.

9. (a) 22 beads Draw a clue board of easy multiples of 9.
(b) 34 rows Use the clue board of easy multiples of 7 in question 7 above.

10. 86 trees Use the clue board of easy multiples of 8 in question 5 above.

11. Answers on next page.

4. Kado makes up bags with 4 avocados in a bag. If he has 136 avocados, how many bags can he fill?

5. A bakery produces 952 loaves of bread and delivers the same number of loaves to 8 grocery stores. How many loaves of bread go to each store?

Always check your answers for division by doing multiplication.

6. Mrs Mbete plans a dinner for 128 guests. She wants to seat 8 guests at a table. How many tables does she need?

7. In an orchard, 168 olive trees are planted in 7 equal rows. How many trees are planted in each row?

8. Calculate.

(a) 272 ÷ 8	(b) 714 ÷ 6
(c) 252 ÷ 7	(d) 240 ÷ 5
(e) 365 ÷ 5	(f) 605 ÷ 5
(g) 714 ÷ 3	(h) 504 ÷ 7
(i) 250 ÷ 5	(j) 750 ÷ 3

9. (a) 198 beads are arranged in 9 equal rows. How many beads are there in each row?

(b) 238 beads are arranged in 7 equal rows. How many beads are there in each row?

10. 688 trees are planted in 8 equal rows. How many trees are there in each row?

CHALLENGE

11. For every 3 red beads in a bead pattern, there are 6 blue beads.

(a) How many red beads are there, if there are 72 blue beads in the pattern?

(b) How many blue beads are there, if there are 270 red beads in the pattern?

Answers (*continued*)

11. (a) 36

There is an equal number of groups of red and blue beads. 72 blue beads = ☐ groups of 6 blue beads. That is 12 groups. 12 groups of 3 red beads = 36.

(b) 540

There is an equal number of groups of red and blue beads. 270 red beads = ☐ groups of 3 red beads. That is 90 groups. 90 groups of 6 blue beads = 540.

12. (a) 17 rows (b) 18 rows

13. (a) R18 (b) 27 spoons

Note about question 14

In this question learners can use the strategies of factorising and seeing which answers they have already worked out (e.g. the clue boards). Note that it is a challenging question. It could be extension work for faster learners.

Another possibility is to let learners work in pairs or in small groups (not more than four learners). You could ask them to find at least two ways (instead of "all the possible answers") but challenge them to see how many ways they can find. Remind learners how to break numbers down into factors (see the "Teaching guidelines" for question 4 in Section 10.1 earlier in this unit).

14. (a) ☐ × ☐ × ☐ = 288. Lists of three numbers that will give the answer 288 when multiplied, for example 1 × 2 × 144 = 288, 2 × 3 × 48 = 288, 3 × 8 × 12 = 288, etc.

1	2	144
1	3	96
1	4	72
1	6	48
1	8	36
1	9	32
1	12	24
1	18	16

2	3	48
2	4	36
2	6	24
2	8	18
2	9	16

3	4	24
3	6	16
3	8	12

4	6	12
4	8	9

(b) Consider learners' plans. A sample explanation is given below.

Divide 288 by one of its factors. Then divide the answer you get by another of its factors. Example: 288 ÷ 2 = 144 and 144 ÷ 4 = 36

So 36 × $\underline{4}$ → $\underline{144}$ × $\underline{2}$ = 288

Carla could have multiplied the numbers 2 and 12 and 144.

15. 112 bags

16. Each child will get 16 coins, and 2 coins will remain.

12. A farmer plants his vegetables in neat rows.

(a) He has 136 bean seedlings and plants 8 seedlings in a row. How many rows are there?

(b) He has 126 pumpkin seeds and plants 7 seeds in a row. How many rows are there?

13. (a) Nina bought 9 plates for R162. How much did one plate cost?

(b) Nina also bought spoons for R162. Each spoon cost R6. How many spoons did she buy?

14. Carla likes to play with numbers. She multiplied three numbers and got 288 as the answer, but now she has forgotten which numbers she used.

(a) Make a list of all the possible answers.

(b) Write down what you did to find the numbers.

15. A citrus farmer hires small trucks to transport her lemons to markets all over the country. The load on a truck may not be more than 900 kg. One bag of lemons has a mass of 8 kg. How many bags of lemons can be transported in one load?

16. One hundred and thirty R1 coins are shared equally between 8 children. How many coins does each child get, and how many coins remain?

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Grade 4 Term 3 Unit 1 Common fractions

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Sections in this unit	**Content**	**Pages in Learner Book**
1.1 Compare fractions	Identifying, naming and comparing fractions of objects and fractions of diagrams	211 to 213
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1.3 Equivalent fractions	Comparing fractions and identifying equivalent fractions from diagrams	215 to 216
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1.6 Compare fractions	Comparing fractions and identifying equivalent fractions	222 to 224

CAPS time allocation	5 hours
CAPS page references	16 and 91

Mathematical background

Like whole numbers, fractions can be represented by number names as well as number symbols.

Twenty-three is the number name and 23 the number symbol for a certain number. Similarly, *three eighths* or *3 eighths* is the number name and $\frac{3}{8}$ the number symbol for a certain number.

The correct way to read a number symbol aloud is to say and think the number name. For example, 723 should be read as *seven hundred and twenty-three*, and not as "*seven two three*". Similarly, the number symbol $\frac{3}{8}$ should be read as *three eighths* and not as "*three over eight*".

The widespread practice of referring to a fraction as "*a number over another number*" should be strictly avoided and strongly discouraged whenever it appears. The language a person uses often strongly influences his/her thinking. Learners who consistently refer to fractions incorrectly, who, for example, refer to or think of $\frac{2}{3}$ as "2 over 3" and $\frac{4}{5}$ as "4 over 5", easily end up believing that $\frac{2}{3} + \frac{4}{5} = \frac{6}{8}$ because 2 + 4 = 6 and 3 + 5 = 8.

Worse, such learners may not even realise that $\frac{2}{3}$ represents 2 thirds (more than half) and $\frac{4}{5}$ represents 4 fifths (also more than half), hence $\frac{2}{3} + \frac{4}{5}$ is more than 1, and $\frac{6}{8}$ is a ridiculous answer for $\frac{2}{3} + \frac{4}{5}$.

1.1 Compare fractions

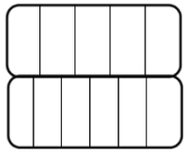
Teaching guidelines

This section is three pages long, but much of the space is taken up by illustrations. Aim to complete it in 1 hour.

Consolidate the fraction name related to different equal parts of different loaves, i.e. thirds, fifths, sixths and tenths. Also ask learners how many half loaves, quarter loaves, thirds of a loaf, fifths of a loaf, etc. make a whole loaf.

Notes on questions

Learners should place the models of the bread cut into pieces next to each other when comparing fraction pieces. For example in question 2(b), if the loaves cut into sixths and fifths respectively are put next to each other you can see the answer.

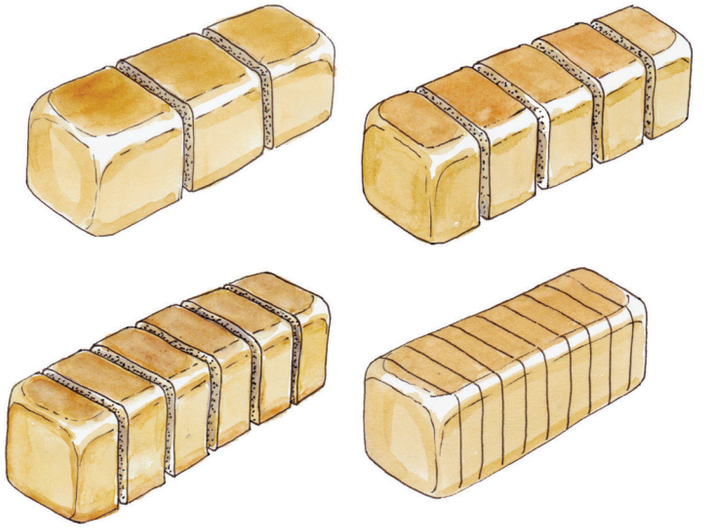


Answers

1. one third
2. (a) 1 sixth (b) 3 sixths (c) 1 third
   (d) equal (e) equal (f) equal
3. (a) 5 tenths (b) 3 fifths (c) 8 eighths (d) 4 eighths

UNIT 1 COMMON FRACTIONS

1.1 Compare fractions



1. Which is more, one third of a loaf or one fifth of a loaf?
2. In each case state which is more.
   (a) 1 sixth or 1 tenth of a loaf (b) 2 fifths or 3 sixths of a loaf
   (c) 1 third or 1 quarter of a loaf (d) 2 sixths or 1 third of a loaf
   (e) 4 fifths or 8 tenths of a loaf (f) 3 thirds or 5 fifths of a loaf
3. (a) Willem eats 5 tenths of a loaf of bread. What part of the loaf is left over?
   (b) Moshanke eats 2 fifths of a loaf of bread. What part of the loaf is left over?
   (c) How many eighths make up a whole loaf?
   (d) How many eighths make up a half of a loaf?

GRADE 4: MATHEMATICS [TERM 3] 211

Critical knowledge

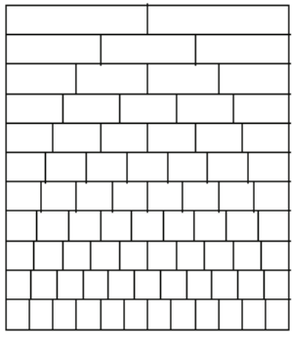
Learners need to know that fractions are equal-sized subdivisions. They need to be able to name different fraction parts, and to know that one whole is 2 halves, 3 thirds, 4 quarters, 5 fifths, 6 sixths, 7 sevenths, 8 eighths, 10 tenths, etc.

Mathematical notes

The images and models of the loaves of bread on page 211 are now translated into fraction bars.

In Grade 4 learners are *not* expected to learn arithmetic rules and procedures for how to add or subtract fractions, how to make equivalent fractions or how to divide wholes into fraction parts. The expectation is rather that learners work with apparatus, diagrams and visual images. This is reflected in the structure of this unit and the solutions modelled in this unit.

Fraction walls are introduced in Section 1.6 on page 222. These are useful tools for visually comparing fractions. However, many Grade 4 learners are confused when given a printed fraction wall with fraction bars showing many different fractions, such as the one alongside. For this reason this kind of fraction wall is only introduced in the Grade 5 Learner Book. In Grade 4 learners are first exposed to loaves of bread cut into different fraction pieces. This image is then translated into fraction bars and fraction measuring strips (rulers). Learners are only introduced to a mini fraction wall towards the end of this unit.



Teaching guidelines

Learners are instructed to draw fraction strips. Encourage them to work quickly but neatly when making the drawings. While it is important that learners think about dividing each bar into equal-size parts (whether quarters, fifths, sixths, etc.), there is no need for their drawings to be 100% accurate. It is not necessary for learners to use a ruler, measure the size of each part or make the lines 100% straight. In fact, it is better if learners do the drawings freehand.

Answers

4. (a) to (d)

   Observe whether learners are able to follow instructions given to produce drawings like those in the Learner Book. Note that the rectangles drawn should be *about the same* length.

Follow the instructions in the questions below to make drawings that show fifths, quarters, eighths and tenths.

Do not use a ruler, but try to draw neatly.

Use a pencil, not a pen.

4. (a) Draw a rectangle more or less like this.

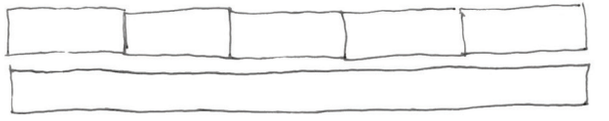


(b) Draw four more rectangles next to it, as close to the same shape and size as you can without using a ruler.



Your five rectangles together form a strip. Each rectangle is one fifth of the whole strip.

(c) Draw a strip of the same length and width, below the one you already have:



(d) Use the first strip as a guide to divide the new strip into fifths as well. Make the lines of your new fifths dark. Draw two more strips, and name your strips as shown below. All your strips must be the same length.

A



B



C

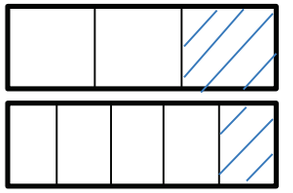


D



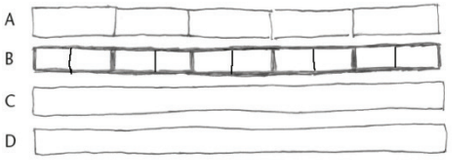
Notes on questions

In questions 6(a) and 7(a) and (b) learners compare fractions that are one fraction part less than 1 whole, i.e. 2 thirds, 4 fifths, 5 sixths, 6 sevenths, 3 quarters. It might be more obvious to learners that 1 third is bigger than 1 fifth than that 2 thirds is smaller than 4 fifths. You can help learners to think by how much each fraction is less than 1, for example 2 thirds is 1 third less than 1; 4 fifths is 1 fifth less than 1.



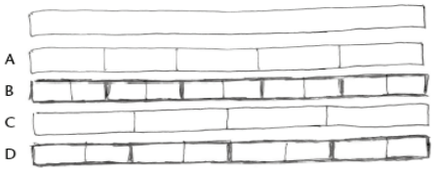
Answers

5. (a) At this point, learners should have a drawing like the one shown alongside.



(b) 10 equal parts; 1 tenth

(c) At this point learners should have a drawing such as the one in question 5(c) of the Learner Book, and shown alongside.



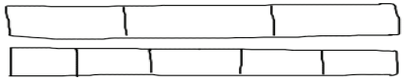
(d) a quarter

(e) an eighth

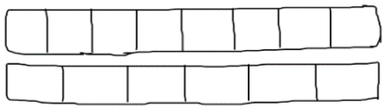
(f) 8 tenths

(g) 8 eighths

6. (a) 4 fifths

(b) 

(c) 2 sixths

7. 

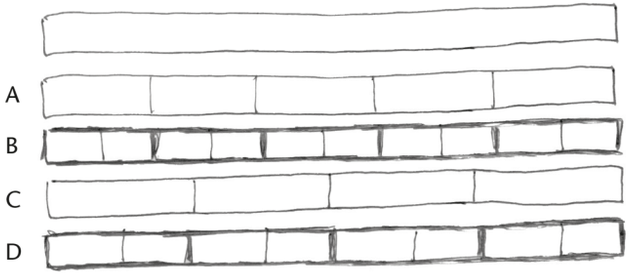
(a) 7 eighths (b) 5 sixths (c) 5 sevenths

8. (a) 3 sevenths (b) 3 fifths

5. (a) Divide each fifth in your Strip B into two equal parts as shown below.

(b) How many equal parts do you now have in Strip B and what can each part be called?

(c) Divide Strip C into quarters, and Strip D into eighths.



(d) Which is bigger, a quarter or a fifth?

(e) Which is bigger, a tenth or an eighth?

(f) How many tenths are the same as four fifths?

(g) How many eighths are a whole strip?

6. (a) Which do you think is bigger, 2 thirds or 4 fifths?

(b) Make drawings like the above to check your answer.

(c) Which is bigger, 3 tenths or 2 sixths?

7. Draw a strip of sixths below your strip of eighths.

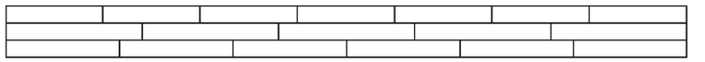
(a) Which do you think is bigger, 5 sixths or 7 eighths?

(b) Which is bigger, 5 sixths or 3 quarters?

8. (a) Which is bigger, 2 fifths or 3 sevenths?

(b) Which is bigger, 3 fifths or 4 sevenths?

(c) Which is bigger, 5 sevenths or 4 sixths?



1.2 Measure with fractions

Mathematical notes

This section builds on all the work done previously on fractions, but takes a similar format to Term 2 Unit 3 Section 3.4, pages 144 to 146.

Teaching guidelines

You can use question 1 to assess whether learners can identify and name fractions.

You can use question 2 and the tinted passage on page 215 for the concept development part of the lesson. Learners can do questions 3, 4, 5(a) and (b), and 6(a) and (b) as classwork. Questions 5(c) to (f) and 6(c) to (f) can be used for additional practice. It may be easier for learners to work with the larger copy of the measuring sticks in the Addendum (pages 454).

Aim to complete this section in 1 hour.

Notes on questions

From question 2 onwards, the ribbons have a whole part and a fraction part.

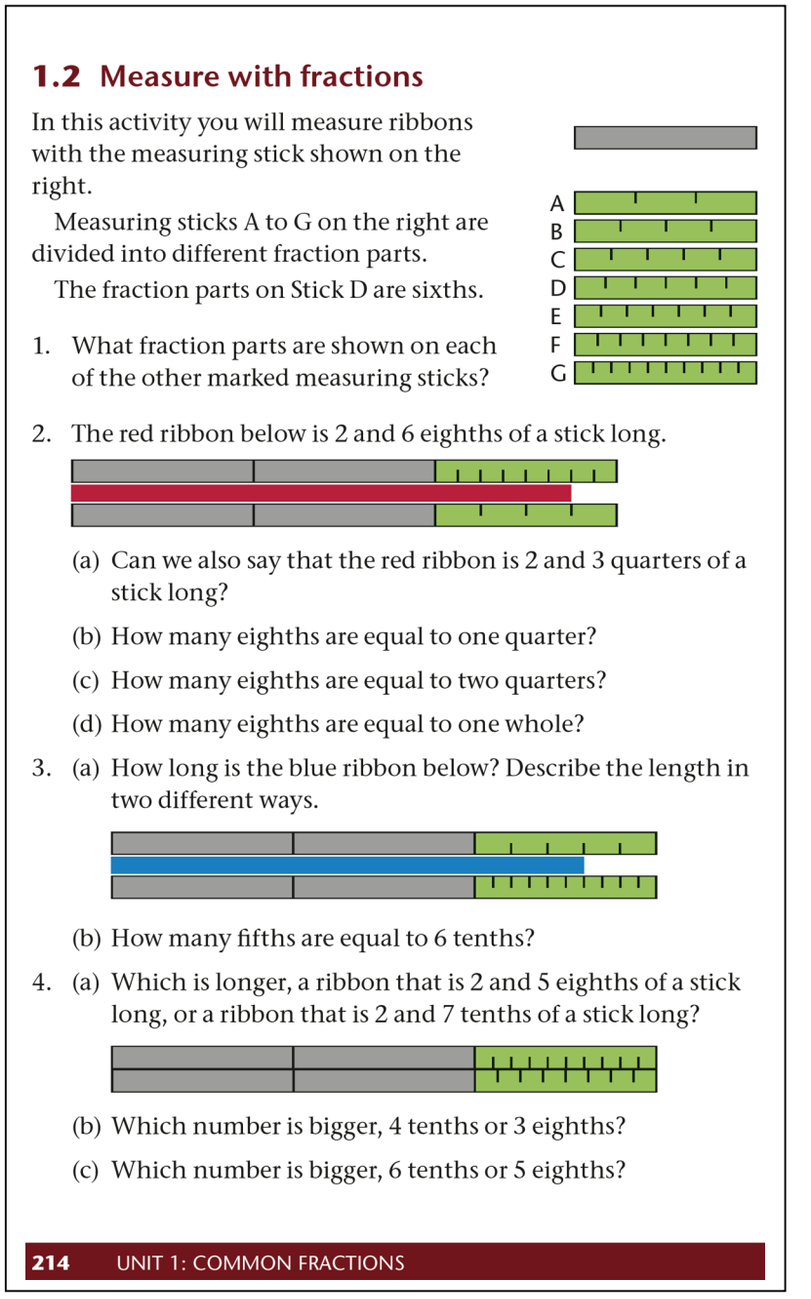
Answers

1. Stick A: thirds Stick B: quarters Stick C: fifths Stick D: sixths
   Stick E: sevenths Stick F: eighths Stick G: tenths

2. (a) Yes (b) 2 eighths (c) 4 eighths (d) 8 eighths

3. (a) 2 and 3 fifths of a stick long or 2 and 6 tenths of a stick long
   (b) 3 fifths

4. (a) A ribbon that is 2 and 7 tenths of a stick long
   (b) 4 tenths
   (c) 5 eighths



1.2 Measure with fractions

In this activity you will measure ribbons with the measuring stick shown on the right.

Measuring sticks A to G on the right are divided into different fraction parts.

The fraction parts on Stick D are sixths.

1. What fraction parts are shown on each of the other marked measuring sticks?

2. The red ribbon below is 2 and 6 eighths of a stick long.

   (a) Can we also say that the red ribbon is 2 and 3 quarters of a stick long?

   (b) How many eighths are equal to one quarter?

   (c) How many eighths are equal to two quarters?

   (d) How many eighths are equal to one whole?

3. (a) How long is the blue ribbon below? Describe the length in two different ways.

   (b) How many fifths are equal to 6 tenths?

4. (a) Which is longer, a ribbon that is 2 and 5 eighths of a stick long, or a ribbon that is 2 and 7 tenths of a stick long?

   (b) Which number is bigger, 4 tenths or 3 eighths?

   (c) Which number is bigger, 6 tenths or 5 eighths?

Answers

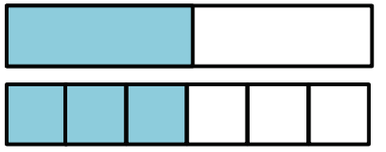
5. (a) $\frac{1}{5}$ (b) $\frac{3}{4}$
   (c) $\frac{4}{6}$ (d) $\frac{2}{3}$
   (e) $\frac{9}{10}$ (f) $\frac{7}{8}$

6. (a) five sixths (b) one third
   (c) five sevenths (d) four eighths
   (e) five eighths (f) nine tenths

1.3 Equivalent fractions

Mathematical notes

Equivalent fractions are fractions that have different names and look different (the way they are written, the way they are represented in diagrams or objects) but have the same value. For example, 1 half and 3 sixths may look different but they have the same value.



Notes on questions

In question 1 the grey bar represents one whole. Each green bar also represent one whole (the different green bars are divided into different fractions). This means that the blue ribbon is more than 1.

Teaching guidelines

You can use question 1, including the diagrams and the information next to the summary bar, as the concept development part of the lesson. Aim to complete this section in 1 hour.

Answers

1. ten sixths or $\frac{10}{6}$ or $1\frac{4}{6}$

A short way to write **one half** is $\frac{1}{2}$.
This way of writing is called the **fraction notation**.
The fraction notation for **two sevenths** is $\frac{2}{7}$.
The fraction notation for **one quarter** or **one fourth** is $\frac{1}{4}$.
The fraction notation for **five sixths** is $\frac{5}{6}$.

5. Write the following numbers in fraction notation:
   (a) one fifth (b) three quarters
   (c) four sixths (d) two thirds
   (e) nine tenths (f) seven eighths

6. Write the following numbers in words:
   (a) $\frac{5}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{7}$
   (d) $\frac{4}{8}$ (e) $\frac{5}{8}$ (f) $\frac{9}{10}$

1.3 Equivalent fractions

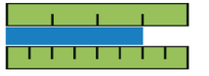
1. Use sixths to state how long this blue ribbon is.





Different ways to describe the same part of a whole are called **equivalent fractions**.

For example, **3 quarters** is equivalent to **6 eighths**.



We can write $\frac{3}{4} = \frac{6}{8}$ to indicate that 3 quarters is equivalent to 6 eighths.

Teaching guidelines

In Grade 4 learners are *not* expected to learn arithmetic rules and procedures for how to make equivalent fractions. The expectation is rather that learners work with apparatus, diagrams and visual images. Learners should use the diagrams at the top of page 216 to answer questions 2 and 3; the diagram at the top of page 214 to answer question 4 and the fraction wall at the top of page 223 to answer question 5.

If you have access to a photocopier you can copy the diagrams on pages 214, 216 and 223 for learners to use (also see Addendum pages 454 to 456). Let them write the names of the fraction parts next to each strip. If you don't have a photocopier, let one learner have their book open to page 214 and another learner to page 216 and later page 223.

Learners could do questions 4(b), (d) and (f), and 5(b) for additional practice.

Mathematical notes

We often compare fractions of the same size wholes. Sometimes we compare fractions of different size wholes, as in question 6.

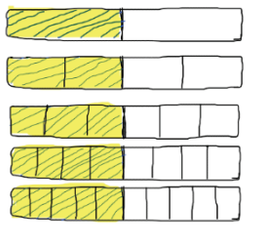
Answers

2. (a) Not equivalent (use Diagram A) (b) Not equivalent (use Diagram A)
   (c) Equivalent (use Diagram A) (d) Equivalent (use Diagram F)
   (e) Equivalent (use Diagram D) (f) Not equivalent (use Diagram C)
   (g) Not equivalent (use Diagram E) (h) Not equivalent (use Diagram G)
   (i) Equivalent (use Diagram C)

3. None. In all cases the first fraction is smaller than or equivalent to the second fraction.

4. (a) $\frac{2}{3}; \frac{3}{4}; \frac{4}{5}$ (b) $\frac{6}{8}; \frac{4}{5}; \frac{5}{6}$ (c) $\frac{3}{8}; \frac{5}{8}; \frac{7}{8}$
   (d) $\frac{3}{5}; \frac{4}{6}; \frac{5}{7}$ (e) $\frac{2}{3}; \frac{7}{10}; \frac{3}{4}$ (f) $\frac{3}{8}; \frac{2}{5}; \frac{4}{7}$

5. (a) Any three of the following: $\frac{2}{4}; \frac{3}{6}; \frac{4}{8}; \frac{5}{10}$
   (b) Any of the three alongside.



6. 3 fifths of the blue ribbon

You may find these diagrams helpful when you answer the questions below.
The diagrams all have the same size.



Diagram A



Diagram B



Diagram C



Diagram D



Diagram E



Diagram F



Diagram G

2. In each case state whether the two fractions are equivalent.
   (a) $\frac{5}{8}$ and $\frac{5}{6}$ (b) $\frac{4}{8}$ and $\frac{4}{6}$ (c) $\frac{4}{8}$ and $\frac{3}{6}$
   (d) $\frac{2}{8}$ and $\frac{1}{4}$ (e) $\frac{1}{5}$ and $\frac{2}{10}$ (f) $\frac{7}{8}$ and $\frac{9}{10}$
   (g) $\frac{5}{7}$ and $\frac{6}{8}$ (h) $\frac{1}{6}$ and $\frac{1}{5}$ (i) $\frac{5}{10}$ and $\frac{4}{8}$

3. In which cases in question 2 is the first fraction bigger than the second fraction?

4. In each case, arrange the fractions from smallest to biggest. The different fractions all refer to the same whole.
   (a) $\frac{4}{5}, \frac{3}{4}$ and $\frac{2}{3}$ (b) $\frac{4}{5}, \frac{6}{8}$ and $\frac{5}{6}$
   (c) $\frac{5}{8}, \frac{3}{8}$ and $\frac{7}{8}$ (d) $\frac{5}{7}, \frac{4}{6}$ and $\frac{3}{5}$
   (e) $\frac{3}{4}, \frac{7}{10}$ and $\frac{2}{3}$ (f) $\frac{2}{5}, \frac{3}{8}$ and $\frac{4}{7}$

5. (a) Write three fractions that are all equivalent to $\frac{1}{2}$.
   (b) Make drawings to show that your answers are correct.

6. Which is longer, $\frac{3}{5}$ of the blue ribbon or $\frac{3}{5}$ of the yellow ribbon?



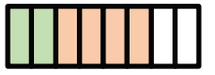
1.4 Calculate with fractions

Teaching guidelines

Learners can use diagrams and/or counting in fractions to calculate the answers to questions 1, 2, 3 and 4. In question 5 they can use fraction bars and colour in pieces as they add. In questions 6 and 7 they can draw pictures and use the relationship between fractions and division.

Possible misconceptions

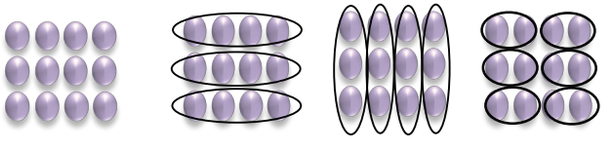
In question 5 and similar questions, when learners do not have a good fraction concept they sometimes add both numerator and denominator. For example, in question 5(a) a learner may mistakenly write $\frac{2}{8} + \frac{4}{8} = \frac{6}{16}$. If learners make this mistake, ask them to draw a fraction bar with eighths and to colour in as they add. They can also change the symbols to words and add 2 eighths plus 4 eighths to see that it gives 6 eighths.



Notes on questions

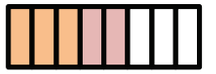
Question 4 is more challenging as learners must first find the fraction of rope that is left over, i.e. 4 sevenths. Then they can work out what 1 seventh of 140 cm is, i.e. 140 cm ÷ 7 = 20 cm, and then multiply 20 cm × 4 = 80 cm to get 4 sevenths of the rope.

In question 6 learners can draw the 12 sweets, and then for (a) and (b) make 3 groups from 12, for (c) make 4 groups from 12, and for (d) make 6 groups from 12.



Answers

1. 5 eighths. Learners could draw a fraction strip to show eighths and colour and count each part – see the diagram alongside.



2. (a) 10 tenths of a metre, or 1 metre (b) 7 eighths of a metre
   (c) 1 and 2 quarters of a metre; $1\frac{2}{4}$ metres; $1\frac{1}{2}$ metres (d) 2 and 4 fifths of a metre
3. 7 sevenths of a metre = 1 metre 4. 80 cm (1 seventh is 20 cm, so 4 sevenths is 80 cm)
5. (a) $\frac{6}{8}$ (b) $\frac{8}{6}$ (c) $\frac{4}{5}$ (d) $\frac{5}{10}$
6. (a) 4 sweets (b) 4 sweets (c) 3 sweets (d) 2 sweets
7. Accept any of the following: (a) $\frac{10}{30}$, 10 thirtieths, $\frac{1}{3}$, 1 third
   (b) $\frac{20}{30}$, 20 thirtieths, $\frac{2}{3}$, 2 thirds (c) $\frac{5}{30}$, 5 thirtieths, $\frac{1}{6}$, 1 sixth

1.4 Calculate with fractions

1. Joe eats 3 eighths of a loaf of bread. He then eats another 2 eighths of the same loaf. What part of the loaf has he eaten altogether?
2. Jenny joins planks of wood to make longer planks. Work out the total length of the plank in each case.
   (a) 3 tenths of a metre and 7 tenths of a metre
   (b) 5 eighths of a metre and 2 eighths of a metre
   (c) two planks that are 3 quarters of a metre each
   (d) 1 and 3 fifths of a metre and 6 fifths of a metre
3. Casper has a piece of rope that is 4 sevenths of a metre long. He has another piece that is 3 sevenths of a metre. How much rope does he have altogether?
4. Thuni has a piece of rope that is 140 cm long. He cuts off 3 sevenths of the rope. How much rope is left over?
5. Calculate:
   (a) $\frac{2}{8} + \frac{4}{8}$ (b) five sixths + three sixths
   (c) $\frac{2}{5} + \frac{2}{5}$ (d) two tenths + three tenths
6. A mother buys a box of sweets. There are 12 sweets in the box.
   (a) The mother wants to share the sweets equally between her 3 children. How many sweets will each child get?
   (b) How many sweets is one third of the box of sweets?
   (c) How many sweets is one quarter of the box of sweets?
   (d) How many sweets is one sixth of the box of sweets?
7. Bonnie has 30 marbles.
   (a) What fraction of all the marbles is 10 marbles?
   (b) What fraction of all the marbles is 20 marbles?
   (c) What fraction of all the marbles is 5 marbles?

Teaching guidelines

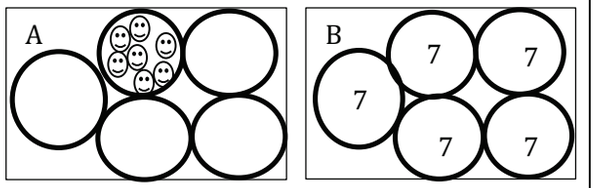
One way to organise your lesson is to let learners do questions 8(a), (b) and (f), 9, 10(a) and 11(a) and (e) as classwork. Questions 8(c) to (e), 10(b), 11(b) to (d), and question 12 can be used for additional practice.

Possible misconceptions

When learners do not have a good fraction concept, they sometimes add both numerator and denominator. In question 11, let learners count in the fraction units to get to the answers.

Notes on questions

Question 9 could be a challenge. Learners need to work out the whole if 1 fifth is 7. One way to do this is to make five groups and first place 7 in one group and then place 7 in each of the other 4 groups. This allows them to see that the class must have $5 \times 7 = 35$ learners.

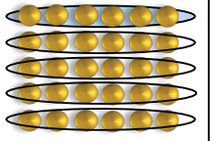


In question 11 learners can simply count in the fraction unit.

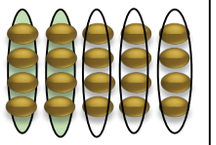
In questions 8 and 10 learners can draw the objects and make groups. These questions link fractions and division.

In question 8 learners need to realise that one fifth means one out of 5 groups, one third means one out of 3 groups, etc. Two sample solutions are provided below:

8. (a) If you divide 30 biscuits into 5 groups, each group is 1 fifth of the biscuits. Each fifth is 6 biscuits.



(f) If you divide 20 chocolates into 5 groups, each group is 1 fifth of the chocolates. One fifth is 4 chocolates.
Two fifths is 8 chocolates.



Answers

8. (a) 6 biscuits (b) 12 marbles (c) 10 children
   (d) 4 minutes (e) 4 eggs (f) 8 chocolates
9. 35 learners
10. (a) 1 quarter or $\frac{1}{4}$ (b) 1 third or $\frac{1}{3}$
11. (a) $\frac{3}{3} = 1$ (b) $\frac{5}{6}$ (c) $\frac{5}{7}$ (d) $\frac{6}{6} = 1$ (e) $\frac{8}{4} = 2$
12. (a) 4 eighths (b) 3 eighths (c) 1 eighth

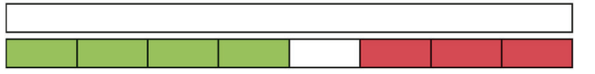
8. Calculate the following:
   (a) one fifth of a box with 30 biscuits
   (b) one third of a bag with 36 marbles
   (c) one quarter of a class of 40 children
   (d) one tenth of a period which is 40 minutes long
   (e) one sixth of a tray which has 24 eggs
   (f) two fifths of a box which has 20 chocolates
9. One fifth of the learners in a class are absent. There are seven learners absent. How many learners are in the class?
10. There are 24 cows in a camp.



   (a) What fraction of all the cows is 6 cows?
   (b) What fraction of all the cows is 8 cows?
11. Write each of the following as one number:
   (a) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ (b) $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$
   (c) $\frac{2}{7} + \frac{3}{7}$ (d) $\frac{1}{6} + \frac{5}{6}$
   (e) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

We know that $\frac{3}{3}$ is the same as 1, so we can write 1 for $\frac{3}{3}$.

12.



   (a) What fraction of the lower strip is green?
   (b) What fraction of the lower strip is red?
   (c) What fraction of the lower strip is white?

1.5 Fraction parts

Teaching guidelines

This is a long section. Aim to complete it in 1 hour. One possibility is to use

- questions 1, 2(d), 3(a), 4, 8 and 10 for concept development,
- questions 2(a) to (c), 3(b) and (c), 5, 7, 9 and 11 for classwork, and
- questions 2(e) to (g), 3(d) to (f) and 6 for additional practice.

You can use the tinted passage on page 219 to introduce this lesson. You can ask learners to count how many rectangles the bar is divided into, and then ask them what fraction of the bar is red and what fraction is yellow.

Then draw their attention to the red lines that divide the bar. Ask learners into how many sections do the red lines divide the bar. Ask what fraction each of these divisions is. Then ask how many quarters are red and how many quarters are yellow. Use this to explain that 3 quarters is equivalent to 6 eighths, and that 1 quarter is equivalent to 2 eighths.

Notes on questions

The bar in question 2(d) has the same proportion of red and yellow as the bar in question 2(c). Although it does not have the same division lines, the same fraction parts are red and yellow.

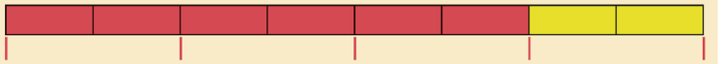
Answers

1. (a) Yes
   (b) 6 eighths
   (c) 2 eighths

2. (a) five sevenths; $\frac{5}{7}$
   (b) four sixths; $\frac{4}{6}$
   (c) four sixths; $\frac{4}{6}$
   (d) four sixths; $\frac{4}{6}$
   (e) three fifths; $\frac{3}{5}$
   (f) three fifths; $\frac{3}{5}$
   (g) six tenths; $\frac{6}{10}$

1.5 Fraction parts

Three quarters of this strip are red.



We can also write in fraction notation:
$\frac{3}{4}$ of this strip is red.

1. (a) Can we also say that 6 eighths of the above strip are red?
   (b) What fraction of the strip below is red?



   (c) What fraction of the strip is yellow?

2. What fraction of each strip below is red? Write your answers in words and in fraction notation.

(a)



(b)



(c)



(d)



(e)



(f)



(g)



GRADE 4: MATHEMATICS [TERM 3] 219

Notes on questions

In question 3 on page 220 and questions 10 and 11 on page 221, string or rope is placed against different kinds of rulers. These help learners to make a transition from understanding fractions as bars or strips to understanding fractions on a number line. In Term 4 Unit 5 learners work with fractions on a number line.

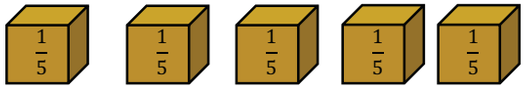
In questions 4, 5 and 6 learners need to link division with making fraction parts.

Teaching guidelines

In question 3, focus learners' attention on counting how many "spaces" each piece of string is divided into. Then ask learners how many of these "spaces" are green parts of the string and how many are red parts of the string.

In question 5 learners could use a combination of drawings and division, as it may be too time-consuming to draw 60 apples.

60 apples packed into 5 boxes → 60 apples ÷ 5 = 12 apples



(a) Each box represents $\frac{1}{5}$ of the whole.

(b) There will be 12 apples in each box.

Answers

3. (a) five sixths; $\frac{5}{6}$ (b) four eighths; $\frac{4}{8}$
   (c) four eighths; $\frac{4}{8}$ (d) three sevenths; $\frac{3}{7}$
   (e) three sixths; $\frac{3}{6}$ (f) three tenths; $\frac{3}{10}$
4. (a) 24 ÷ 6 = 4 people (b) 4 litres
5. (a) one fifth; $\frac{1}{5}$ (b) 12 apples
6. (a) 8 people (b) one eighth; $\frac{1}{8}$

3. What part of each piece of string is red? Write your answers in words and in fraction notation.

(a)



(b)



(c)



(d)



(e)



(f)



4. 24 ℓ of water is shared equally between a number of people. Each person gets one sixth of the water.
   (a) How many people share the water?
   (b) How much water does each person get?
5. 60 apples are packed into 5 boxes. The same number of apples are put into each box.
   (a) What fraction of the apples is put into each box?
   (b) How many apples are put into each box?
6. R40 is divided equally between a number of people. Each person gets R5.
   (a) How many people share the money?
   (b) What fraction of the money does each person get?

Notes on questions

These questions are challenging. Learners will need substantial time and some learners may need individual support.

There is a strong focus on the relationship between fractions and division.

Teaching guidelines

Learners might find questions 10 and 11 easier than questions 7, 8 and 9, partly because they provide visual support. You could start with questions 10 and 11.

Grade 4 learners are *not* expected to learn how to simplify fractions by "cancelling". Instead they could use the connection between division and fractions.

Encourage learners to ask: "*What do I already know that can help me here?*" This will help them to link different sub-sections of questions, link previous questions with later questions, and link different parts of mathematics.

In questions 8(c) and 9(a) and (b) learners need to recognise what the whole is. For example, in question 8(c) the whole is 700 g, in question 9(a) the whole is 800 kg, in question 9(b) the whole is 200 kg.

Answers

7. (a) 2 × 8 kg = 16 kg

   (b) one eighth; $\frac{1}{8}$ (because there are 8 equal portions)

8. (a) 700

   (b) 140 (Learners are expected to realise that 140 + 140 + 140 + 140 + 140 is five equal groups of 140, so each group is 1 fifth.)

   (c) Learners need to recognise that you could divide the total amount of flour, i.e. 560 g + 140 g = 700 g by the amount of white bread flour, i.e. 140 g. Once learners see that the total amount of flour is 700 g they should recognise from 8(b) that 140 g is one fifth ($\frac{1}{5}$). If not, learners can find how many portions of 140 g you get in 700 g by dividing 700 g ÷ 140 g = 5. So 140 g is one part out of five, or 1 fifth ($\frac{1}{5}$).

9. (a) Altogether the mixture is 200 kg + 600 kg = 800 kg. Some learners may recognise that there are 4 lots of 200 kg in 800 kg; if not, they can divide 800 kg ÷ 200 kg = 4. So 200 g is one part of 4 parts, or one quarter ($\frac{1}{4}$).

   (b) The total mass of the mixture is 200 kg. 200 kg ÷ 25 kg = 8. So 25 kg is one eighth ($\frac{1}{8}$) of 200 kg.

7. A bag of flour is divided into 8 portions of 2 kg each.

   (a) How much flour was in the bag, before it was divided into portions?

   (b) What fraction of the whole bag is each portion?

8. (a) Calculate 140 + 140 + 140 + 140 + 140.

   (b) How much is one fifth of 700?

   (c) 140 g of white bread flour is mixed with 560 g of wholewheat flour to make bread dough. What fraction of the flour mixture is white bread flour?

9. What fraction of each mixture is cement?

   (a) 200 kg of cement is mixed with 600 kg of sand.

   (b) 25 kg of cement is mixed with 175 kg of sand.

10. The ruler below is marked in millimetres.
    What fraction of this piece of rope is green?



11. What fraction of each piece of rope is green?

(a)


(b)


(c)

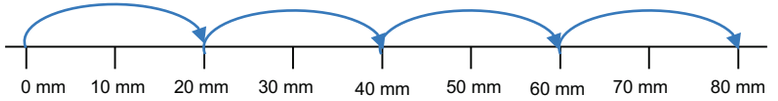

(d)


Answers *(continued)*

10. Learners could take a number of approaches here.

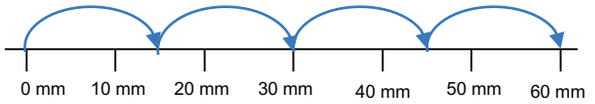
    They can convert the millimetres to centimetres and think of the green part as 2 cm out of 8 cm, or two eighths ($\frac{2}{8}$).

    They can think of the ruler as a number line and ask: "How many 20 mm in 80 mm?"

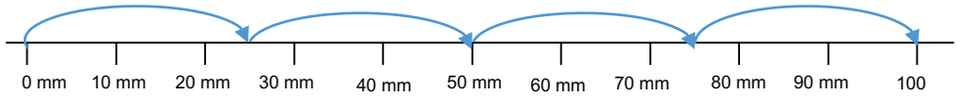


    There are four 20s in 80, so 20 mm is one quarter ($\frac{1}{4}$) of 80 mm.

11. (a) The green part is 15 mm out of 60 mm. There are 4 groups of 15 in 60.
    15 mm is a quarter ($\frac{1}{4}$) of 60 mm.



    (b) The green part of the rope is 25 mm out of 100 mm. There are four 25s in 100.
    You can show this on a number line.



    You could also simply divide 100 ÷ 25 = 4. So 25 mm is one out of 4 parts.
    The green part of the rope is $\frac{1}{4}$ of the length of the rope.

    (c) There are three 30s in 90. So 30 mm is $\frac{1}{3}$ of 90 mm.

    (d) Here learners can convert millimetres to centimetres.
    The green part is 3 cm out of 8 cm. This is $\frac{3}{8}$ of the length of the rope.

7. A bag of flour is divided into 8 portions of 2 kg each.
   (a) How much flour was in the bag, before it was divided into portions?
   (b) What fraction of the whole bag is each portion?
8. (a) Calculate 140 + 140 + 140 + 140 + 140.
   (b) How much is one fifth of 700?
   (c) 140 g of white bread flour is mixed with 560 g of wholewheat flour to make bread dough. What fraction of the flour mixture is white bread flour?
9. What fraction of each mixture is cement?
   (a) 200 kg of cement is mixed with 600 kg of sand.
   (b) 25 kg of cement is mixed with 175 kg of sand.
10. The ruler below is marked in millimetres.
    What fraction of this piece of rope is green?



11. What fraction of each piece of rope is green?

(a)


(b)


(c)


(d)


1.6 Compare fractions

Notes on questions

Questions 1 and 2 are revision of diagrammatic representation of fractions. The aim is that they help to prepare learners to compare fractions in the later questions.

Question 9 deals with fractions of collections of objects. Show learners how to use diagrams and the relationship between division and fractions to make sense of the sub-questions. Worked solutions using this approach are provided alongside question 9 in this guide.

Teaching young learners rules to find fractions of collections generally confuses them and often makes them fearful of fractions as a topic. The curriculum does *not* require the teaching of rules to do calculations with fractions in Grade 4.

Question 10 is a challenge. Learners need to construct their own diagrams. Perhaps not all learners need to attempt this question or all parts of this question.

Teaching guidelines

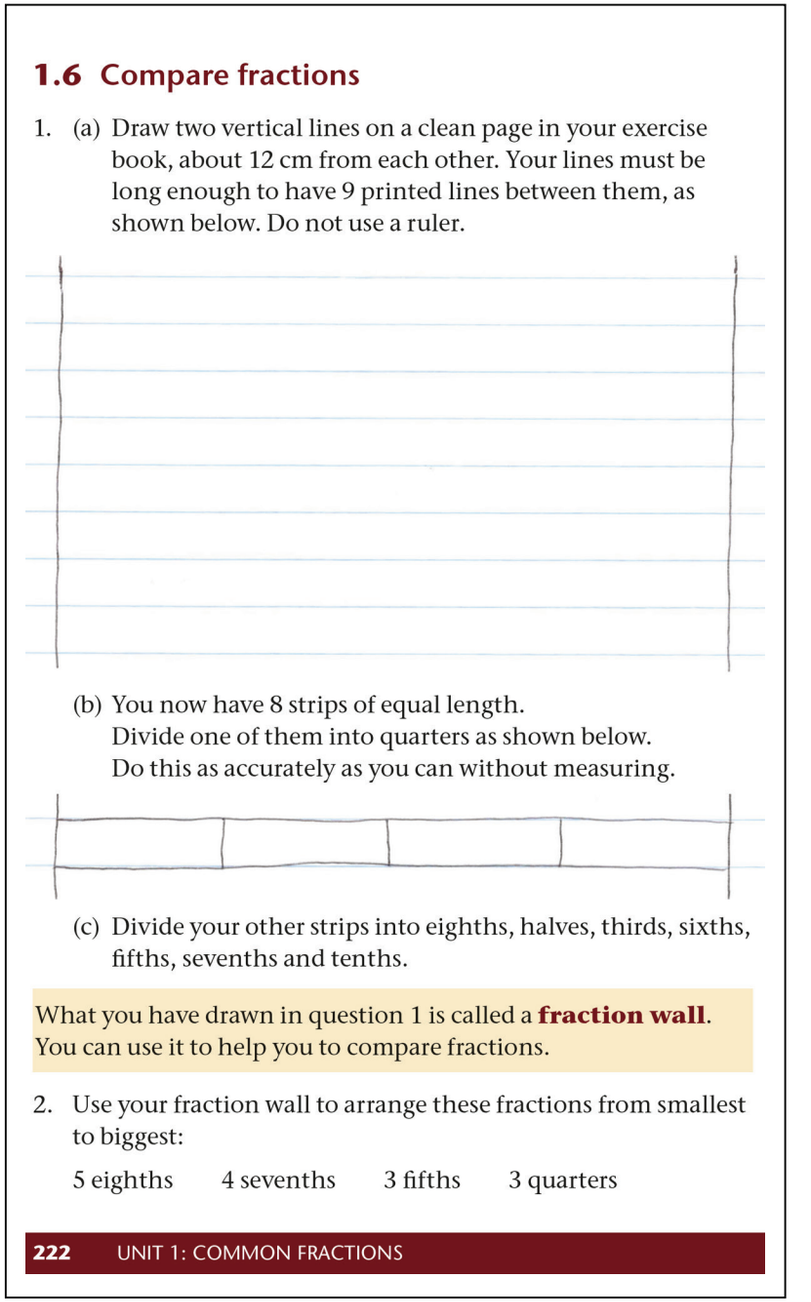
This is a long unit. If you run out of time, you can defer Section 1.6 until the fourth term.

In question 1 learners should be kept focused on the fact that fractions are equal divisions, for example quarters are 4 equal parts, sevenths are 7 equal parts, etc. However, their drawings do not need to be 100% accurate. So learners should work neatly but quickly and not worry about using a ruler to measure or draw the lines.

In question 2 learners should refer to the fraction wall they have drawn to get their answers.

Answers

1. Practical preparation of a fraction wall
2. 4 sevenths; 3 fifths; 5 eighths; 3 quarters



1.6 Compare fractions

1. (a) Draw two vertical lines on a clean page in your exercise book, about 12 cm from each other. Your lines must be long enough to have 9 printed lines between them, as shown below. Do not use a ruler.

(b) You now have 8 strips of equal length.
Divide one of them into quarters as shown below.
Do this as accurately as you can without measuring.

(c) Divide your other strips into eighths, halves, thirds, sixths, fifths, sevenths and tenths.

What you have drawn in question 1 is called a **fraction wall**.
You can use it to help you to compare fractions.

2. Use your fraction wall to arrange these fractions from smallest to biggest:

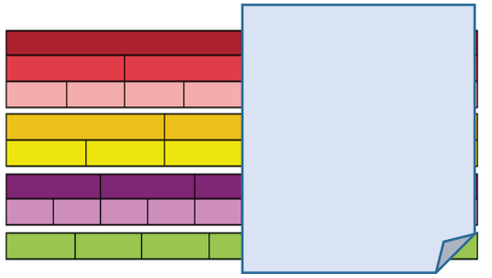
5 eighths 4 sevenths 3 fifths 3 quarters

222 UNIT 1: COMMON FRACTIONS

Teaching guidelines

Grade 4 learners are *not* expected to use formulae or arithmetic procedures to find equivalent fractions. They are expected to use diagrams as an aid to identifying equivalent fractions.

Learners should use the fraction wall on page 223 (or the larger copy in the Addendum on page 456) to answer questions 3 to 8. In order to find equivalent fractions or to compare fractions, they can place a ruler or anything with a straight edge against the end of one of the fraction parts and read off which fractions are equivalent, or whether the fraction they are comparing to is more or less than the one against which the ruler is placed. For example, in the diagram alongside a piece of paper is placed against 1 half. This allows learners to see that 1 half is equal to 2 quarters, 4 eighths, 3 sixths and 5 tenths.



Answers

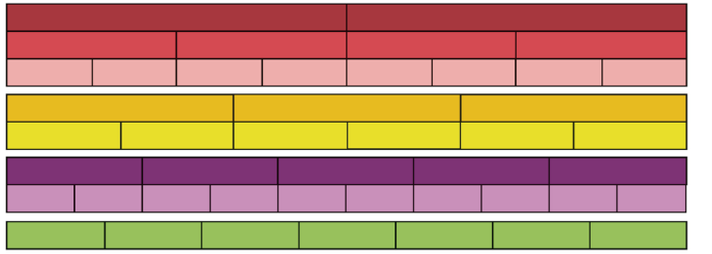
3. Learners use the accurate fraction wall to check their answer to question 2.
4. (a) $\frac{3}{6}$ (b) three quarters (c) equal (d) seven eighths
   (e) $\frac{2}{5}$ (f) equal (g) $\frac{2}{3}$ (h) two thirds
5. (a) $\frac{3}{8}; \frac{1}{2}; \frac{5}{8}; \frac{3}{4}$ (b) $\frac{3}{7}; \frac{3}{6}; \frac{3}{5}; \frac{3}{4}$ (c) $\frac{1}{4}; \frac{1}{3} = \frac{2}{6}; \frac{2}{5}$ (d) $\frac{2}{7}; \frac{2}{6}; \frac{3}{8}; \frac{3}{7}$
6. (a) Yes (b) $\frac{4}{6}$
7. (a) $\frac{4}{6}$ (b) $\frac{2}{6}$ (c) $\frac{2}{8}$ (d) $\frac{1}{2}$ or $\frac{4}{8}$ or $\frac{3}{6}$ or $\frac{5}{10}$
8. Any three of: $\frac{2}{4}; \frac{4}{8}; \frac{3}{6}; \frac{5}{10}$

Mathematical notes

In Section 1.4 learners started working with fractions of collections of objects such as sweets, marbles, biscuits, children in a class, eggs in a tray and cattle in a herd. In question 9 on page 224 learners continue to work with fractions of collections of objects. This is further extended in Term 4 Unit 5. You may like to read Term 4 Unit 5 before you teach question 9.

In question 9 learners work with beads arranged in rows and columns to form an array. Learners have worked with arrays when doing multiplication and division. In question 9 they link taking a fraction of a collection of objects with division and multiplication.

3. Use this accurate fraction wall to check your answer to question 2.



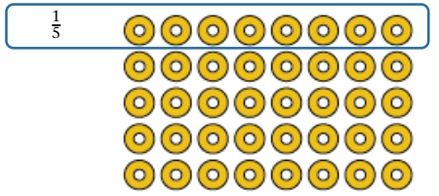
4. Which is more? You may use the above fraction wall to answer this question.
   (a) $\frac{3}{7}$ or $\frac{3}{6}$ (b) two thirds or three quarters
   (c) $\frac{4}{8}$ or $\frac{3}{6}$ (d) six sevenths or seven eighths
   (e) $\frac{2}{5}$ or $\frac{1}{3}$ (f) three sixths or four eighths
   (g) $\frac{3}{5}$ or $\frac{2}{3}$ (h) two fifths or two thirds
5. Arrange the fractions from the smallest to the biggest.
   (a) $\frac{1}{2}; \frac{3}{4}; \frac{5}{8}; \frac{3}{8}$ (b) $\frac{3}{6}; \frac{3}{5}; \frac{3}{7}; \frac{3}{4}$
   (c) $\frac{2}{6}; \frac{1}{3}; \frac{1}{4}; \frac{2}{5}$ (d) $\frac{3}{8}; \frac{2}{7}; \frac{3}{7}; \frac{2}{6}$
6. (a) Is $\frac{6}{8}$ equivalent to $\frac{3}{4}$?
   (b) Write a fraction that is equivalent to $\frac{2}{3}$.
7. Write a fraction that is equivalent to:
   (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{4}$
8. Write three different fractions that are all equivalent to $\frac{1}{2}$.

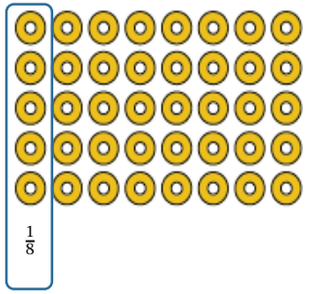
Teaching guidelines

You can introduce question 9 by focusing learners' attention on the rows and columns. Ask: *"How many rows of beads are there in the picture?", "How many columns of beads are there in the picture?", "How many beads are there altogether?", "What fraction is one row of the beads?", "What fraction is two rows of the beads?", "What fraction is three rows of the beads?", "What fraction is four rows of the beads?", "What fraction is five rows of the beads?"*

Then turn the question around and ask: *"How many beads is $\frac{1}{5}$ of the beads, $\frac{2}{5}$ of the beads?"*, etc. Each row is 1 out of 5 rows, or 1 fifth of the beads. So $\frac{1}{5}$ of the beads is 8 beads.

Repeat this for the columns. Each column is 1 out of 8 columns, or 1 eighth of the beads.





Learners can use division to work out the unit fraction, i.e. $\frac{1}{5}; \frac{1}{8}; \frac{1}{4}$, and then multiply to get the other fractions required.

Answers

9. (a) $\frac{1}{8}$ of 40 beads = 5 beads (1 column of beads in the picture/array)

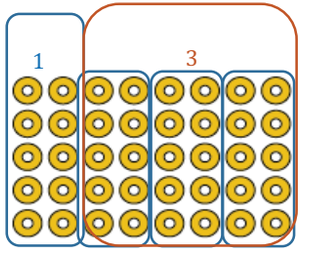
   (b) $\frac{3}{8}$ of 40 beads = 3 × 5 beads = 15 beads (3 columns of beads in the picture)

   (c) $\frac{3}{5}$ of 40 beads = 3 × $\frac{1}{5}$ of the beads = 3 × 8 beads = 24 beads (3 rows of beads)

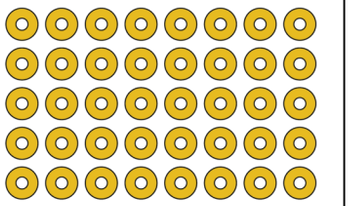
   (d) $\frac{3}{5}$ of 40 beads, i.e. 24 beads, is 9 more beads than $\frac{3}{8}$ of 40 beads, i.e. 15 beads.

   (e) $\frac{5}{8}$ of 40 beads = 5 × 5 beads = 25 beads. $\frac{5}{8}$ of 40 beads is more than $\frac{3}{5}$ of 40 beads (24 beads). $\frac{5}{8}$ of 40 beads is 1 more bead than $\frac{3}{5}$ of 40 beads.

   (f) $\frac{1}{4}$ of 40 beads = 10 beads. $\frac{3}{4}$ of 40 beads = 3 ×10 beads = 30 beads. $\frac{5}{8}$ of 40 beads = 5 × 5 = 25 beads.
   $\frac{3}{4}$ of 40 beads is more than $\frac{5}{8}$ of 40 beads.
   $\frac{3}{4}$ of 40 beads is 5 more beads than $\frac{5}{8}$ of 40 beads.



9. (a) How many beads is one eighth of 40 beads?



   (b) How many beads is three eighths of 40 beads?

   (c) How many beads is three fifths of 40 beads?

   (d) Which is more, $\frac{3}{8}$ of 40 beads or $\frac{3}{5}$ of 40 beads? How many more is it?

   (e) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{5}$ of 40 beads? How many more is it?

   (f) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{4}$ of 40 beads? How many more is it?

10. (a) Which is more, $\frac{3}{8}$ of 80 beads or $\frac{3}{8}$ of 40 beads? How many more is it?

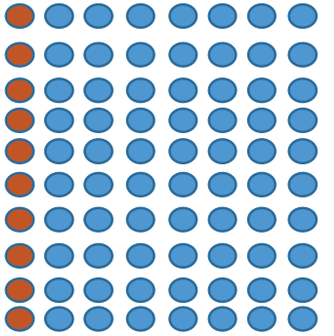
   (b) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{5}$ of 50 beads? How many more is it?

   (c) Which is more, $\frac{7}{8}$ of 80 beads or $\frac{4}{5}$ of 80 beads? How many more is it?

   (d) Which is more, $\frac{5}{8}$ of 120 beads or $\frac{2}{3}$ of 120 beads? How many more is it?

   (e) Which is more, $\frac{5}{6}$ of 90 beads or $\frac{4}{5}$ of 150 beads? How many more is it?

Notes on questions



Question 10 is a challenge. Perhaps it is not necessary for all learners to attempt this question or all sub-sections of this question.

A diagram of the 80 beads is not provided in the Learner Book. You could alert learners to the fact that 80 beads are double 40 beads, and encourage them to use this when they do their drawing and also when they do the calculations.

It is useful for learners to first work out a unit fraction of the collections, for example $\frac{1}{8}; \frac{1}{5}; \frac{1}{6}; \frac{1}{3}$ and then to multiply to get other fractions, for example $\frac{3}{8}; \frac{4}{5}; \frac{5}{6}; \frac{2}{3}$.

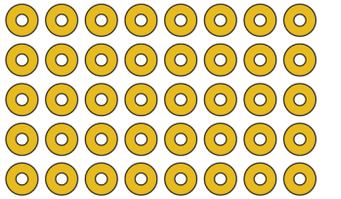
In 10(d) and (e) learners might not want to draw the collections of 120, 90 and 150 beads. They could instead use the relationship between division and fractions to find unit fractions and then multiply as needed. In question 10(d) they can also use the fact that 120 is 3 × 40 beads. In 10(e) they can use the fact that 150 beads is 3 × 50 beads.

Answers

10. (a) $\frac{3}{8}$ of 80 = 30 beads and is 15 more than $\frac{3}{8}$ of 40 beads, which is 15 beads.

Learners already know from question 9(d) above that $\frac{3}{8}$ of 40 beads = 3 × 5 beads = 15 beads.
Learners could draw pictures to work this out. They could draw an 8 × 10 array to work out what $\frac{1}{8}$ of 80 beads is (=10 beads) and multiply by 3 to get $\frac{3}{8}$ of 80 beads (= 30 beads).
Learners could also calculate as follows: $\frac{1}{8}$ of 80 = 80 ÷ 8 = 10.
So $\frac{3}{8}$ of 80 beads = 3 × 10 = 30 beads. So $\frac{3}{8}$ of 80 beads is 15 more beads than $\frac{3}{8}$ of 40 beads.

Another approach is to say we know (from question 9(b)) that $\frac{3}{8}$ of 40 beads = 15 beads. 80 is double 40, so $\frac{3}{8}$ of 80 beads = 30 beads. So $\frac{3}{8}$ of 80 beads is 15 more beads than $\frac{3}{8}$ of 40 beads.



9. (a) How many beads is one eighth of 40 beads?
   (b) How many beads is three eighths of 40 beads?
   (c) How many beads is three fifths of 40 beads?
   (d) Which is more, $\frac{3}{8}$ of 40 beads or $\frac{3}{5}$ of 40 beads? How many more is it?
   (e) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{5}$ of 40 beads? How many more is it?
   (f) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{4}$ of 40 beads? How many more is it?
10. (a) Which is more, $\frac{3}{8}$ of 80 beads or $\frac{3}{8}$ of 40 beads? How many more is it?
   (b) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{5}$ of 50 beads? How many more is it?
   (c) Which is more, $\frac{7}{8}$ of 80 beads or $\frac{4}{5}$ of 80 beads? How many more is it?
   (d) Which is more, $\frac{5}{8}$ of 120 beads or $\frac{2}{3}$ of 120 beads? How many more is it?
   (e) Which is more, $\frac{5}{6}$ of 90 beads or $\frac{4}{5}$ of 150 beads? How many more is it?

Answers *(continued)*

10. (b) Learners already know from question 9(e) that $\frac{5}{8}$ of 40 beads = 25 beads.

Learners can work out that $\frac{1}{5}$ of 50 beads = 50 ÷ 5 = 10 beads, or they could draw a picture to show this. So $\frac{3}{5}$ of 50 beads = 3 × 10 beads = 30 beads.

So $\frac{3}{5}$ of 50 beads is 5 more beads than $\frac{5}{8}$ of 40 beads.

(c) Learners already know from question 9(e) that $\frac{1}{8}$ of 40 beads = 5 beads.

So $\frac{7}{8}$ of 40 beads = 7 × 5 beads = 35 beads.

$\frac{1}{5}$ of 80 beads = 80 ÷5 = 16 beads. So $\frac{4}{5}$ of 80 beads = 4 × 16 beads = 64 beads.

So $\frac{4}{5}$ of 80 beads is 29 more beads than $\frac{7}{8}$ of 40 beads.

(d) $\frac{1}{8}$ of 120 beads = 120 ÷ 8 = 15 beads. So $\frac{5}{8}$ of 80 beads = 5 × 15 beads = 75 beads.

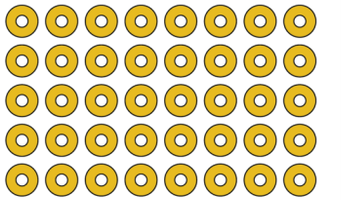
$\frac{1}{3}$ of 120 beads = 120 ÷ 3 = 40 beads. So $\frac{2}{3}$ of 80 beads = 2 × 40 beads = 80 beads.

So $\frac{2}{3}$ of 120 beads is 5 more beads than $\frac{5}{8}$ of 120 beads.

(e) $\frac{1}{6}$ of 90 beads = 90 ÷ 6 = 15 beads. So $\frac{5}{6}$ of 90 beads = 5 × 15 beads = 75 beads.

$\frac{1}{5}$ of 150 beads = 150 ÷ 5 = 30 beads. So $\frac{4}{5}$ of 150 beads = 4 × 30 beads = 120 beads.

So $\frac{4}{5}$ of 150 beads is 45 more beads than $\frac{5}{6}$ of 90 beads.



9. (a) How many beads is one eighth of 40 beads?

(b) How many beads is three eighths of 40 beads?

(c) How many beads is three fifths of 40 beads?

(d) Which is more, $\frac{3}{8}$ of 40 beads or $\frac{3}{5}$ of 40 beads? How many more is it?

(e) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{5}$ of 40 beads? How many more is it?

(f) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{4}$ of 40 beads? How many more is it?

10. (a) Which is more, $\frac{3}{8}$ of 80 beads or $\frac{3}{8}$ of 40 beads? How many more is it?

(b) Which is more, $\frac{5}{8}$ of 40 beads or $\frac{3}{5}$ of 50 beads? How many more is it?

(c) Which is more, $\frac{7}{8}$ of 80 beads or $\frac{4}{5}$ of 80 beads? How many more is it?

(d) Which is more, $\frac{5}{8}$ of 120 beads or $\frac{2}{3}$ of 120 beads? How many more is it?

(e) Which is more, $\frac{5}{6}$ of 90 beads or $\frac{4}{5}$ of 150 beads? How many more is it?

Grade 4 Term 3 Unit 2 Capacity and volume

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 Measure in millilitres	Developing a sense of the volume size of a millilitre and the relationship between millilitres and litres	225 to 226
2.2 Volume and capacity	Differentiating between volume and capacity	227 to 230
2.3 Litre and millilitre	Consolidating the relationship between litres and millilitres Calculating and solving problems with litres and millilitres	231 to 235
2.4 Measuring and reading capacity and volume	Measuring and reading capacity calibrations at numbered and unnumbered intervals	236 to 237

CAPS time allocation	6 hours
CAPS page references	26 and 88 to 90

Mathematical background

Capacity/volume, length, mass, perimeter and area are different properties of objects. When we measure these properties of objects we are describing how much of that property they have; we are describing the property in terms of a numerical value.

When we measure the capacity of containers we are allocating a numerical value to how much that container can hold. This allows us to compare and order containers in terms of what they can hold, for example a 2 ℓ milk bottle can hold more than a 1 ℓ milk bottle. Giving numerical values to capacity/volume allows us to do calculations, for example to calculate how many 150 ml cups of juice you can pour from a 2 ℓ bottle of juice.

Learners go through four stages when learning to measure:

1. Identifying and understanding the property they are measuring. This is done with capacity in the Foundation Phase.
2. Comparing and ordering examples of a particular measure (see Section 2.1). This is done with capacity in the Foundation Phase.
3. Using informal or non-standard units to measure (see question 1 of Section 2.1). This is done with capacity in the Foundation Phase.
4. Using formal or standard units to measure (see Section 2.1 from question 2 onwards, and Sections 2.2 to 2.4). This is the focus of this unit.

Millilitres and litres are the standard units of capacity/volume for Grade 4 learners. Learners sometimes find it difficult to read formal measuring instruments calibrated in standard units (see Section 2.4).

Resources

This topic involves a number of practical activities for which various resources are needed. These resources are listed in the various sections.

2.1 Measure in millilitres

Mathematical notes

Learners often see containers that hold 1 ℓ, so they may have a sense of how much 1 ℓ is. However, learners seldom see 1 ml, so they have a poor sense of how much 1 ml is. Learners see 750 ml, 500 ml, 330 ml and 200 ml containers much more frequently, but may not link the capacity of the containers with the products.

Resources

Measuring or medicine teaspoons, preferable one for each group of learners.

Clay or play dough. You can find recipes for play dough on the internet.

Water, sand, salt, sugar, flour, etc.

Mugs that can hold 250 ml. (You can also cut the top off small plastic bottles that can hold more than 250 ml to make "cups" that can hold 250 ml.)

Empty 1 ℓ bottles or boxes.

Teaching guidelines

It is best to do the activities in this section practically. Since learners will be working with water (or sand, salt, sugar, flour, etc.) you might want to do the activities outside rather than in the classroom.

In all the questions in this section, let learners estimate first, then do the activity, i.e. measure. In question 1 learners can also use sand, salt, flour or water.

You can ask learners to make small cubes of clay with sides 1 cm each. This is 1 ml of clay.

Answers

1. If learners do this practically, accept any number close to 50. Learners can calculate the answer afterwards: 250 ml ÷ 5 = 50 ml, since 5 × 50 ml = 250 ml.

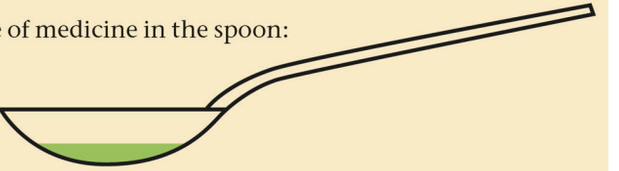
UNIT 2 CAPACITY AND VOLUME

2.1 Measure in millilitres

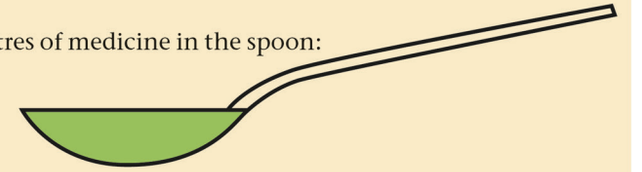
The photo shows about **1 millilitre** of milk.



1 millilitre of medicine in the spoon:



5 millilitres of medicine in the spoon:

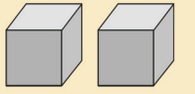


A full cup of tea is about 250 millilitres of tea.

1. Approximately how many teaspoonfuls of sugar are needed to fill one cup with sugar?

The **symbol for millilitre** is **ml**.

A small ice cube with side lengths of 1 cm is 1 ml of ice.



GRADE 4: MATHEMATICS [TERM 3] 225

Teaching guidelines

It is best to do questions 2, 3 and 4 practically. As stated earlier, since learners will be working with water (or sand, salt, sugar, flour, etc.) you might want to do these activities outside rather than in the classroom.

Remind learners always to estimate first. Then they do the activity and measure.

In questions 2 and 3 learners can first make a 5 ml lump of clay (using a teaspoon as a measure). Then they can make a clay pencil. Then ask them to break the pencil up into lumps of approximately 5 ml to assess how accurate their estimate was. In question 3 learners do not have to make a model of the whole book to check their answer. They can make a model of a fraction of the book, for example $\frac{1}{8}$. Then they can work out how much clay it would take to make a model of the whole book.

In question 4 learners can pour from a 1 ℓ bottle into a 200 ml cup to check their answer. They can also use a 250 ml mug and mark off approximately $\frac{4}{5}$ of it.

Possible misconceptions

A measuring cup and a large mug hold 250 ml. Most teacups hold less than this: some hold less than 200 ml.

Answers

2. About 20 ml to 50 ml
3. About 750 ml to 1 000 ml
4. 5 glasses (5 × 200 ml = 1 000 ml)
5. (a) Some learners may write 100 ml because they are thinking about the 200 ml glass in question 4. Some may write 125 ml because they are thinking about a 250 ml cup (see page 225). An approximate answer could be 100 ml.
   (b) Some learners may write 400 ml because they are thinking about the 200 ml glass in question 4. Some may write 500 ml because they are thinking about a 250 ml cup (page 225). Approximate answers could be 400 ml to 500 ml.

2. About how many millilitres of clay do you think you need to make a clay model of your pencil?
3. About how many millilitres of clay do you think you need to make a clay model of this book?

Milk is sold in containers of various sizes.

1 litre is 1 000 millilitres.





These pictures are much smaller than actual 1-litre containers.

4. How many glasses can you fill with 200 ml of milk from a full 1-litre container of milk?
5. (a) Approximately how many millilitres water do you drink if you drink half a cup of water?
   (b) Approximately how many millilitres water do you drink if you drink two full cups of water?



2.2 Volume and capacity

Mathematical notes

In this section learners continue to work with millilitres. They distinguish between volume and capacity.

The initial focus is on everyday objects that do not have gradation lines (also called graduation lines). Then learners read the volume of liquids in millilitres at numbered gradation lines. The capacity and shape of the measuring containers illustrated vary.

Resources

Empty 330 ml can.

Food colouring or ink.

Small glass.

Teaching guidelines

In questions 1 to 3 learners are shown a shorter, wider can that holds more than a taller, narrower glass. They are expected to read the capacity of the can, and to judge that the glass is half-filled, and then work out how much juice this is in millilitres.

You can demonstrate this situation. Bring a 330 ml can to class. Fill it with water coloured with food colouring or ink. Try to find a small glass. It does not have to be a 100 ml glass. Tell learners the capacity of the glass. Half fill the glass. Ask learners how much liquid is in the glass and how much is left in the can.

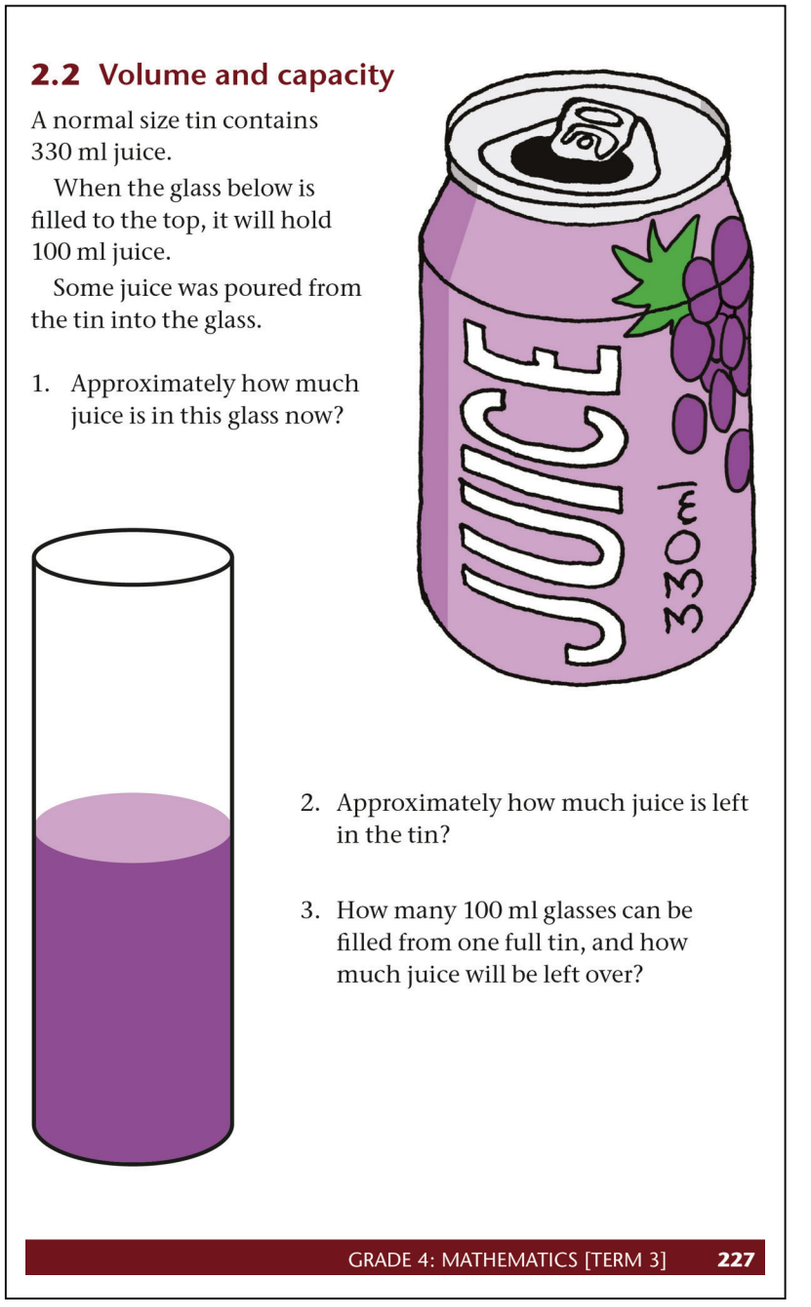
Ask learners how much juice the can in the picture can hold. Let them answer the rest of the questions on page 227.

Possible misconceptions

Young learners often think that taller containers hold more. They often do not take the width of the container into account. It is useful to let learners have repeated experience of short, wide containers that hold more than tall, thin containers. This is illustrated on pages 229 and 230.

Answers

1. About 50 ml (since it is about half of 100 ml).
2. About 280 ml (because 330 ml – 50 ml = 280 ml).
3. 3 glasses of juice, and 30 ml will be left in the tin.



2.2 Volume and capacity

A normal size tin contains 330 ml juice.

When the glass below is filled to the top, it will hold 100 ml juice.

Some juice was poured from the tin into the glass.

1. Approximately how much juice is in this glass now?

2. Approximately how much juice is left in the tin?

3. How many 100 ml glasses can be filled from one full tin, and how much juice will be left over?

GRADE 4: MATHEMATICS [TERM 3] 227

Mathematical notes

"**Capacity** is the amount of substance that an object can hold or the amount of space inside the object.

Volume is the amount of space that an object occupies.

So a bottle can have a 1 litre capacity, but it may not be filled to its full capacity. It could for example, only contain a volume of 250 ml."

DBE (2011). *Curriculum and Assessment Policy Statement Grades 4–6 Mathematics*. Pretoria. Government Printer, page 90

Teaching guidelines

Here you want to teach and stress the difference between volume and capacity.

Learners can do a practical activity. You might prefer them to work outside as they will be working with water. Give each group a syringe (without a needle) filled with coloured liquid and a small bowl with a layer of coloured liquid. It is cheaper to use syringes than measuring jugs. There must be more water in the bowl than in the syringe. Ask learners which has more water. Let them check by emptying the syringe and then using it to work out how much liquid is in the bowl.

Talk with learners about what is the same and what is different about the two measuring jugs shown on page 228. Both jugs are the same size: they have the same capacity. Both jugs have the same number of gradation lines. Jug A has each gradation line numbered. On Jug B every fifth gradation line is numbered (50, 100, 150, 200). The jugs hold different amounts of juice: they have different volumes of juice.

Point out that the container at the bottom of the page also has a capacity of 200 ml.

Notes on questions

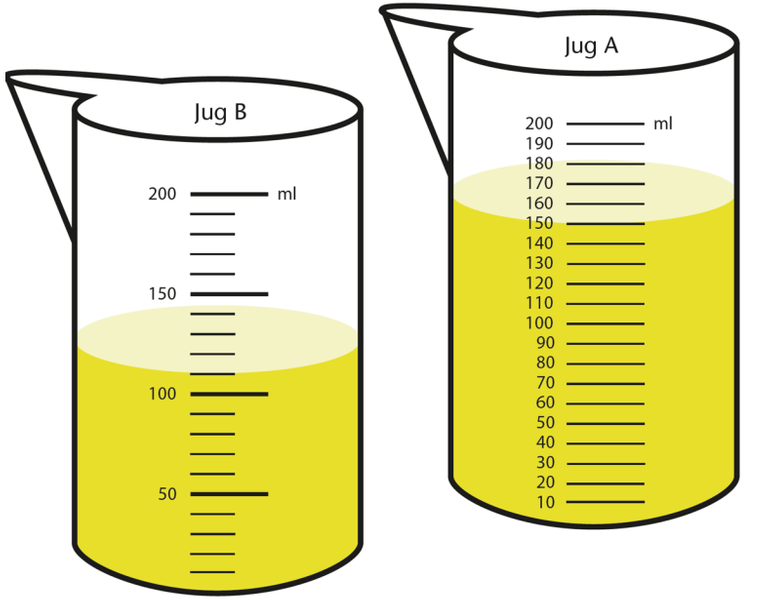
How can learners establish that the squat container at the bottom of the page has less juice than jug B? They should work out that it is half-filled. They can first estimate and then use their rulers to measure that it is 2 cm high and filled to 1 cm. So it is half-filled. Half of 200 ml is 100 ml. There is 110 ml of juice in Jug B.

Answers

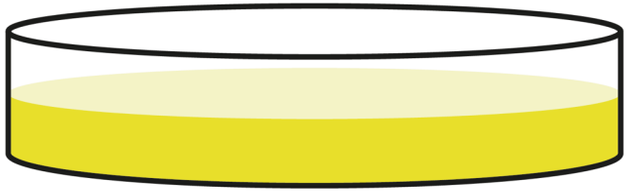
4. Less or the same as in Jug B

Each of these jugs can hold 200 ml of liquid (or sand or salt or sugar). We say the **capacity** of each jug is 200 ml.

There is 110 ml of juice in Jug B, and 150 ml of juice in Jug A. We say the **volume** of the juice in Jug B is 110 ml, and the volume of the juice in Jug A is 150 ml.



4. Do you think the juice in the 200 ml container below is less than or more than the juice in Jug B?



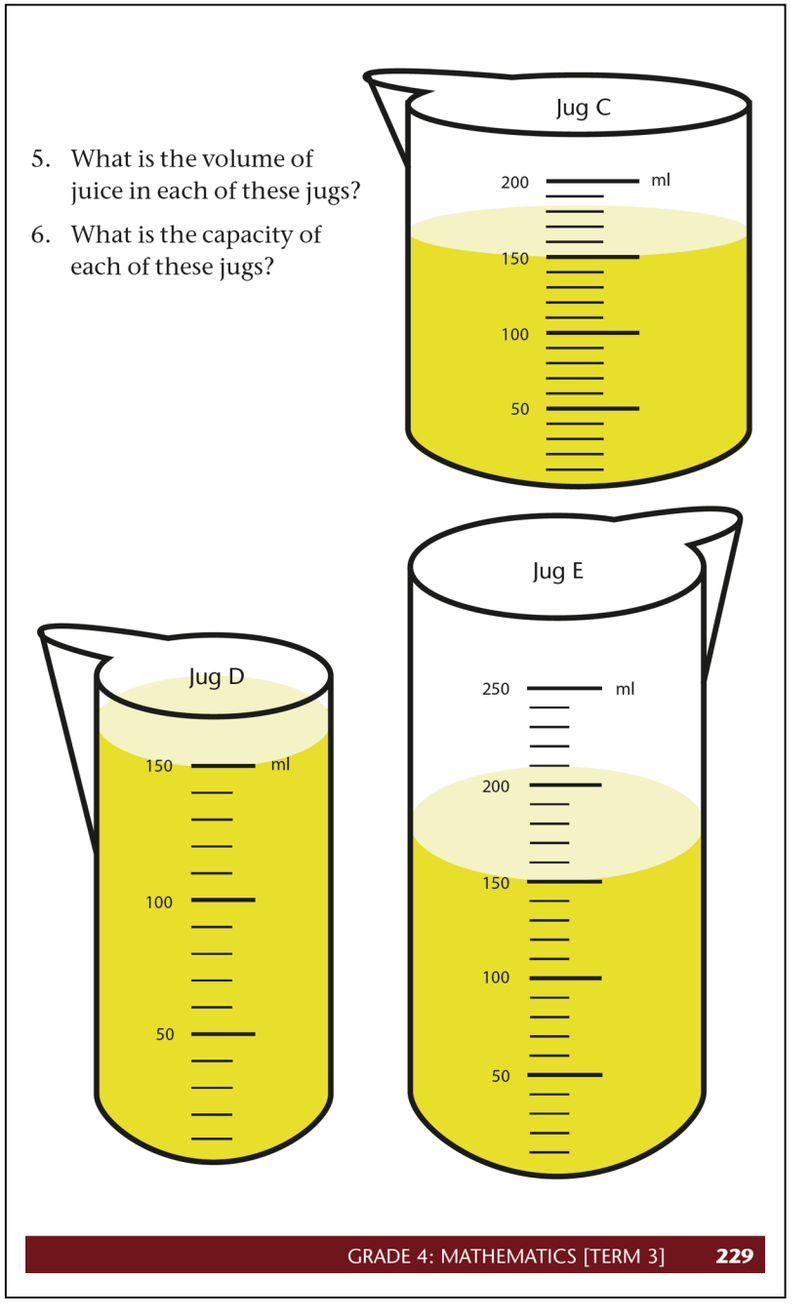
Teaching guidelines

Here the focus is on the difference between volume and capacity. All the jugs on page 229 have the same volume of juice, but they have different capacities. If they were filled to the top mark, they would hold different amounts of liquid.

Also point out to learners that although Jug D is taller than Jug C, it has a smaller capacity.

Answers

5. Jug C: 150 ml
   Jug D: 150 ml
   Jug E: 150 ml
6. Jug C: 200 ml
   Jug D: 150 ml
   Jug E: 250 ml



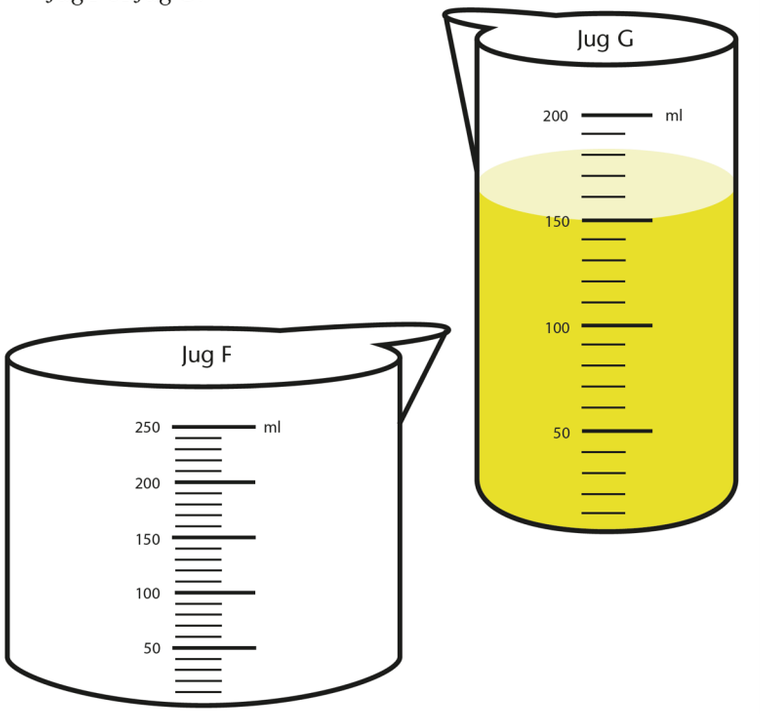
Teaching guidelines

Once learners have answered questions 7 and 8, you can again point out that in this case the shorter jug has the larger capacity.

Answers

7. 250 ml
8. 200 ml
9. (a) Yes
   (b) 100 ml
10. Jug F
11. 860 ml of juice (150 ml + 110 ml +150 ml +150 ml +150 ml + 150 ml)
12. Jugs A, C, D, E and G (they all contain 150 ml of juice)
13. Jugs E and F

7. What is the capacity of Jug F?
8. What is the capacity of Jug G?
9. (a) Can all the juice in Jug G be poured into Jug F?
   (b) How much more juice can be poured into Jug F?
10. Which jug is bigger, Jug F or Jug G?



11. How much juice is there altogether in Jugs A to G?
12. Which of the jugs contains the biggest volume of juice?
13. Which of the jugs have the biggest capacity?

2.3 Litre and millilitre

Mathematical notes

In this section learners work with the relationship between litres and millilitres. Note that while Foundation Phase learners ought to have worked with millilitres and have worked with litres, they were not expected to know the relationship between these units of measurement.

Milli means "one thousandth of": 1 litre contains 1 000 millilitres.

Many learners leave the Intermediate Phase without a good understanding of measurement. This is reflected in the Annual National Assessments (ANAs). Proposed interventions listed include:

> "Carry out practical demonstrations to show the relationship between a litre and a millilitre ... For instance the relationship between a litre and millilitres can be demonstrated by practically allowing learners to measure the same volume of water using two containers calibrated differently: one container calibrated in millilitres and the other calibrated in litres and then allow them to make conclusions."

DBE (2014). *Annual National Assessment. 2013 Diagnostic Report and 2014 Framework for Improvement.* Page 27 (see also page 33).

Teaching guidelines

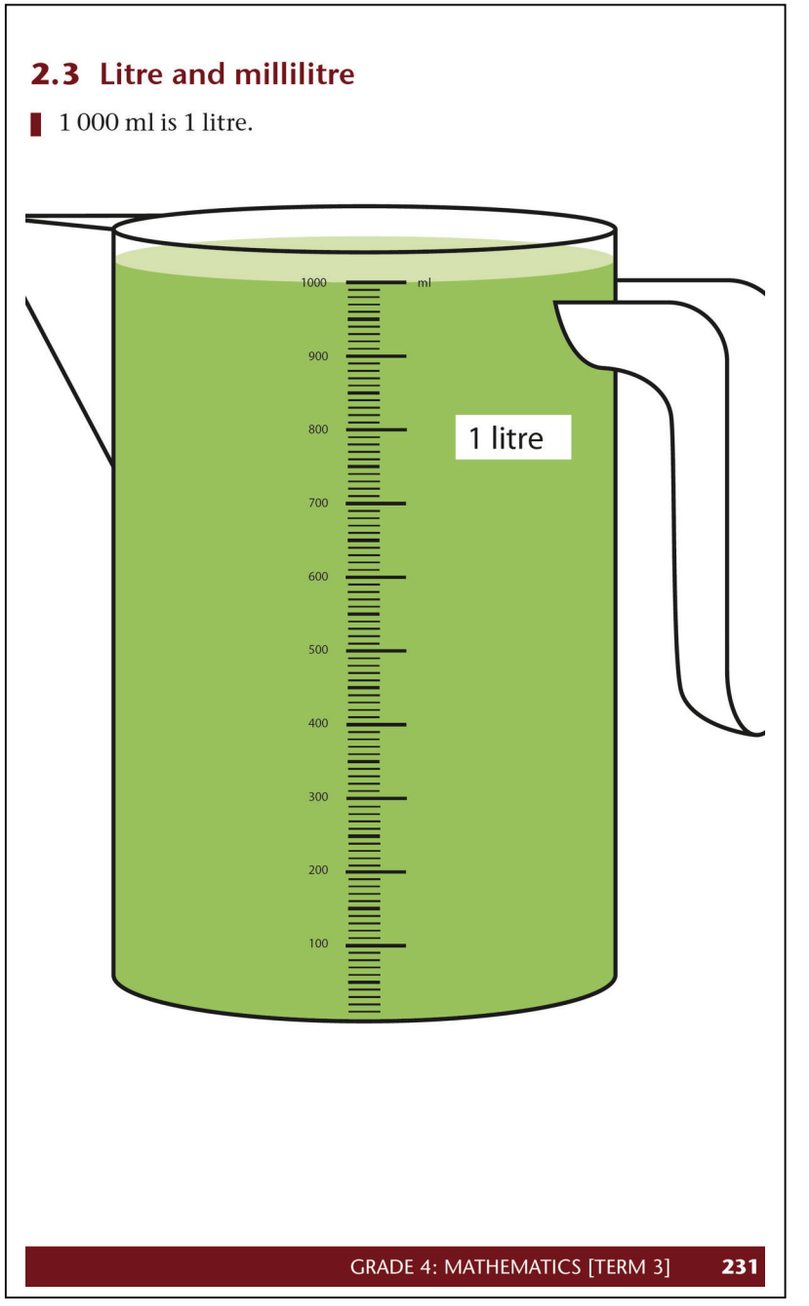
Page 231 states that 1 000 ml is 1 litre. Learners easily forget this when they are only told it.

You can let learners work in groups and fill a jug calibrated in litres to the 1 ℓ mark. Then let them pour this water into a jug calibrated in millilitres. If the second jug has a capacity of 1 000 ml they will see that 1 ℓ is 1 000 ml. However, if the jug holds less than 1 000 ml, learners will have to fill it and pour out the water repeatedly while making notes of how many times they fill it. The same effect can be achieved by learners repeatedly filling a 200 ml cup or a 500 ml bottle, and pouring this into a 1 ℓ bottle.

The picture on page 231 is a visual reminder of the relationship between litres and millilitres.

Possible misconceptions

In Grade 4 learners are only expected to work with litres and millilitres. These are the units of capacity/volume that they are most likely to encounter in their daily lives. Because learners are not exposed to other units they may wrongly assume that these are the only units of capacity/volume in the metric system.



In the unit on length in this Teacher Guide (Term 2 Unit 4 Section 4.4) we showed that there are a range of units between kilometres and millimetres. A range of units also exist between kilolitres and millilitres, namely hectolitres, decalitres, litres, decilitres and centilitres. Each unit of a higher power contains 10 of the adjacent unit of a lower power.

Teaching guidelines

Here learners work with parts and fractions of litres. This helps to consolidate the relationship between litres and millilitres.

You might want to suggest that learners work in pairs. One learner keeps page 231 of their Learner Book open while the other keeps the questions on page 232 open.

Answers

1. (a) 250 ml (a quarter is half of a half; half a litre is 500 ml)
   (b) 125 ml
2. 3 quarters of a litre (Learners can estimate this off the jug on page 231, or they can add 3 quarters of a litre, i.e. 250 ml + 250 ml + 250 ml.)
3. (a) 3 000 ml (b) 2 250 ml (c) 3 500 ml (d) 1 750 ml
4. 5 portions
5. (a) 500 ml (125 ml + 125 ml + 125 ml + 125 ml = 500 ml and 1 000 ml – 500 ml = 500 ml)
   (b) 4 portions
   (c) 125 ml (1 000 ml ÷ 8 = 125 ml)
   (Learners have seen in (a) that 4 × 125 ml = 500 ml, so 8 × 125 ml = 1 000 ml.
   So, 125 ml = $\frac{1}{8}$ of 1 000 ml, or = $\frac{1}{8}$ of 1 litre.)
   (d) 375 ml
   (e) 375 ml
   (f) 16 portions (16 eighths = 2 wholes)

500 ml is half of a litre.

1. (a) How many millilitres make up a quarter of a litre?
   (b) How many millilitres make up half of a quarter-litre?
2. What part of a litre is 750 ml?
3. How many millilitres are each of the following?
   (a) 3 litres
   (b) $2\frac{1}{4}$ litres
   (c) $3\frac{1}{2}$ litres
   (d) $1\frac{3}{4}$ litres

$3\frac{1}{2}$ litres can also be written as 3,5 litres.

4. How many portions of 200 ml milk can be poured from a full 1-litre container of milk?
5. Four portions of 125 ml each are poured from a full 1-litre container of milk.
   (a) How much milk is left in the container?
   (b) How many more portions of 125 ml each can be poured from the container?
   (c) How many millilitres is one eighth of a litre?
   (d) How many millilitres is three eighths of a litre?
   (e) How many millilitres is one eighth of three litres?
   (f) How many eighths-of-a-litre portions can be poured from a full 2-litre container of milk?



Teaching guidelines

Most bits of mathematics are connected. Learners sometimes do not connect bits of information, or bits from one question with another question. This slows down their work rate, and their success rate in Mathematics.

In the questions on page 233, encourage learners to use the answers they get to help them with subsequent questions.

Answers

6. 200 (because 1 000 ml ÷ 5 ml = 200)
7. Consider and discuss learners' estimates.
   Spoon A: 25 ml (given)
   Spoon B: about 15 ml
   Spoon C: about 10 ml
   Spoon D: about 5 ml
   Spoon E: about 2 ml
8. (a) 100 spoonfuls of 10 ml
   (Learners can use the information in question 6, and note that if there are 200 × 5 ml in 1 ℓ, then there are 100 × 10 ml in 1 1 ℓ, or they can say 1 000 ml ÷ 10 = 100 ml)
   (b) 20 spoonfuls of 10 ml (200 ml ÷ 10 ml = 20)
   (c) 25 spoonfuls of 10 ml (Using the information in (b): 20 + 5; or 250 ml ÷ 10 ml = 25)
   (d) 40 spoonfuls of 5 ml (Using the information in (b): if 20 spoonfuls of 10 ml make 200 ml, then 40 spoonfuls of 5 ml make 200 ml; or 200 ml ÷ 5 ml = 40)
   (e) 20 tablespoonfuls (300 ml ÷ 15 ml = 20)

6. If a teaspoon holds 5 ml, how many teaspoonfuls of honey will fill a 1-litre container?



Tablespoons have a capacity of about 15 ml. Cooks and nurses use spoons of several different sizes.



7. The biggest of these measuring spoons has a capacity of 25 ml. Estimate the capacity of each of the other spoons.
8. (a) How many 10 ml spoonfuls of honey will you need to fill a 1-litre container?
   (b) How many 10 ml spoonfuls of honey will you need to fill a container marked 200 ml?
   (c) How many 10 ml spoonfuls of honey will you need to fill a container marked 250 ml?
   (d) How many 5 ml spoonfuls of honey will you need to fill a container marked 200 ml?
   (e) How many tablespoonfuls of honey will you need to fill a container marked 300 ml?

Teaching guidelines

There are many questions here. Learners could do questions 9, 10(a), (b), (e) and (g), 11(a), (d), (e) and (h), 12(a), (c) and (e), and 13(a), (c), (e), (g) and (i) in class. The rest can be used for additional practice.

Note that in questions 13(g) to (j) and in question 14 learners will need to convert litres to millilitres, or millilitres to litres.

Answers

9. No

10. (a) 2 000 ml (b) 5 000 ml (c) 9 000 ml (d) 3 000 ml
    (e) 1 500 ml (f) 250 ml (g) 3 750 ml (h) 2 250 ml

11. (a) 3 ℓ (b) 8 ℓ (c) $2\frac{1}{2}$ ℓ (d) $\frac{1}{2}$ ℓ
    (e) $4\frac{1}{4}$ ℓ (f) $\frac{3}{4}$ ℓ (g) 6 ℓ (h) $5\frac{1}{4}$ ℓ

12. (a) 1 ℓ and 750 ml (b) 3 ℓ and 503 ml
    (c) 8 ℓ and 649 ml (d) 4 ℓ and 50 ml
    (e) 9 ℓ and 98 ml (f) 12 ℓ and 5 ml

13. (a) 1 ℓ and 397 ml (b) 2 ℓ and 40 ml
    (c) 1 ℓ and 210 ml (d) 0 ℓ and 247 ml
    (e) 1 ℓ and 221 ml (f) 0 ℓ and 970 ml
    (g) 1 ℓ and 442 ml (h) 1 ℓ and 609 ml
    (i) 1 ℓ and 531 ml (j) 3 ℓ and 376 ml

14. 3 ℓ and 750 ml

9. Would your answers for question 8 be different if the questions were about sugar instead of honey?

The **symbol for litre** is **ℓ**.

10. How many millilitres are each of the following?

    (a) 2 ℓ (b) 5 ℓ (c) 9 ℓ (d) 3 ℓ
    (e) 1,5 ℓ (f) $\frac{1}{4}$ ℓ (g) $3\frac{3}{4}$ ℓ (h) $2\frac{1}{4}$ ℓ

11. How many litres are each of the following? Give your answer as a fraction if necessary.

    (a) 3 000 ml (b) 8 000 ml (c) 2 500 ml (d) 500 ml
    (e) 4 250 ml (f) 750 ml (g) 6 000 ml (h) 5 250 ml

1 600 ml can be written as 1 ℓ and 600 ml.

12. Write in litres and millilitres, as shown in the example above.

    (a) 1 750 ml (b) 3 503 ml
    (c) 8 649 ml (d) 4 050 ml
    (e) 9 098 ml (f) 12 005 ml

13. Write the answers of the following as ℓ + ml.

    (a) 575 ml + 822 ml (b) 1 734 ml + 306 ml
    (c) 3 784 ml – 2 574 ml (d) 802 ml – 555 ml
    (e) 7 785 ml – 6 564 ml (f) 1 772 ml – 802 ml
    (g) 4 ℓ – 2 558 ml (h) 7 ℓ – 5 391 ml
    (i) 5 ℓ + 406 ml – 3 875 ml (j) 3 ℓ + 683 ml – 2 ℓ + 1 693 ml

14. Ansie bought three different bottles of vinegar. According to the labels, the bottles contain the following volumes of vinegar: 2,5 ℓ, 800 ml and 450 ml.

    How many litres plus millilitres of vinegar did Ansie buy in total?

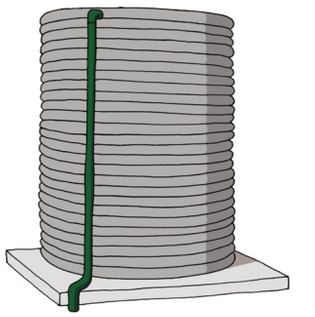
Teaching guidelines

There are many questions here. Learners could do questions 18 and 19 in class, and questions 15, 16 and 17 for additional practice.

Answers

15. 8 ℓ and 100 ml (2$\frac{1}{2}$ litres + 2$\frac{1}{2}$ litres + 750 ml + 750 ml + 750 ml + 850 ml)
16. 1 ℓ and 50 ml (150 ml × 7 = 1 050 ml)
17. 10 chocolate cakes (2 000 ml ÷ 200 ml = 10)
18. (a) 10 ℓ and 621 ml
    (b) 10 ℓ and 620 ml (550 ml + 6 250 ml + 3 820 ml = 10 620 ml)
    (c) 10 ℓ and 600 ml (500 ml + 6 300 ml + 3 800 ml = 10 600 ml)
    (d) 11 ℓ (1 litre + 6 litres + 4 litres = 11 litres)
    (e) The answer in (b).
    (f) Rounding off to the nearest 10 is closer to the exact calculation than rounding off to the nearest 100 or 1 000.
19. (a) 1 006 ℓ + 942 ml
    (b) 4 777 ℓ and 58 ml

15. On a Monday morning Katy sold milk in the farm stall. The buyers brought their own containers. She filled two containers with 2$\frac{1}{2}$ ℓ milk each, three containers with 750 ml each and one container with 850 ml milk. How many litres + millilitres milk did she sell?
16. Peter makes milk puddings. He needs 150 ml milk for each pudding. How many litres + millilitres milk does he need for 7 puddings?
17. Jacob has 2 ℓ milk. For one chocolate cake he needs 200 ml milk. How many chocolate cakes can he bake?
18. (a) Add these volumes: 545 ml + 6 253 ml + 3 823 ml. Write the answer as ℓ + ml.
    (b) Round the volumes in (a) off to the nearest 10 ml. Add the rounded numbers.
    (c) Round the volumes in (a) off to the nearest 100 ml. Add the rounded numbers.
    (d) Round the volumes in (a) off to the nearest litre. Add the rounded numbers.
    (e) Which one of the three additions with rounded numbers in (b), (c) and (d) is closest to your answer in (a)?
    (f) Discuss what you can learn from your answer in (e).
19. There is 5 784 ℓ water in the tank. Dadla uses 1 006 ℓ to water the vegetable garden and the fruit trees. He uses another 942 ml to water a pot plant.
    (a) How much water does he use?
    (b) How much water is left in the tank? Write your answer in litres + millilitres.



2.4 Measuring and reading capacity and volume

Mathematical notes

This section helps to prepare learners to read the volume of liquids at unnumbered intervals on graduated containers.

Comments on the Grade 6 Mathematics 2013 ANAs include:

> "Majority of learners could not determine the correct intervals on calibrations of a jug. From their responses it is evident that they were mainly exposed to the intervals of 10 and/or 20 and not 25."

Proposed interventions include:

> "Use a number line to enable learners to understand and determine intervals other than 10 and 20."

DBE (2014). *Annual National Assessment. 2013 Diagnostic Report and 2014 Framework for Improvement.* Pages 39 and 40.

Page 89 of the CAPS also encourages the use of number lines.

Possible misconceptions

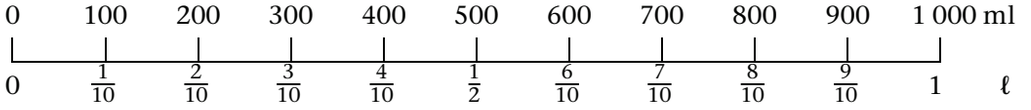
Because there are 10 unnumbered intervals between each numbered interval on a ruler (10 millimetres to each centimetre), learners sometimes think that there are 10 unnumbered intervals between *any* numbered interval on measuring jugs and scales.

Teaching guidelines

Learners are taught to count the unnumbered intervals and then work out the value of each of these intervals.

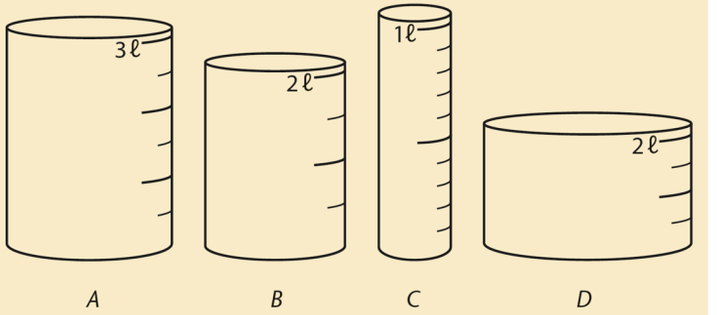
Answers

1. Example for Container C:



2.4 Measuring and reading capacity and volume

Measuring containers are marked in different ways.



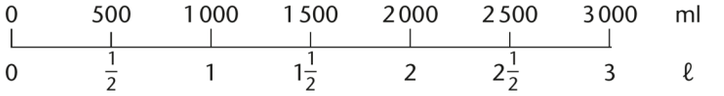
Container A is marked to show 6 equal parts. The top mark reads 3 ℓ. We can calculate what the other marks should read by dividing 3 ℓ into 6 equal parts.

3 ℓ = 3 000 ml and 3 000 ÷ 6 = 500

The lowest mark would therefore indicate 500 ml or $\frac{1}{2}$ ℓ.

1. Use a ruler to draw the right side of Container C horizontally in your book.

   Here is the line for Container A:



   Write what each mark on your line indicates, in litres and in millilitres, as in the above example.

Teaching guidelines

Remind learners of the method shown in the tinted passage on page 236: to find the value of each unnumbered interval you count the total number of intervals and then divide it into the total capacity. For example, in question 2(a):
1 000 ml ÷ 5 = 200 ml, so each interval is 200 ml. You can say 1 ℓ ÷ 5 = $\frac{1}{5}$ ℓ, so each interval is $\frac{1}{5}$ ℓ.

Question 2(f) is more complicated as the number line does not start at 0. Learners first need to subtract 400 from 2 000: 2 000 – 400 = 1 600. They then count the number of intervals, which is 8. Then they divide: 1 600 ml ÷ 8 = 200 ml.

Answers

2. (a) x = 200 ml = $\frac{1}{5}$ ℓ y = 400 ml = $\frac{2}{5}$ ℓ z = 600 ml = $\frac{3}{5}$ ℓ

   (b) x = 250 ml = $\frac{1}{4}$ ℓ y = 500 ml = $\frac{2}{4}$ ℓ z = 750 ml = $\frac{3}{4}$ ℓ

   (c) x = 600 ml = $\frac{6}{10}$ ℓ y = 1 200 ml = $\frac{12}{10}$ ℓ or $1\frac{2}{10}$ ℓ z = 1 600 ml = $\frac{16}{10}$ ℓ or $1\frac{6}{10}$ ℓ

   (d) x = 500 ml = $\frac{1}{2}$ ℓ y = 1 500 ml = $1\frac{1}{2}$ ℓ z = 4 500 ml = $4\frac{1}{2}$ ℓ

   (e) x = 250 ml = $\frac{1}{4}$ ℓ y = 750 ml = $\frac{3}{4}$ ℓ z = 1 500 ml = $1\frac{1}{2}$ ℓ

   (f) x = 800 ml = $\frac{8}{10}$ ℓ or $\frac{4}{5}$ ℓ

   y = 1 400 ml = $\frac{14}{10}$ ℓ or $1\frac{2}{5}$ ℓ

   z = 1 800 ml = $\frac{18}{10}$ ℓ or $1\frac{4}{5}$ ℓ

3. (a) 1 125 + 125 → 1 250 + 125 → 1 375 + 2 125 = 3 500 ml = $3\frac{1}{2}$ ℓ

   (b) 500 + 250 + 150 + 250 = 1 150 ml = 1 ℓ and 150 ml

4. (a) $\frac{1}{2} + \frac{1}{2} \rightarrow 1 + \frac{1}{2} \rightarrow 1\frac{1}{2} + \frac{1}{2} \rightarrow 2 + \frac{1}{2} = 2\frac{1}{2}$ ℓ = 2 500 ml

   (b) $\frac{1}{4} + \frac{1}{4} \rightarrow \frac{2}{4}$ or $\frac{1}{2} + \frac{1}{4} \rightarrow \frac{3}{4} + \frac{1}{4} + \frac{1}{4} = 1\frac{1}{4}$ ℓ = 1 250 ml

   (c) $1\frac{1}{2} + \frac{1}{4} \rightarrow 1\frac{3}{4} + \frac{1}{4} = 2$ ℓ = 2 000 ml

   (d) $\frac{3}{4} + \frac{3}{4} \rightarrow 1\frac{1}{2} + \frac{3}{4} \rightarrow 2\frac{1}{4} + \frac{3}{4} \rightarrow 3$ ℓ = 3 000 ml or

   $\frac{3}{4} + \frac{3}{4} \rightarrow \frac{6}{4} + \frac{3}{4} \rightarrow \frac{9}{4} + \frac{3}{4} \rightarrow \frac{12}{4}$ or 3 ℓ = 3 000 ml

2. Write the numbers, for litres and for millilitres, that should be at the marks at x, y and z on each line below.

   For example, you can write x = 200 ml = $\frac{1}{5}$ ℓ.

   (a) 0 — x — y — z — 1 000 ml

   (b) 0 — x — y — z — 1 000 ml

   (c) 0 — x — y — z — 2 000 ml

   (d) 0 — x — y — z — 5 000 ml

   (e) 0 — x — y — z — 2 000 ml

   (f) 400 ml — x — y — z — 2 000 ml

3. Count in millilitres.

   (a) 1 125 + 125 → ____ + 125 → ____ + 2 125 = ____ ml = ____ ℓ

   (b) 500 + 250 + 150 + 250 = ____ ml = ____ ℓ and ____ ml

4. Count in fractions of a litre.

   (a) $\frac{1}{2} + \frac{1}{2} \rightarrow$ ____ + $\frac{1}{2} \rightarrow$ ____ + $\frac{1}{2} \rightarrow$ ____ + $\frac{1}{2} \rightarrow$ ____ ℓ = ____ ml

   (b) $\frac{1}{4} + \frac{1}{4} \rightarrow$ ____ + $\frac{1}{4} \rightarrow$ ____ + $\frac{1}{4} + \frac{1}{4} =$ ____ ℓ = ____ ml

   (c) $1\frac{1}{2} + \frac{1}{4} \rightarrow$ ____ + $\frac{1}{4} =$ ____ ℓ = ____ ml

   (d) $\frac{3}{4} + \frac{3}{4} \rightarrow$ ____ + $\frac{3}{4} \rightarrow$ ____ + $\frac{3}{4} \rightarrow$ ____ ℓ = ____ ml

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Counting	Counting in groups of 10 into the hundreds and bridging to 1 000	238 to 239
3.2 Represent, order and compare numbers	Counting verbally and on number lines, and representing numbers in symbols and words	240 to 241
3.3 Even and odd numbers	Exploring even and odd numbers	241 to 242

CAPS time allocation	1 hour
CAPS page references	14 and 92

Mathematical background

Counting grouped objects, or in this case grouped objects in a picture, helps learners to develop a sense of numbers up to and beyond 1 000. Working with the structure of larger numbers, i.e. the way they are built up of thousands, hundreds, tens and units, also allows learners to develop a sense of larger numbers.

Once learners have a feel for larger numbers, it is easy for them to order and compare these numbers.

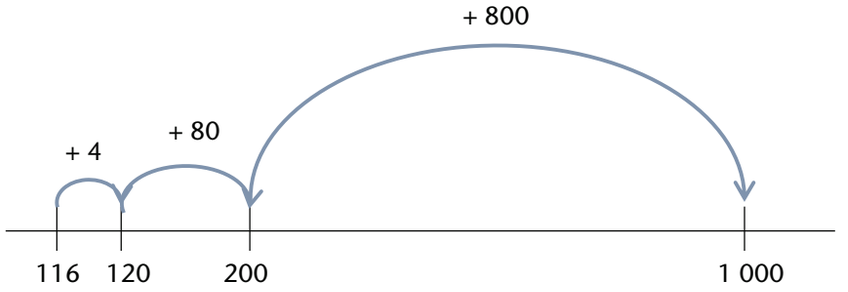
3.1 Counting

Teaching guidelines

Only 1 hour is allocated to this topic in Term 3. Try to cover this section in 15 minutes. There are only two questions. If learners count objects in ones, they will waste a lot of time, and also not develop a sense of bigger numbers.

To help learners to focus on counting in groups, you can ask them: *"How many apples are in each tray?"*, *"Do all the trays have the same number of apples?"* The aim is for learners to count in 10s to 110 and add the extra 6 apples (in question 1(a)). Learners can also count in 20s up to 100 and add the extra 10 and 6 apples.

Making up or bridging to the next 10, 100 and 1 000 develops learners' sense of numbers and operations. One way for learners to imagine getting from 116 apples to 1 000 apples in question 1(b) is to add on using an empty number line. This is illustrated below:



Answers

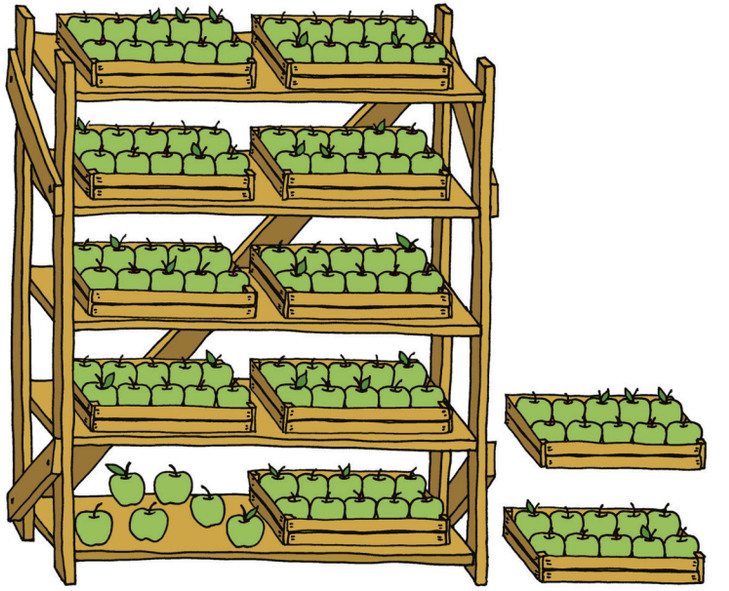
1. (a) 116 apples (b) 884 apples

UNIT 3 WHOLE NUMBERS

3.1 Counting

In this section you will count apples. The apples are packed in boxes.

1. (a) How many apples are shown on this page?

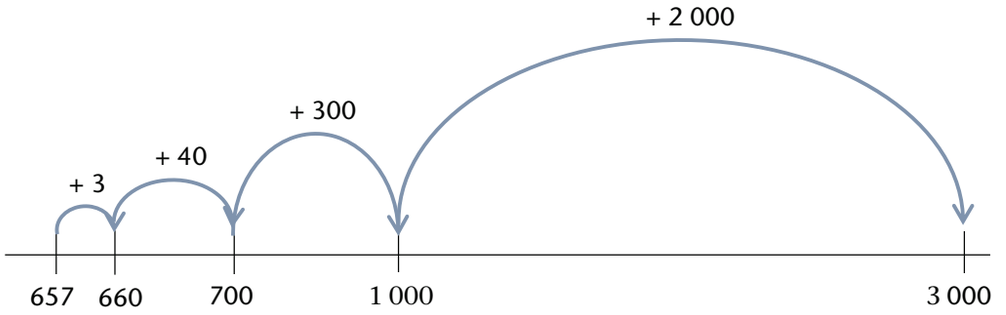


(b) How many apples should be added to this page to make 1 000 apples?

Teaching guidelines

In question 2(a) trays of 10 apples are shown. Check that learners understand that each tray has 10 apples. Ask them how many trays of apples are in each row. This will allow them to count rows in 50s until they reach 500. Learners can group the remaining trays into two more groups of 100 and 50 respectively (or they can count in groups of 10 and get 150) to get 650 apples on trays. They can then add the 7 loose apples. Learners might also count two rows at a time, i.e. count in 100s until they reach 500.

One way for learners to imagine getting from 657 apples to 3 000 apples is to add on using an empty number line and filling up or bridging to the next 10, 100 and 1 000. This can develop learners' sense of numbers and operations. This is illustrated below:



Answers

2. (a) 657 apples (b) 2 343 apples

2. (a) How many apples are shown on this page?



(b) How many apples should be added to this page to make 3 000 apples?

GRADE 4: MATHEMATICS [TERM 3] **239**

3.2 Represent, order and compare numbers

Teaching guidelines

Try to move quickly through Section 3.2: aim to cover this section in 20 minutes. One possibility is to use

- questions 1, 2 and 3 for mental mathematics,
- question 8 for concept development,
- questions 4(a), (c) and (f), 5, 6(c), (e) and (f), and 7(a), (b) and (d) for classwork, and
- questions 4(b), (d) and (e), 6(a), (b) and (d), and 7(c) for additional practice.

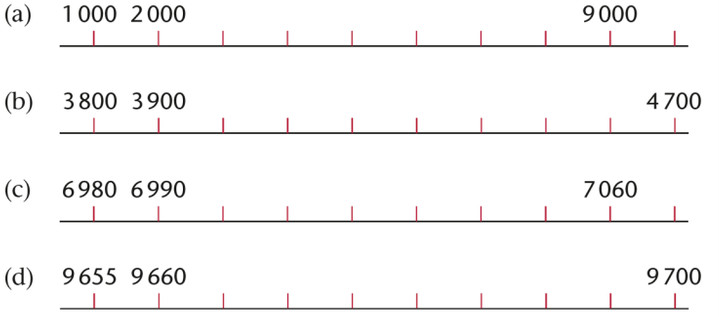
To save time for learners, you can copy the number lines provided in the Addendum (page 457). They can use these number lines for question 1.

Answers

1. (a) 3 000 4 000 5 000 6 000 7 000 8 000 9 000 10 000
   (b) 4 000 4 100 4 200 4 300 4 400 4 500 4 600
   (c) 7 000 7 010 7 020 7 030 7 040 7 050 7 060 7 070
   (d) 9 665 9 670 9 675 9 680 9 685 9 690 9 695
2. (a) 783 883 983 1 083 1 183 1 283 1 383 1 483 1 583
   (b) 1 875 1 900 1 925 1 950 1 975 2 000 2 025 2 050
   (c) 2 883 2 908 2 933 2 958 2 983 3 008 3 033 3 058 3 083
   (d) 2 983 2 986 2 989 2 992 2 995 2 998 3 001 3 004 3 007
   3 010 3 013
3. (a) 4 288 (b) 3 860 (c) 1 009 (d) 5 997
   (e) 3 981 (f) 956 (g) 4 200 (h) 2 468
4. (a) six thousand one hundred and fifty-four
   (b) nine thousand six hundred and fifty
   (c) eight thousand and thirty
   (d) one thousand three hundred and eleven
   (e) two thousand two hundred and twenty-two
   (f) nine thousand and nine

3.2 Represent, order and compare numbers

1. Which numbers are missing on the number lines below? Write them in the correct order in your book.



2. In each case write the numbers as you count.
   (a) Count in hundreds from 783 to 1 583.
   (b) Count in 25s from 1 875 to 2 050.
   (c) Count in 25s from 2 883 to 3 083.
   (d) Count in 3s from 2 983 to 3 013.

3. Write the numbers.
   (a) 1 more than 4 287
   (b) 2 less than 3 862
   (c) 10 more than 999
   (d) 3 less than 6 000
   (e) 2 000 more than 1 981
   (f) 500 less than 1 456
   (g) half of 8 400
   (h) double 1 234

4. Write the number names.
   (a) 6 154
   (b) 9 650
   (c) 8 030
   (d) 1 311
   (e) 2 222
   (f) 9 009

Teaching guidelines

When doing question 8, first alert learners to the fact that there are 10 small intervals between each large interval (just as there are on a ruler). Learners can first count in 10s to work out the numbers at the larger intervals. This will give them the answers to questions (a), (b), (f) and (i). Learners can then count the smaller intervals to work out the answers to the other questions: (c), (d), (e), (g), (h) and (j).

Possible misconceptions

Some learners may still write 9 000 9 instead of 9 009 for question 6(c), and 7 000 50 instead of 7 050 for question 6(f), and 5 000 20 instead of 5 020 for question 7(d).

Acknowledge that this is the way we say the numbers and that learners understand the parts of the numbers well, and that they can use this when writing the expanded notation. However, also use place value cards, for example placing the 9 over the last zero of the 9 000, and explain that this shows how we make and write the number symbol.

Answers

5. 9 650; 9 009; 8 030; 6 154; 2 222; 1 311

6. (a) 9 657 (b) 1 311 (c) 9 009 (d) 5 329 (e) 2 909 (f) 7 050

7. (a) 3 763 (b) 7 205 (c) 2 936 (d) 5 020

8. (a) 6 720 (b) 6 740 (c) 6 756 (d) 6 793 (e) 6 826
   (f) 6 850 (g) 6 875 (h) 6 901 (i) 6 910 (j) 6 924

3.3 Even and odd numbers

Answers

1. (a) 4 (b) 6 (c) 8 (d) 10
2. (a) 5 (b) 7 (c) 9 (d) 11

Teaching guidelines

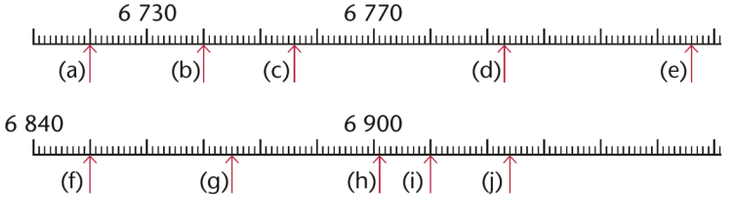
You could ask 12 learners to come to the front of the classroom and stand in two rows of 6 each. Ask each learner to stand opposite a learner in the other row without stepping out of line. Ask them to take the hand of the learner opposite them. Ask the class whether each learner has a partner. Add learners to one or both lines and repeat the question. For example, ask one more learner to join one line and ask again whether each learner has a partner. Explain that if each learner is paired off, then there is an even number of learners. If one learner is not paired off, he/she is the odd one out; the number is odd.

5. Write the numbers in question 4 from biggest to smallest.

6. Write the number symbols of the numbers that are given in expanded notation.
   (a) 9 000 + 600 + 50 + 7 (b) 1 000 + 300 + 10 + 1
   (c) 9 000 + 9 (d) 5 000 + 300 + 20 + 9
   (e) 2 000 + 900 + 9 (f) 7 000 + 50

7. Write the number symbols.
   (a) three thousand seven hundred and sixty-three
   (b) seven thousand two hundred and five
   (c) two thousand nine hundred and thirty-six
   (d) five thousand and twenty

8. Write the numbers indicated by the arrows.



3.3 Even and odd numbers

1. Calculate each of the following.
   (a) 2 + 2 (b) 2 + 2 + 2
   (c) 2 + 2 + 2 + 2 (d) 2 + 2 + 2 + 2 + 2

2. Add 1 to each of your answers in question 1.

All the numbers that can be formed by adding 2s together, like you did in question 1, are called **even numbers**.

When you add 1 to any even number, you get an **odd number**.

Follow this with the explanation provided in the tinted passage on page 241. Ask two more learners to join the other line, and repeat the question. Keep adding or taking away learners from one or both lines. Each time ask the class whether there is an even or odd number of learners.

Notes on questions

Question 8 is a challenge. Learners who complete questions 1 to 7 before the rest of the class can work on this. Ask learners to explore a number of possibilities, which they can share with the class. However, we cannot prove something is true just by giving many cases where it is true. Ask learners to give reasons why they think it is always true or not always true.

In preparation for working with learners on this question, you can watch a video clip (see web address below) from a Grade 3 Mathematics class in Michigan, showing 10 minutes of a longer discussion about even and odd numbers. A boy named Sean comments that he has noticed something special about the number six. He claims that it could be even and it could be odd. Sean explains his idea and the class goes on to discuss it, raising other perspectives, counter-arguments and questions. Watch until the end of the video to see a girl named Ofala produce a definition of what an odd number is. This definition resonates with the one provided in the tinted passage at the bottom of page 241. Note that this video will not interest most Grade 4 learners, but should interest you as a teacher.

http://hdl.handle.net/2027.42/65013 or

http://deepblue.lib.umich.edu/handle/2027.42/65013 Mathematics Teaching and Learning to Teach, University of Michigan. (2010). In Sean Numbers-Ofala [Online].

Answers

3. Learners count in twos and write all the numbers down:

   2 460 2 462 2 464 2 466 2 468 2 470 2 472 2 474

   2 476 2 478 2 480 2 482 2 484 2 486 2 488 2 490

4. Learners' own answers, e.g. 2 461; 2 459; 2 465; 2 483 (odd numbers)
5. 20 22 24 26 28 30 32 34 36 38
6. 19 21 23 25 27 29 31 33 35 37
7. (a) 7 231 7 233 7 235 7 237 7 239 7 241 7 243 7 245 7 247 7 249

   (b) 7 232 7 234 7 236 7 238 7 240 7 242 7 244 7 246 7 248
8. Yes

3. Count in twos from 2 460 up to 2 490 and write down the numbers while you count.
4. Write down five numbers between 2 460 and 2 490 that you left out (didn't use) when you counted in question 3.

The numbers you left out when you did question 3 are *odd numbers*.

5. The first nine even numbers are given below:

   2 4 6 8 10 12 14 16 18

   Write down the next ten even numbers.
6. The first nine odd numbers are given below:

   1 3 5 7 9 11 13 15 17

   Write down the next ten odd numbers.
7. (a) Write all the odd numbers between 7 230 and 7 250.

   (b) Write all the even numbers between 7 230 and 7 250.
8. Is it true that when you add any two odd numbers the answer is always an even number?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Addition and subtraction facts and skills	Making new addition and subtractions facts from given number facts	243 to 246
4.2 Practise addition and subtraction	Addition and subtraction using building up and breaking down, and transfer	247 to 248
4.3 Find some real information	Solving word problems	249

CAPS time allocation	4 hours
CAPS page references	14 to 15, 69 to 71 and 93

Mathematical background

Learners continue to engage with three methods of addition in Grade 4:

A. Adding by filling up multiples of 10 and 100.
B. Adding by breaking down one number (the smaller one) into place value parts and adding the parts one by one.
C. Adding by breaking down both numbers, rearranging and adding up the answers of the parts.

Learners also engage with two methods of subtraction in Grade 4:

A. Subtracting by filling up multiples of 10 and 100 (subtraction by addition).
B. Subtracting by breaking down both numbers, rearranging and adding up the answers of the parts.

Learners can only use these methods effectively if they know the addition and subtraction bonds for units and for multiples of ten and hundred well, or can quickly reconstruct these facts.

Learners should develop confidence to make new number facts from number facts they know or are given. They will do this when they reconstruct number facts, when they change subtraction number sentences to addition number sentences and vice versa (to check calculations or to make them easier) and when they use transfer (carrying or borrowing).

Learners should also continue to use estimation to check whether their answers are reasonable.

4.1 Addition and subtraction facts and skills

Teaching guidelines

Aim to cover this section in 2 hours. One possibility is to use

- questions 1 to 4, 10 and 12 for mental mathematics,
- the tinted passages on pages 243 to 245 for concept development,
- questions 5(a) and (b), 6, 7(a), (c), (e) and (g), 8(a), (c), (e) and (g), 9(a), (b), (e) and (f), and 13 for classwork, and
- questions 5(c) to (e), 7(b), (d), (f) and (h), 8(b), (d), (f) and (h), 9(c) and (d), and 11 for additional practice.

Use the tinted passages on page 243 to help learners understand how to break numbers down into multiples of 100.

Possible misconceptions

Learners may think that multiples of 10 are limited to 10, 20, 30, 40, 50, 60, 70, 80, 90, 100. But 11 × 10, 12 × 10, 15 × 10, 32 × 10, etc. are also multiples of 10.

Answers

Remember that the numbers can be added in any order.

1. 100 + 900; 200 + 800; 300 + 700; 500 + 500
2. Any five of the following:

100 + 2 400	600 + 1 900	1 000 + 1 500
200 + 2 300	700 + 1 800	1 100 + 1 400
300 + 2 200	800 + 1 700	1 200 + 1 300
400 + 2 100	900 + 1 600	1 300 + 1 200
500 + 2 000		

3. Any five of the following (there are many other possibilities):

3 000 = 100 + 900 + 2 000	3 000 = 400 + 700 + 1 900
3 000 = 200 + 800 + 2 000	3 000 = 500 + 600 + 1 900
3 000 = 300 + 700 + 2 000	3 000 = 300 + 900 + 1 800
3 000 = 400 + 600 + 2 000	3 000 = 400 + 800 + 1 800
3 000 = 500 + 500 + 2 000	3 000 = 500 + 700 + 1 800
3 000 = 200 + 900 + 1 900	3 000 = 600 + 600 + 1 800
3 000 = 300 + 800 + 1 900	3 000 = 600 + 700 + 1 700

UNIT 4

WHOLE NUMBERS: ADDITION AND SUBTRACTION

4.1 Addition and subtraction facts and skills

100 can be formed by adding 30 and 70:

100 = 30 + 70

100 can be formed in different ways by adding two multiples of 10, for example:

100 = 20 + 80 100 = 50 + 50

100 = 40 + 60 100 = 70 + 30

3 000 can be formed in different ways by adding two multiples of 100, for example:

3 000 = 1 400 + 1 600

3 000 = 2 000 + 1 000

We can also say:

3 000 can be **expressed** as the **sum** of two multiples of 100.

30 = 3 × 10
70 = 7 × 10
Numbers like 30 and 70 are called **multiples** of 10.

200 is also a multiple of 10 because 200 = 20 × 10.

1. 1 000 = 600 + 400. Write number sentences to describe four other ways in which 1 000 can be formed by adding two multiples of 100.
2. Write number sentences to describe five different ways in which 2 500 can be formed by adding two different multiples of 100.

2 500 can be formed by adding three different multiples of 100, for example:

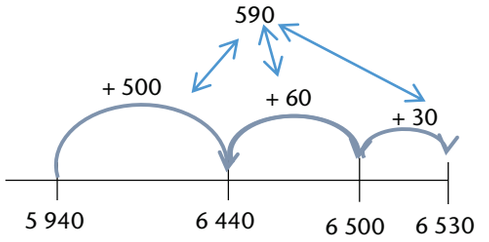
2 500 = 1 000 + 800 + 700 2 500 = 900 +1 100 + 500

2 500 = 600 + 1 500 + 400 2 500 = 100 + 200 + 2 200

3. Write number sentences to describe five different ways in which 3 000 can be formed by adding three different multiples of 100.

Possible sources of error

When learners break down the second number to add, especially when adding on to the next multiple of 1 000, 100 or 10, they may lose track of parts of the number. It can help them to keep an image of the whole, the parts they have added, and what remains to be added. For example, in question 7(f) they add 590. They can make an arrow diagram to show that first they add 500, then 60, then 30. You can combine this with using a number line.



Answers

4. There are many correct possibilities. Consider learners' number sentences.

5. (a) 870 + 130 → 1 000 + 2 600 → 3 600 + 400 → 4 000 + 3 234 = 7 234
   (b) 1 700 + 300 → 2 000 + 4 000 → 6 000 + 1 450 → 7 450 + 550 = 8 000
   (c) 920 + 280 → 1 200 + 1 800 → 3 000 + 1 700 → 4 700 + 200 = 4 900
   (d) 900 + 800 → 1 700 + 70 → 1 770 + 60 → 1 830 + 8 = 1 838 + 5 = 1 843
   (e) 900 + 70 → 970 + 8 → 978 + 800 → 1 778 + 60 → 1 838 + 5 = 1 843

6. Learners may set out this calculation in many different ways, for example:
   230 + 400 → 630 + 20 → 650 + 80 → 730 + 130 → 860 + 60 = 920 or
   230 + 420 + 80 + 130 + 60 = 200 + 400 + 100 + 30 + 20 + 80 + 30 + 60 = 700 + 30 + 100 + 30 + 60 = 800 + 120 = 920

7. (a) 670 + 280 → 670 + 200 → 870 + 30 → 900 + 50 = 950
   (b) 870 + 460 → 870 + 400 → 1 270 + 30 → 1 300 + 30 = 1 330
   (c) 740 + 690 → 740 + 600 → 1 340 + 60 → 1 400 + 30 = 1 430
   (d) 1 240 + 690 → 1 240 + 600 → 1 840 + 60 → 1 900 + 30 = 1 930
   (e) 8 460 + 330 → 8 460 + 300 → 8 760 + 30 = 8 790
   (f) 5 940 + 590 → 5 940 + 500 → 6 440 + 60 → 6 500 + 30 = 6 530
   (g) 6 660 + 840 → 6 660 + 800 → 7 460 + 40 = 7 500
   (h) 3 780 + 770 → 3 780 + 700 → 4 480 + 20 → 4 500 + 50 = 4 550

4. Write number sentences to describe five different ways in which each of the following can be formed by adding three different multiples of 100.
   (a) 2 300 (b) 3 500
   (c) 5 000 (d) 4 200

5. Copy and complete the following.
   (a) 870 + . . . → 1 000 + . . . → 3 600 + . . . → 4 000 + . . . = 7 234
   (b) . . . + 300 → 2 000 + 4 000 → . . . + . . . → 7 450 + . . . = 8 000
   (c) 920 + . . . → 1 200 + . . . → 3 000 + . . . → 4 700 + . . . = 4 900
   (d) 900 + 800 → . . . + 70 → . . . + 60 → . . . + 8 → . . . + 5 = ...
   (e) 900 + 70 → . . . + 8 → . . . + 800 → . . . + 60 → . . . + 5 = ...

6. Calculate 230 + 420 + 80 + 130 + 60.

To calculate 340 + 280 we can shift 200 from the 280 to the 340:

200
340 + 280 = 540 + 80

The 80 can now be added in steps, by filling up to 600 first:

540 + 60 → **600** + 20 = 620

So 340 + 280 = 620.

7. Transfer the hundreds part of the second number to the first number in each case, and then add on to find the answer.
   (a) 670 + 280 (b) 870 + 460
   (c) 740 + 690 (d) 1 240 + 690
   (e) 8 460 + 330 (f) 5 940 + 590
   (g) 6 660 + 840 (h) 3 780 + 770

Transfer is another word for shift.

Teaching guidelines

We can't know every number fact off by heart. People who are good at mathematics are good at making new number facts from given or known number facts.

Use the first tinted passage on page 245 to show learners how to form subtraction facts from addition facts. This is possible because addition and subtraction are inverse operations.

Use the second tinted passage on page 245 to show learners how they can make new number sentences from those that are given by adding or subtracting a multiple of thousand to or from one of the numbers, and to or from the answer, for example 800 + 700 = 1 500 means that 5 800 + 700 = 6 500. Learners can also add multiples of a hundred or ten to one number and to the answer, for example 800 + 700 = 1 500 means that 820 + 700 = 1 520. This extends work done in previous terms; see for example Term 1 Unit 3 Section 3.2, page 29.

Remember, learners have already worked with making new addition and subtraction number facts by changing all units to tens or hundreds or thousands, for example 3 *tens* + 5 *tens* = 8 *tens*, 3 *hundreds* + 5 *hundreds* = 8 *hundreds*, 3 *thousands* + 5 *thousands* = 8 *thousands;* see Term 1 Unit 3 Section 3.4, page 34.

Answers

8. (a) 950 – 670 = 280 950 – 280 = 670
   (b) 1 330 – 870 = 460 1 330 – 460 = 870
   (c) 1 430 – 740 = 690 1 430 – 690 = 740
   (d) 1 930 – 1 240 = 690 1 930 – 690 = 1 240
   (e) 8 790 – 8 460 = 330 8 790 – 330 = 8 460
   (f) 6 530 – 5 940 = 590 6 530 – 590 = 5 940
   (g) 7 500 – 6 660 = 840 7 500 – 840 = 6 660
   (h) 4 550 – 3 780 = 770 4 550 – 770 = 3 780

9. (a) 800 + 700 = 1 500 1 500 – 800 = 700 1 500 – 700 = 800
   (b) 1 800 + 700 = 2 500 2 500 – 1 800 = 700 2 500 – 700 = 1 800
   (c) 2 800 + 500 = 3 300 3 300 – 2 800 = 500 3 300 – 500 = 2 800
   (d) 4 600 + 900 = 5 500 5 500 – 4 600 = 900 5 500 – 900 = 4 600
   (e) 5 970 + 50 = 6 020 6 020 – 5 970 = 50 6 020 – 50 = 5 970
   (f) 5 700 + 500 = 6 200 6 200 – 5 700 = 500 6 200 – 500 = 5 700

10. (a) 4 900 (b) 800
    (c) 700 (d) 600

You can form two subtraction facts from any addition fact.
For example, if you know that 1 800 + 700 = 2 500, you also know that 2 500 – 1 800 = 700 and 2 500 – 700 = 1 800.

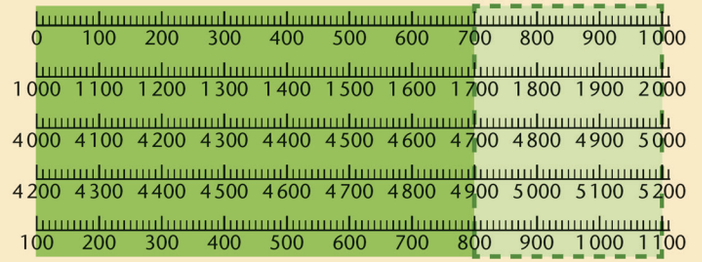
8. Form two subtraction facts from each of the addition facts you formed in question 7.

9. Complete each addition fact and form two subtraction facts from it.
   (a) 800 + 700 (b) 1 800 + 700
   (c) 2 800 + 500 (d) 4 600 + 900
   (e) 5 970 + 50 (f) 5 700 + 500

You can easily form new subtraction facts from a subtraction fact you already know.

For example if you know that **1 000 – 300 = 700**, it is easy to form the following facts and others too:

2 000 – 300 = 1 700 5 000 – 300 = 4 700
1 000 – 300 = 700 **1 000 – 300 = 700**
+ 1 000 + 1 000 + 4 000 + 4 000
2 000 – 300 = 1 700 5 000 – 300 = 4 700



10. How much is each of the following?
    (a) 5 200 – 300 (b) 1 100 – 300
    (c) 1 100 – 400 (d) 1 100 – 500

Mathematical notes

Learners will use their skill at making new number facts from given or known number facts when they use transfer (see e.g. page 248). It is even more useful when doing subtraction.

Notes on questions

Before doing question 12, remind learners that they should always think: *"What do I already know that can help me here?"* For example, the answer to 12(f) should help them with the answer to 12(g); the answer to 12(i) should help them to answer 12(j). Likewise the answer to 12(j) will help them find the answer to 12(k); 12(k) will help with 12(l); 12(l) with 12(m); 12(m) with 12(n), etc.

Answers

11. There are many, many more possible answers. Check all learners' calculations.

(a) 1 000 – 400 = 600
1 200 – 600 = 600
2 000 – 600 = 1 400
1 400 – 600 = 800
2 100 – 600 = 1 500

(b) 1 000 – 500 = 500
1 200 – 500 = 700
1 900 – 500 = 1 400
2 000 – 500 = 1 500
1 400 – 500 = 900

(c) 1 000 – 200 = 800
1 100 – 200 = 900
1 200 – 200 = 1 000
2 000 – 200 = 1 800
2 100 – 200 = 1 900

(d) 1 600 – 1 100 = 500
1 700 – 1 100 = 600
1 800 – 1 100 = 700
1 900 – 1 100 = 800
2 100 – 1 100 = 1 000

(e) 1 600 – 600 = 1 000
1 500 – 600 = 900
1 400 – 600 = 800
1 300 – 600 = 700
1 200 – 600 = 600

(f) 3 400 – 3 000 = 400
3 300 – 400 = 2 900
3 200 – 400 = 2 800
3 100 – 400 = 2 700
2 900 – 400 = 2 500

12. (a) 2 600 (b) 4 200 (c) 5 100 (d) 6 900
(e) 7 700 (f) 7 300 (g) 6 300 (h) 2 500
(i) 6 700 (j) 6 500 (k) 3 500 (l) 3 200
(m) 3 100 (n) 3 000 (o) 3 800 (p) 4 800
(q) 8 500 (r) 3 800 (s) 2 800 (t) 3 800

13. Learners complete the number sentences they have written down. See question 12 above for the answers.

11. Make five subtraction facts from each of the following facts.

(a) 1 000 – 600 = 400 (b) 1 000 – 500 = 500
(c) 1 000 – 800 = 200 (d) 1 600 – 600 = 1 000
(e) 1 600 – 1 000 = 600 (f) 3 400 – 400 = 3 000

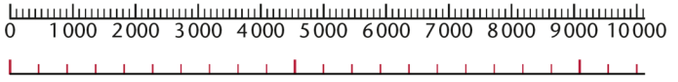
12. Write the answers for the questions that you can do quickly. Copy the number sentences for which you *cannot* give the answers quickly.

(a) 3 100 – 500 = ... (b) 4 500 – 300 = ...
(c) 5 500 – 400 = ... (d) 7 400 – 500 = ...
(e) 8 500 – 800 = ... (f) 8 100 – 800 = ...
(g) 8 100 – 1 800 = ... (h) 3 700 – 1 200 = ...
(i) 7 200 – 500 = ... (j) 7 200 – 700 = ...
(k) 7 200 – 3 700 = ... (l) 7 200 – 4 000 = ...
(m) 7 200 – 4 100 = ... (n) 7 200 – 4 200 = ...
(o) 4 400 – 600 = ... (p) 5 700 – 900 = ...
(q) 9 300 – 800 = ... (r) 5 700 – 1 900 = ...
(s) 3 600 – 800 = ... (t) 4 500 – 700 = ...

When you cannot quickly find the answer for a simple subtraction, you may think of another fact that may help you.

For example, when you cannot quickly find out how much 9 300 – 5 800 is, you may think of 9 300 – 6 000. If you know how much 9 300 – 6 000 is, you know that 9 300 – 5 800 is 200 more.

13. Work on the sentences that you wrote down in question 12. You can think of subtraction facts that you know. You can also think of steps on the number line.



4.2 Practise addition and subtraction

Teaching guidelines

Aim to cover this section in 1 hour. One possibility is to use

- questions 1(a) and (h), and the tinted passages on page 248 for concept development,
- questions 1(b) and (f), 2, 3, 4, 6 and 7 for classwork, and
- questions 1(c), (d), (e) and (g), and 5 for additional practice.

Learners should be able to check addition calculations by doing subtraction, and subtraction calculations by doing addition. They should be asked to do this regularly, so that it becomes a habit. Question 5 does this. Learners should also estimate their answers by rounding off to the specified place value parts before adding or subtracting. Estimated answers will help them to see whether their answers are reasonable.

Answers

1. Estimated answers: (a) 8 000 (b) 8 000 (c) 9 000 (d) 10 000
   (e) 2 000 (f) 3 000 (g) 0 (h) 3 000

   Learners may use other methods when calculating the answers, such as breaking down both numbers into place value parts (and using transfer): see pages 248, 197 and 194.

   (a) 3 467 + 5 231 → 3 467 + 5 000 → 8 467 + 200 → 8 667 + 30 → 8 697 + 1 = 8 698

   (b) 4 736 + 3 263 → 4 736 + 3 000 → 7 736 + 200 → 7 936 + 60 → 7 996 + 3 = 7 999

   (c) 4 891 + 4 119 → 4 891 + 4 000 → 8 891 + 100 → 8 991 + 10 → 9 001 + 9 = 9 010

   (d) 3 714 + 6 156 → 3 714 + 6 000 → 9 714 + 100 → 9 814 + 50 → 9 864 + 6 = 9 870

   (e) 9 653 – 7 643 → 9 653 – 7 000 → 2 653 – 600 → 2 053 – 40 → 2 013 – 3 = 2 010

   (f) 6 487 – 3 397 → 6 487 – 3 000 → 3 487 – 300 → 3 187 – 87 → 3 100 – 3 → 3 097 – 7 = 3 090

   (g) 8 345 – 7 558 → 8 345 – 7 000 →1 345 – 300 → 1 045 – 200 → 845 – 50 → 795 – 5 → 790 – 3 = 787

   (h) 5 352 – 1 963 → 5 352 – 1 000 → 4 352 – 300 → 4 052 – 52 → 4 000 – 600 → 3 400 – 11 = 3 389

2. Estimated answers: (a) 3 000 (b) 3 000
   Calculated answers: (a) 3 002 (b) 2 992
3. Yes. If not, learners should redo their calculations.
4. 60 written mistakenly as 80 (line 2 expansion).
   100 and 1 000 were not transferred to their correct places (line 4). (*See next page.*)

4.2 Practise addition and subtraction

1. First estimate the answers to the nearest thousand.
   Then calculate the answers.
   (a) 3 467 + 5 231 (b) 4 736 + 3 263
   (c) 4 891 + 4 119 (d) 3 714 + 6 156
   (e) 9 653 – 7 643 (f) 6 487 – 3 397
   (g) 8 345 – 7 558 (h) 5 352 – 1 963

2. First estimate the answers to the nearest thousand.
   Then calculate the answers.
   (a) 765 + 427 + 388 + 628 + 794
   (b) 626 + 792 + 425 + 763 + 386

3. Is your answer for question 2(a) ten more than your answer for question 2(b)?
   If the difference between the answers is not ten, you have made a mistake somewhere.
   In this case, go back to your work in question 2, find your mistakes, and correct your work.

4. Is the calculation below correct or are there mistakes?

   2 376 = 2 000 + 300 + 70 + 6
   5 669 = 5 000 + 600 + 80 + 9
   2 376 + 5 669 = 7 000 + 900 + 150 + 15
   = 7 035

   Describe any mistakes you find, and then do the calculation correctly.

5. Check your answers for questions 1(e), (f), (g) and (h) by doing addition. If some of your answers are wrong, go back to your work, find your mistakes and correct them.

Answers *(continued)*

4. 2 376 = 2 000 + 300 + 70 + 6
   5 669 = 5 000 + 600 + 60 + 9
   2 376 + 5 669 = 7 000 + 900 + 130 + 15 = 7 000 + 900 + 100 + 30 + 15 = 8 045
5. Responses depend on their initial calculations.

Teaching guidelines

Although there are only two questions, this page involves a substantial amount of work in the classroom. The first tinted passage consolidates learners' understanding of the breaking-down and building-up method of addition. The second tinted passage introduces a more effective way of recording the calculation. This recording format is also a step towards adding in columns (which learners will do in Grade 5).

Answers

6. 4 758 + 2 765
   4 758 = 4 000 + 700 + 50 + 8
   2 765 = 2 000 + 700 + 60 + 5
   4 000 + 2 000 = 6 000
   700 + 700 = 1 400
   50 + 60 = 110
   8 + 5 = 13
   = 6 000 + 1 400 + 110 + 13
   Transfer 10 to 110
   = 6 000 + 1 400 + 120 + 3
   Transfer 100 to 1 400
   = 6 000 + 1 500 + 20 + 3
   Transfer 1 000 to 6 000
   = 7 000 + 500 + 20 + 3
   = 7 523
7. 4 758 + 2 765 = (4 000 + 700 + 50 + 8) + (2 000 + 700 + 60 + 5)
   = (4 000 + 2 000) + (700 + 700) + (50 + 60) + (8 + 5)
   = 6 000 + 1 400 + 110 + 13
   = 6 000 + 1 400 + 120 + 3
   = 6 000 + 1 500 + 20 + 3
   = 7 000 + 500 + 20 + 3
   = 7 523

To calculate 5 726 + 2 685 you can break both numbers down, work with the parts of the same kind, and then build the answer up:

5 726 = 5 000 + 700 + 20 + 6
2 685 = 2 000 + 600 + 80 + 5

5 000 + 2 000 = 7 000
700 + 600 = 1 300
20 + 80 = 100
6 + 5 = 11

5 726 + 2 685
= 7 000 + 1 300 + 100 + 11
Transfer 10 to 100
= 7 000 + 1 300 + 110 + 1
Transfer 100 to 1 300
= 7 000 + 1 400 + 10 + 1
Transfer 1 000 to 7 000
= 8 000 + 400 + 10 + 1
= 8 411

6. Calculate 4 758 + 2 765 in the above way, and also write your work down in the way it is done above.

A different way to show the above thinking is to use brackets to indicate that certain calculations are thought of separately or done before others. The above thinking can be written like this:

5 726 + 2 685 = (5 000 + 700 + 20 + 6) + (2 000 + 600 + 80 + 5)
= (5 000 + 2 000) + (700 + 600) + (20 + 80) + (6 + 5)
= 7 000 + 1 300 + 100 + 11
= 7 000 + 1 300 + 110 + 1
= 7 000 + 1 400 + 10 + 1
= 8 000 + 400 + 10 + 1
= 8 411

7. Rewrite the work you did to calculate the answer to question 6 but this time, make use of brackets.

4.3 Find some real information

Teaching guidelines

Aim to cover this section in 1 hour. One possibility is to use

- questions 4 and 5 for concept development,
- questions 2, 3 and 6 for classwork, and
- questions 1, 7 and 8 for additional practice.

As suggested, you can use questions 4 and 5 for concept development. This does *not* mean writing out the calculations on the board as worked examples. Rather read the questions to learners and ask them to explain the question in their own words. Ask questions like: *"What does this question ask you to do?", "What useful information have you been given?", "What will you do now?"* Let learners do the actual calculations.

Possible misconceptions

Learners may not understand that when trees in a timber plantation are harvested, it means that they are cut down. In question 3 you can help learners to focus on the fact that they are asked: *"How many trees are still standing,"* which implies that some trees are not still standing, i.e. they have been cut down so that their wood can be used.

Notes on questions

In question 4 check that learners understand that kudu (like cattle) are cows (adult female), bulls (adult male) or calves (juvenile, i.e. not yet adult).

Encourage learners to always ask if there are any words that they do not know or understand.

Answers

1. 9 km and 920 m
2. R1 882
3. 5 058 trees
4. 898 kudu bulls
5. R9 810
6. (a) 6 541 people
   (b) 1 595 people
7. 1 171 learners
8. 2 175 students

4.3 Find some real information

1. While practising, an athlete ran 5 253 m on a Saturday and 4 667 m on a Sunday. How far did he run during the weekend? Give your answer in kilometres + metres.
2. Riana invested R7 755 for 10 years. Then she was paid out R9 637. How much money did she receive as interest? (In other words, how much more than R7 755 did she receive?)
3. In a plantation, 3 492 of the 8 550 trees have been harvested. How many trees are still standing?
4. At the last count, there were 3 104 kudus in a provincial nature reserve. The cows and calves made up 2 206. How many of the kudus were bulls?
5. Bobby withdrew R6 025 from his savings account, leaving him with R3 785 in the account. How much money did he have before the withdrawal?
6. On a Friday evening, 2 473 people attended a music festival and on the Saturday 4 068 people attended.
   (a) How many people went to the festival during that weekend?
   (b) How many more people were at the festival on Saturday than on Friday?
7. This year, 9 104 learners will be writing the ANA tests in the primary schools of a certain town. Last year, 7 933 learners wrote the tests. How many more learners will write the test this year than last year?
8. At Sun College, Zulu is the home language of 5 879 of the 8 054 students. How many students do not speak Zulu as their home language?

Grade 4 Term 3 Unit 5 Viewing objects

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 What you see from where you are	Objects look different when viewed from different positions	250 to 251
5.2 Looking from different positions	More about the way objects look when seen from different positions	252 to 254

CAPS time allocation	2 hours
CAPS page references	23 and 94

Mathematical background

This unit is about noticing how the same object can look very different when it is viewed from different positions. This awareness is important for developing spatial sense of three-dimensional objects. When working with three-dimensional objects, learners need to be able to imagine the whole object from a drawing that shows only part of the object. This unit starts to focus learners' attention on the fact that objects look different from different positions. This lays the foundation for being able to interpret the whole object from the features that are shown in a drawing or photograph that is taken from a particular position.

It is also important when one has to draw a three-dimensional object, especially if the object is not a simple one. One will then draw the object as seen from a number of different positions. Together the drawings become a useful tool to understand the total spatial form of the object. Such diagrams are routinely used in the technical fields (e.g. civil and mechanical engineering) during the design process.

Critical knowledge

It is critical that learners can imagine the whole object from drawings or photographs of particular views of the object.

It is also critical that learners can link, with reasons, particular views with the position of the viewer relative to the object.

Resources

Everyday objects such as teapots, mugs, teacups and milk jugs.

5.1 What you see from where you are

Mathematical notes

This section introduces the importance of being able to imagine what an object looks like from different positions. In this section it is only necessary to be able to decide which of a given set of drawings corresponds to a particular viewing position.

Resources

Everyday objects such as teapots, mugs, teacups and milk jugs. It is better to use objects that look different from the front, back and sides. It is difficult to distinguish a back view from a front and side view in objects such as drinking glasses, bottles, bricks, etc.

Teaching guidelines

Spend about 1 hour on this section.

You could begin by asking learners to draw an object from different positions. You could set the learners up in small groups around a table on which you place an object. Some may stand over the object and some may sit below it, while others sit around it. Ask each learner to draw the object as they see it. Once all learners have done their drawings, let them compare their drawings. Let learners shift positions to allow them to confirm the view seen by other members in the group.

An alternative or additional activity is to go into the school's parking lot and let learners stand in front, behind and on each side of a car. Different groups of learners can stand around different cars. Each learner draws the view they see. In the classroom you can collect all the drawings and hand them to different learners. Then ask all those with front views to stand in front of the classroom, showing the drawing they have. All those with drawings done from the right of the car (driver's side) can stand on the right-hand side of the classroom, etc. Choose particular drawings and ask learners to explain how they know that it is a front, back or side view. This helps learners to focus on how the features look different from certain angles. It also develops the language necessary to talk about it.

You can use the tinted passage on page 250 to confirm that what you see of an object or creature looks different from different positions.

Always ask learners to explain why they have linked a particular person with a particular view.

Answers

1. (a) Drawing B (b) Drawing A

UNIT 5 VIEWING OBJECTS

5.1 What you see from where you are

What you see of something depends on from where you look at it.

When a bird sits on the ground and you look down at it, you may see this:



A

When you look up at the bird when it sits high up in a tree, you may see this:



B

When the same bird sits on a low branch, so that you do not have to look up or down to see it, you may see this:



C

1. (a) Which of Drawings A, B and C show what you will see when the bird is higher from the ground than your head is?

   (b) Which of the drawings show what you will see when your head is higher from the ground than the bird is?

Possible misconceptions

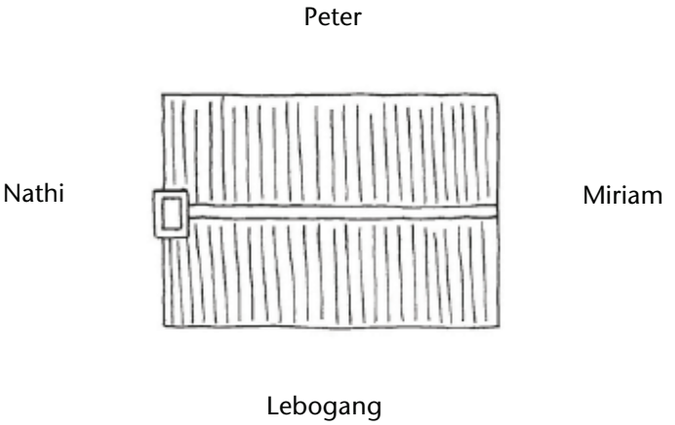
Some views are easier to distinguish than others. For example, in question 4 it is easy to see that Nathi stood at the end of the house where the whole chimney shows along the wall (a side view) and that Miriam stood at the other side (she can only see the chimney sticking out above the roof). It is much more difficult to work out who was standing in front of the house and who was standing behind it. To place Peter and Lebogang correctly, learners need to think about whose view has the chimney on the right-hand side and whose view has it on the left-hand side.

Notes on questions

If learners struggle to identify Lebogang's view and Peter's view correctly, ask them questions like: *"When Lebogang looks at the house, is the chimney on her left or right?"*, *"Where can you place her around the top view of the house so that the chimney will be on that side?"*

Answers

2. Mary and Jane are sitting on opposite sides of the table looking at the teacup.
3. Sibu was sitting on the floor on the same side of the table as Mary, looking up at the cup *or* Sibu is small enough to be standing with his head below the table. He is close to the table and on the same side of it as Mary.
4.



2. Mary and Jane sit at a table. There is a teacup on the table. They both made drawings of the teacup. Why do you think their drawings are so different?



Mary's drawing

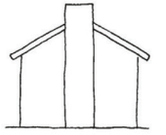


Jane's drawing

3. This is how Mary's little brother Sibu saw the teacup. Where do you think he was, when he looked at the teacup?



4. Nathi, Lebogang, Peter and Miriam all look at the same house. This is what each one sees:



Nathi



Lebogang

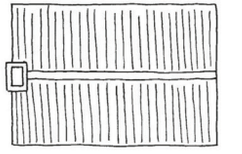


Peter



Miriam

Make a rough drawing to show how a bird will see the house from above. Write the names of the four friends on your drawing to show where they were when they looked at the house.



5.2 Looking from different positions

Mathematical notes

The ideas in the previous section are formalised here. There are certain views that are more useful than others. Top and bottom views are helpful, as well as side views showing each face, or side views that show the symmetry of the object. Sometimes we may have to portray an object from a less obvious position. In this case the properties of the object will not necessarily show up.

Teaching guidelines

You should spend about 1 hour on this section.

Again, time and resources permitting, allow learners to draw actual objects. However, now engage them about which positions are the most useful to show up the properties of the object (its faces, symmetry, etc.). Alternatively, engage them about how an object may appear from a particular position (a greater challenge).

When learners link a particular person with a particular view, always ask them to explain how they know that this person saw that view. For example, how do you know that Peter saw view (b) (the back view) of the teapot, or how do you know that the person in the tree saw the view of the bakkie shown in question 2.

Try to bring similar objects to class. If learners find it difficult to link pictures to the position of the viewer, you can use your cell phone to photograph objects from the suggested views. This will help learners to develop the skill of linking views with the position of the viewer.

Possible misconceptions

As mentioned in the previous section, some views are easier to distinguish than others. In questions 1 and 3 the side views are more difficult. For example, to place Nathi and Lebogang correctly in question 1, learners need to think about whose view of the teapot has the handle on the right-hand side and the spout on the left-hand side, and whose view has the handle on the left-hand side and the spout on the right-hand side.

Similarly, in question 3 learners need to think about who will see the side view of the bakkie as portrayed.

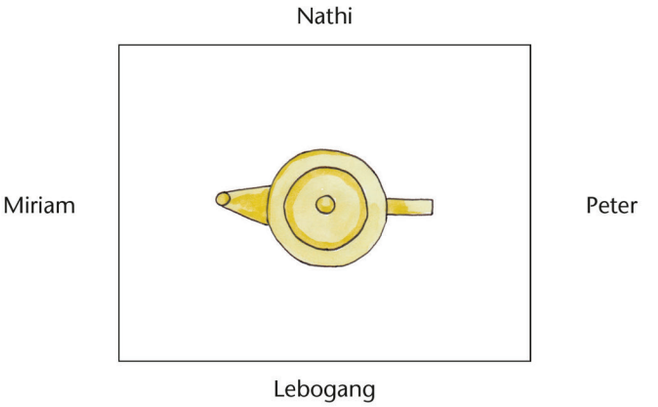
In question 4 learners will need to think carefully about whether they will see the cup handle on the left from the position of the yellow dot or the position of the green dot.

Answers

1. (a) Nathi (b) Peter
   (c) Miriam (d) Lebogang

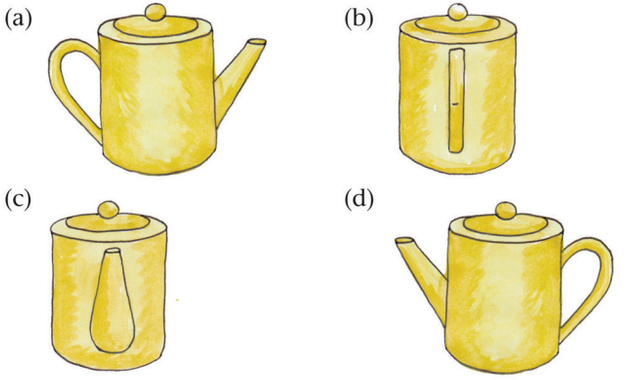
5.2 Looking from different positions

Nathi, Lebogang, Peter and Miriam sit around a table.
This is a **top view** or **plan** of the teapot and the table.



Each of the four friends makes a drawing of how they see the teapot from where they sit.

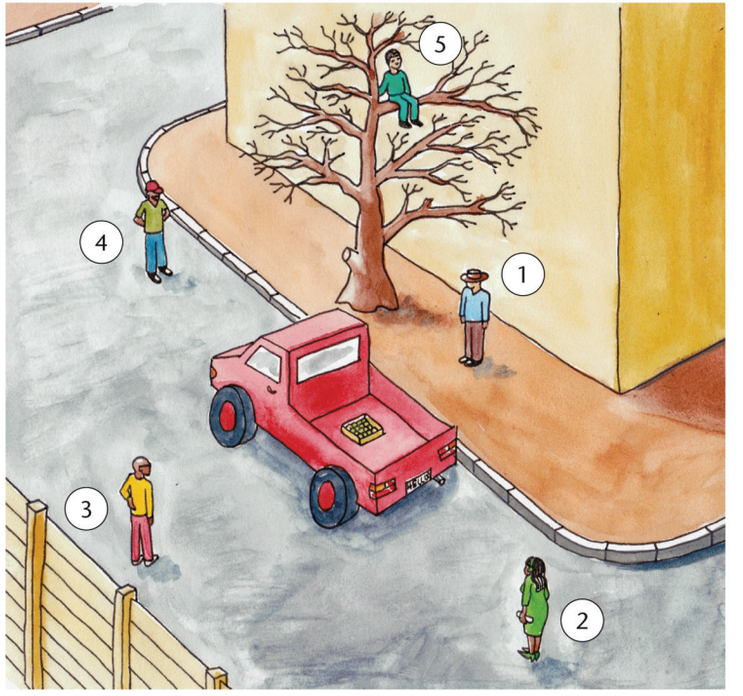
1. Who drew each picture?



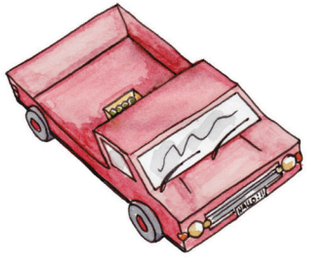
Answers

2. Person number 5

Five people look at a bakkie.



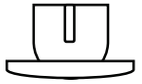
2. Which person sees the bakkie like this?



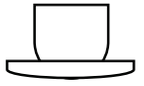
Answers

3. (a) Person number 1
   (b) Person number 4
   (c) Person number 2
   (d) Person number 3

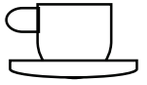
4. (a) From the red dot view



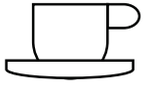
(b) From the blue dot view



(c) From the yellow dot view

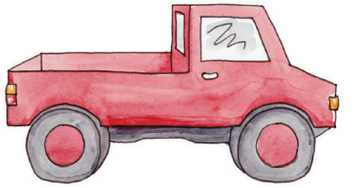


(d) From the green dot view

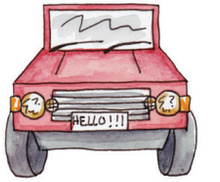


3. For each drawing, write down the number of the person who will see the bakkie as shown.

(a)



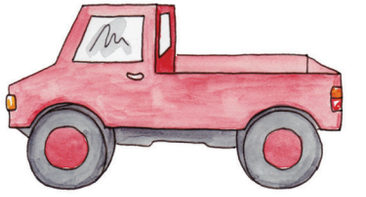
(b)



(c)



(d)



4. This is a top view of a cup and saucer.



(a) Draw the cup as you would see it from the side of the red dot.

(b) Draw the cup as you would see it from the side of the blue dot.

(c) Draw the cup as you would see it from the side of the yellow dot.

(d) Draw the cup as you would see it from the side of the green dot.

Grade 4 Term 3 Unit 6 Properties of two-dimensional shapes

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
6.1 Classify 2-D figures	Shapes with straight sides, curved sides and lines of symmetry	255 to 256
6.2 Draw 2-D figures	Practise drawing common 2-D shapes using grid paper	257

CAPS time allocation	4 hours
CAPS page references	21 to 22, 59 to 61 and 94

Mathematical background

The mathematical work here extends and consolidates the work done on two-dimensional shapes in Term 1 Unit 8, and the concept of symmetry covered in Term 2 Unit 8.

All two-dimensional figures (2-D figures) are made up of sides. These sides may be straight or curved lines. Many figures also have one or more corners. These are where two sides meet to form a pointed part. Although we are focusing on closed figures, we sometimes encounter figures that have gaps between their sides. These are called open figures.

The sides of 2-D figures may have specific lengths. It is easy to use grid paper to mark off lengths in units along the two grid directions. For this to be sensible, the grid has to consist of squares. This is called a square grid.

Resources

Learners will need 2 or 3 sheets of square grid paper. If you have a photocopier, you can copy the square grid paper provided in the Addendum (page 433).

6.1 Classify 2-D figures

Mathematical notes

This section focuses on the sides of 2-D figures: these can be curved or straight and may form corners where they meet.

Teaching guidelines

Spend about $1\frac{1}{2}$ hours on this section.

The skills being developed in this section are important. In question 1 learners practise the skills developed in Term 2 Unit 8 *Symmetry*. Allow time (if at all possible) to discuss symmetry in detail again. Some shapes may have symmetrical properties while others may be "almost symmetrical". In real life, objects are usually "almost symmetrical", but not perfectly symmetrical, for example the human body, a car seen directly from the front or from behind, etc. We often pretend that these objects are perfectly symmetrical even though they are not.

The other important skill is classifying or grouping figures according to their properties. In the previous units on two-dimensional figures, classification was always in the background. In this unit we are being more explicit about it. Classification is about properties that are the same and properties that differ. Engage your learners in discussions about properties that are shared by two or more shapes, and properties that are specific to certain shapes. Pose questions such as: "*What is the same about these figures?*", "*How do these figures differ?*", "*What shape does the figure have?*", "*Is this shape a special form of that shape?*", etc. Learners should be encouraged to justify their responses to such questions by referring to specific properties (shape, lengths of sides, symmetry, etc.).

When learners do question 2, let one learner have their book open on page 255 and the learner sitting next to him or her have it open on page 256. This way they will be able to see the questions and the figures they refer to at the same time.

Answers

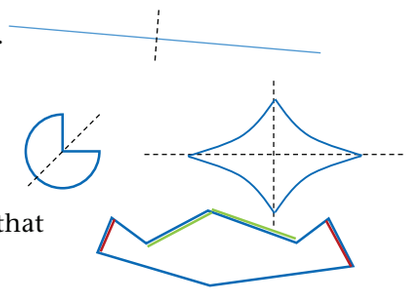
1. (a) Learners make their own sketches of the lines and figures on page 255.

   (b) Line A: symmetry line is a perpendicular straight line through the midpoint of the line.

   Line B: no line of symmetry

   Figure C: no line of symmetry

   Figures D and Figure E: see alongside.

   (c) Figure C. Learners can measure the pairs of sides marked in red or green alongside, to see that they are different lengths.

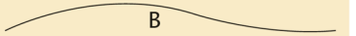
UNIT 6 PROPERTIES OF TWO-DIMENSIONAL SHAPES

6.1 Classify 2-D figures

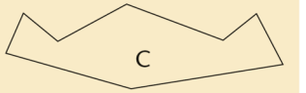
This is a piece of straight line:



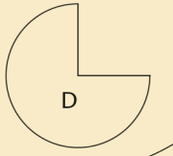
This is a piece of curved line:



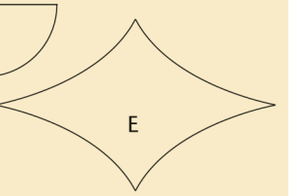
This figure has only straight sides:



This figure has two straight sides and one curved side:



This figure has curved sides only:



1. (a) Some of the above figures are precisely symmetrical. Quickly make a rough drawing of each of them. Do not use a ruler.

   (b) Draw the lines of symmetry on your drawings.

   (c) Which of the above figures is approximately symmetrical, but not quite?

2. State which of the figures on the next page

   (a) have curved sides only. (b) have straight sides only.

   (c) are triangles. (d) are hexagons.

   (e) are pentagons. (f) are quadrilaterals.

   (g) have straight and curved sides.

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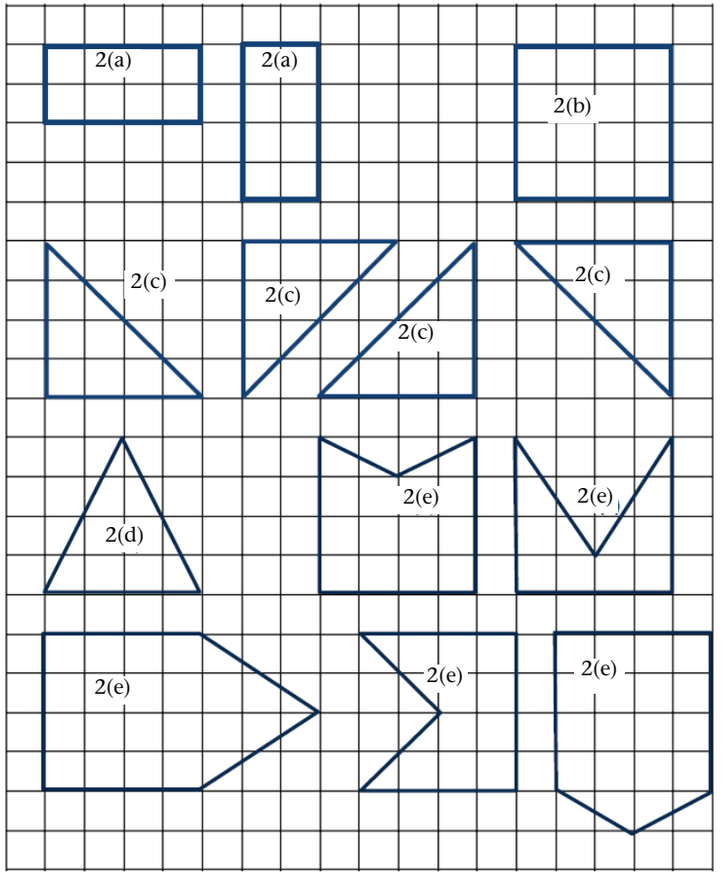
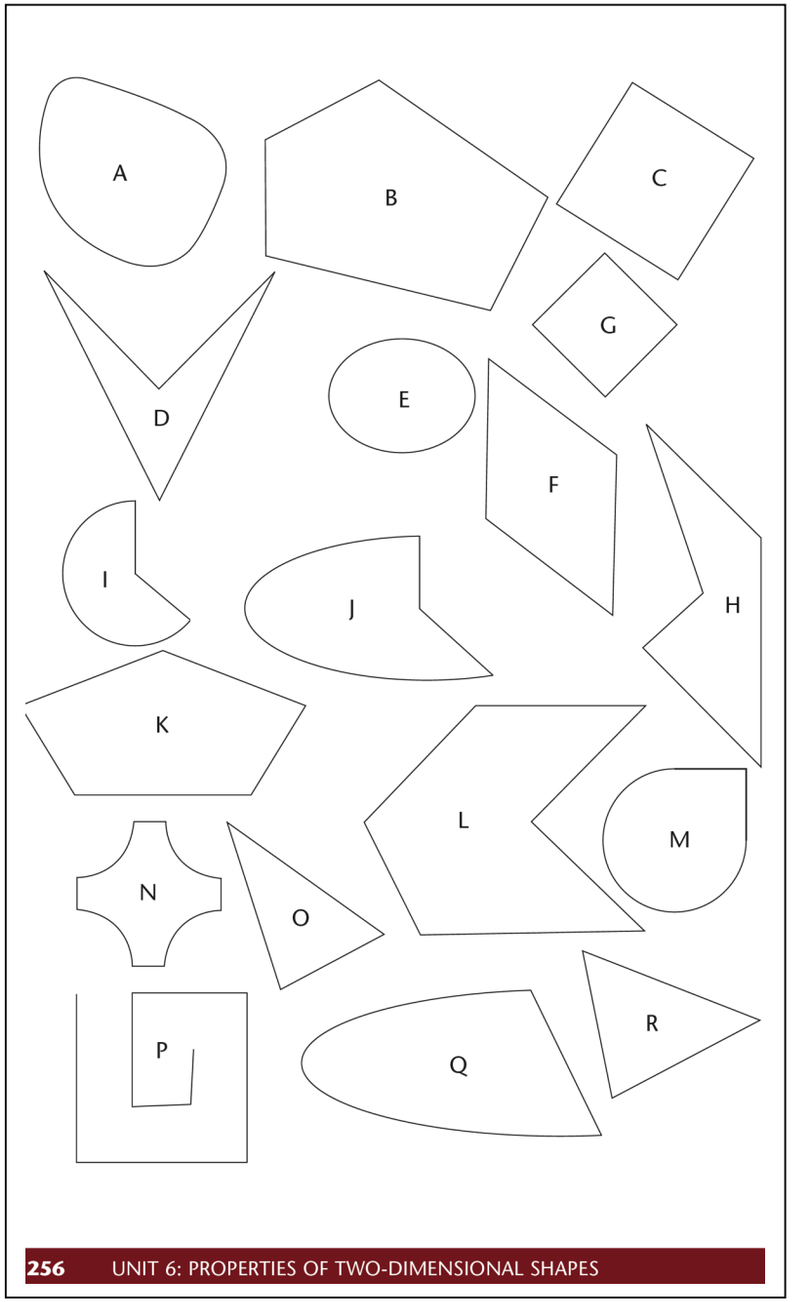
Answers (*continued*)

2. (a) Figures A, E
   (b) Figures B, C, D, F, G, H, K, L, O, P, R
   (c) Figures O, R
   (d) Figure L
   (e) Figures B, H, K
   (f) Figures C, D, F, G
   (g) Figures I, J, N, M, Q

Answers to Section 6.2: questions 2(a), (b), (c), (d) and (e)





6.2 Draw 2-D figures

Mathematical notes

In order to draw figures correctly, learners need to be familiar with the properties of the most common shapes. Remember that a square is a special rectangle: see "Mathematical notes" on page 104 and "Teaching guidelines" on page 106 of this Teacher Guide.

Resources

Learners will need 2 or 3 sheets of square grid paper. If you have a photocopier, you can copy the square grid paper provided in the Addendum (page 433).

Notes on questions

Spend about $2\frac{1}{2}$ hours on this section: the discussions in question 1 and the drawing of the figures will take long. The explanations in question 1 are important. Learners may produce a range of different but equivalent instructions. Discussing and comparing the different instructions for a particular shape will be a very rewarding activity. Once learners have discussed and agreed on the best instructions, they can look back to the descriptions of figures given in the summary bar on page 100. They can write down the best descriptions in their exercise books.

In question 2 it is essential that each learner has access to well-drawn square grids. Drawing the grids may prove very time-consuming, so it would be a good idea to have plenty of extra copies of grid paper available, if at all possible.

In questions 2(j) to (n) ask learners to draw the lines of symmetry. It might be challenging for learners to draw the figures with the required lines of symmetry, but they have already seen similar examples on page 192.

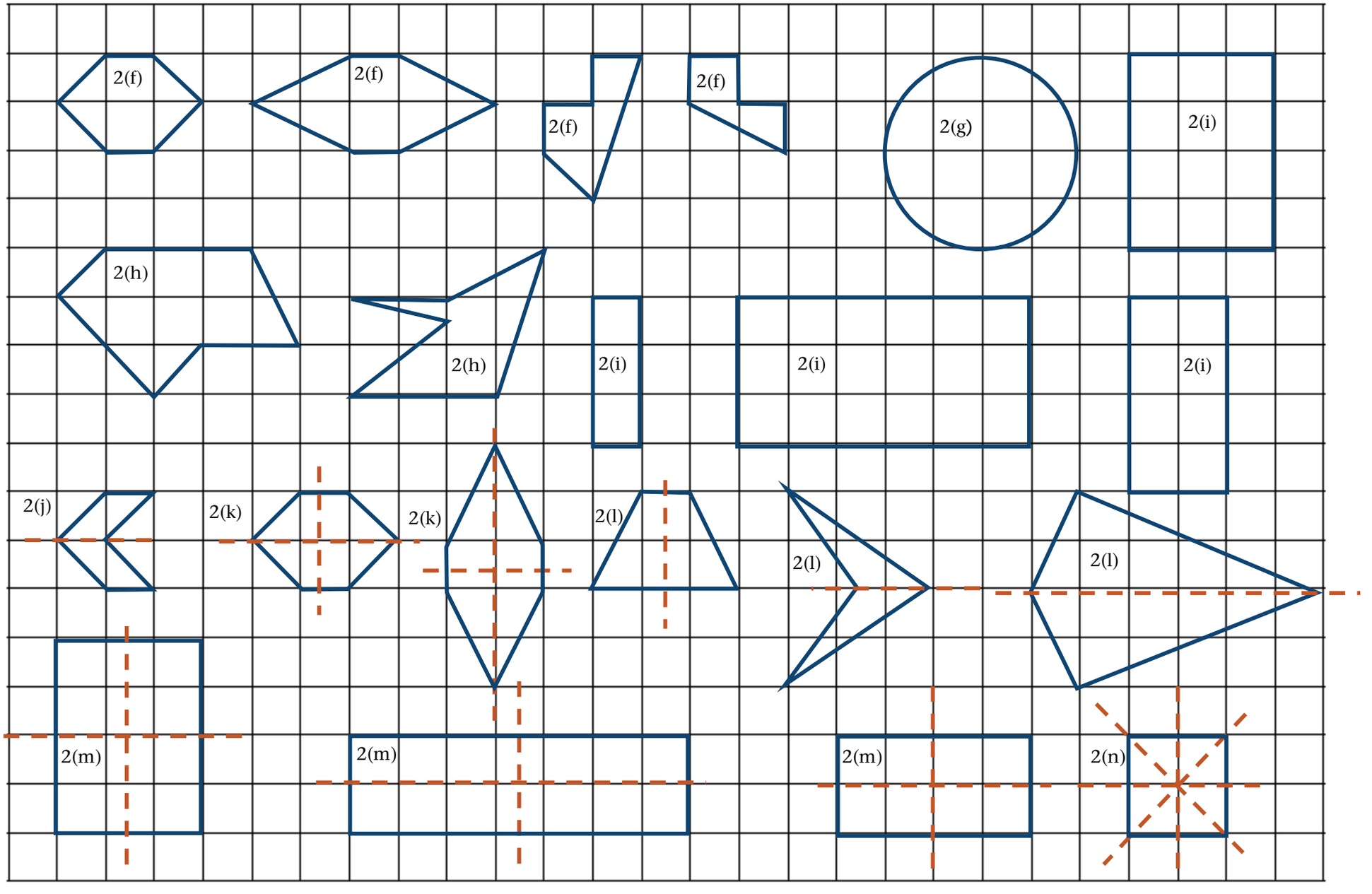
Encourage a group discussion about the different approaches followed by learners. It is very important to let them compare drawings and to check if they agree with the different ways a drawing has been done.

Answers

1. (a) Draw a closed shape with three straight sides.
   (b) Draw a closed shape with four straight sides and with all corners the same size.
   (c) Draw a closed shape with four straight sides and with opposite sides equal.
   (d) Draw a closed shape with five straight sides.
   (e) Draw a closed shape with four straight sides where the length of the sides and the size of the corners differ.
2. (a) to (n) Learners' own drawings. Some examples were provided on the previous page, and some are on the following page. Note that figures can be arranged in different orientations, i.e. face different directions (see 2(c)). There are many more possible answers for 2(e), (f), (h) to (l) and (m), and the squares in 2(n) could be different sizes.

6.2 Draw 2-D figures

1. Write an explanation for a Grade 3 learner of how to draw each of the following figures:
   (a) a triangle
   (b) a square
   (c) a rectangle that is not a square
   (d) a pentagon
   (e) a quadrilateral that is not a square or a rectangle
2. Draw the following figures on grid or dot paper:
   (a) a rectangle with two sides each 4 units long, and two sides each 2 units long
   (b) a rectangle with each side 4 units long
   (c) a triangle with two sides each 4 units long
   (d) a triangle with one side 4 units long; the top corner of the triangle is 4 units above this side
   (e) a pentagon with three sides each 4 units long
   (f) a hexagon with four of the sides equal
   (g) a circle that is 4 units wide at its widest (Hint: choose a dot or a place where the grid lines on your paper cross to be the centre of the circle)
   (h) a hexagon with all the sides a different length
   (i) a rectangle that is not a square
   (j) a hexagon with only one line of symmetry
   (k) a hexagon with two lines of symmetry
   (l) a quadrilateral with only one line of symmetry
   (m) a quadrilateral with only two lines of symmetry
   (n) a quadrilateral with four lines of symmetry


2(f)
2(f)
2(f)
2(f)
2(g)
2(i)
2(h)
2(h)
2(i)
2(i)
2(i)
2(j)
2(k)
2(k)
2(l)
2(l)
2(l)
2(m)
2(m)
2(m)
2(n)

Grade 4 Term 3 Unit 7 Data handling

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 Reading data in tables	Comparing numbers in different categories	258 to 259
7.2 Reading data in bar graphs	Estimating frequencies on bar graphs, using data to support what you are saying	259
7.3 Reading data in pie charts and in bar graphs	Comparing and interpreting trends and proportions, using data to support what you are saying	260 to 261

CAPS time allocation	7 hours
CAPS page references	30 to 31 and 95 to 96

Mathematical background

There is a difference between opinions and facts. The numbers we give are facts; the way we interpret the numbers is opinion. We must use the facts to support our opinions.

Most of the mathematics that learners have done requires them to give a single number as an answer. In statistics we try to find a single number to answer a question, but the number must be representative of all the numbers that we have gathered as data. In data handling we are looking at trends. Different representations help us to analyse these trends.

To reiterate what was stated in Term 1, data handling differs from mathematics in several ways:

- **The answer to data questions is in the information from lots of data gathered.**

  Data handling is necessary where measurements and frequencies vary, and therefore one measurement cannot provide accurate information about a situation. Lots of different data can be confusing, so we organise the data that we collect in different ways to get the "picture" of the situation.
- **The numbers we use in data handling always have some unit of measurement, or some description of a category they belong to.**

  In mathematics, learners mostly work with abstract numbers. In data handling the numbers must be interpreted in a context. The number 2 can be 2 cm or 2 rhinos, depending on the question.
- **Data questions are always answered with a story about the context.**

  Data handling starts when we need to answer a question about a situation where the property we are looking at varies. The numerical answers we get through data handling must be interpreted to answer the question about the situation.

7.1 Reading data in tables

Teaching guidelines

Prepare the table on the board or on a poster for use in class discussions.

Aim to spend about 2 hours on this section.

In questions 1(a) and (b) it is important to stress the word "about": learners are not asked *exact* numbers. Learners' estimates may differ and lead them to interpret the statements in questions 1(a) and (b) differently. Whether learners agree or disagree depends on how much difference they are willing to see as "almost the same".

Notes on questions

The data are realistic. You may visit www.wessa.org.za or other relevant websites to get the latest data. Adapt the answers to fit the new data.

Answers

1. (a) Agree – Reason: 15 + 52 + 17 + 57 + 4 + 3 + 38 + 1 = 187. In total 333 (146 + 187) rhinos were killed in 2010. Half of 333 is about 167. The 146 killed in the Kruger National Park is 21 less than 167. So it is fair to say that *about half* of all the rhinos killed in 2010 in South Africa were killed in the Kruger National Park.

   Some learners may differ as they may feel that the difference between 146 and 167 is too big to say "about half". What is important is the reason they give for their opinion.

   (b) Learners' opinions may differ, because some learners may feel that 52 and 57 are about the same, while others may feel that 52 and 57 are not about the same. What is important is the reasons learners give for their answers.
   Agree – Reason: Because the range of rhinos killed is from 0 to 146, 52 and 57 can be considered to be similar amounts, or almost the same, or about the same.
   Disagree – Reason: There were 52 rhinos killed in Limpopo and 57 rhinos killed in North West; the numbers clearly differ.

   (c) Opinions may differ. Reason: Since 2012 the Western Cape seems to be fighting rhino poaching successfully. Killings seem to be on the increase in the Northern Cape. Overall the killings in these two provinces are less than in the other provinces.

UNIT 7 DATA HANDLING

7.1 Reading data in tables

To understand data, we have to understand the situations that give the data. Hold a class discussion about the problem of rhino poaching. Ask your Social Sciences teacher to help you understand where the Kruger National Park is.

Data about rhino poaching

Rhino deaths	2010	2011	2012	2013	2014
Kruger National Park	146	252	425	606	827
Gauteng	15	3	1	8	5
Limpopo*	52	80	59	114	110
Mpumalanga*	17	31	28	92	83
North West	57	21	77	87	65
Eastern Cape	4	11	7	5	15
Free State	3	4	0	4	4
KwaZulu-Natal	38	34	66	85	99
Western Cape	0	6	2	0	1
Northern Cape	1	0	0	0	5

* The parts of Limpopo and Mpumalanga that are not in the Kruger National Park

[Source: www.wessa.org.za]

1. Study the table to say if you agree with the statements below. Explain why you say so.

   (a) About half of all the rhinos killed in 2010 in South Africa were killed in the Kruger National Park.

   (b) In 2010 about the same number of rhinos were killed in Limpopo and in North West.

   (c) Western Cape and Northern Cape do not seem to have a problem with rhino poaching.

Answers

2. (a) Number of rhinos killed per year. Add the numbers in each column.

2010	2011	2012	2013	2014
2010	2011	2012	2013	2014

(b) Number of rhinos killed per province between 2010 and 2014. Add the numbers in each row.

Kruger National Park	0
Gauteng	00
Limpopo	0
Mpumalanga	0
North West	0
Eastern Cape	0
Free State	0
KwaZulu-Natal	0
Western Cape	0
Northern Cape	0

(c) The problem is getting worse. More and more rhinos are killed every year. The biggest increase was between 2012 and 2013. In 2014 there were almost four times as many rhinos poached as in 2010.

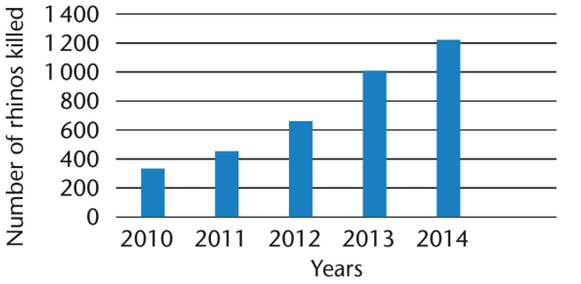
2. (a) If you have not yet done so, calculate how many rhinos were killed in total every year from 2010 to 2014.

(b) Calculate the number of rhinos killed in each of the provinces between 2010 and 2014.

(c) Do you think the problem of rhino poaching is getting worse or better? Explain why you say so.

7.2 Reading data in bar graphs

We draw graphs to help us see quickly how to compare data. Bar graphs are not as accurate as data in tables, so we have to estimate carefully when we read graphs.



1. (a) Give the graph a heading. Write the heading in your book. Your heading must tell what the graph is about.

(b) Estimate the number of rhinos killed each year.

(c) How many rhinos were killed altogether from 2010 to 2014?

(d) How many more rhinos were killed in 2014 than in 2013?

(e) If you look at the graph, how many rhino killings do you expect in 2015? Explain why you say so.

2. Write a short paragraph to tell the story about rhino poaching in South Africa between 2010 and 2014.

7.2 Reading data in bar graphs

Mathematical notes

Bar graphs are not as accurate as the data in the tables. Use the scale to estimate as accurately as possible.

Teaching guidelines

Aim to spend about 1 hour on this section. Prepare the graphs on the board or on posters for use in class discussions. Demonstrate how to compare lengths of bars by fitting shorter bars over longer bars to estimate proportions.

Notes on questions

Learners should use the data from Section 7.1, question 2 and Section 7.2, question 1 to write their paragraphs. They should look for trends in the data. For example, they should look to see whether the data is increasing or decreasing and if there were any dramatic increases or decreases over any period. They can also state what the maximum and/or minimum data points are.

Possible misconceptions

Learners may think bars that end between scale markings are read in fractions, e.g. 2010: 200 and a half. Help them to understand the scale.

Answers

1. (a) Number of rhinos killed in South Africa between 2010 and 2014

   (b) 2010: about 350
   2011: about 450
   2012: about 650
   2013: about 1 000
   2014: about 1200

   (c) Using the estimates above: about 3 650.

   (d) Using the estimates above: about 200 more.

   (e) We can expect even more killings in 2015 than in 2014. If you compare the 2013 and 2014 numbers, you could estimate 1 400. If you look back to 2012 you could estimate this to be more, perhaps in the region of 1 500.

2. The number of rhinos poached from 2010 to 2014 increased each year. The biggest increase, 350 more killings, occurred between 2012 and 2013. In 2014, the number of rhinos poached was almost four times the number of rhinos poached in 2010.

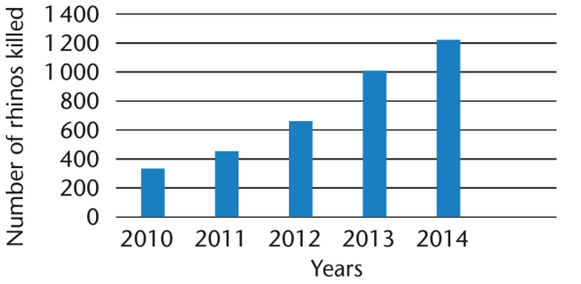
2. (a) If you have not yet done so, calculate how many rhinos were killed in total every year from 2010 to 2014.

   (b) Calculate the number of rhinos killed in each of the provinces between 2010 and 2014.

   (c) Do you think the problem of rhino poaching is getting worse or better? Explain why you say so.

7.2 Reading data in bar graphs

We draw graphs to help us see quickly how to compare data. Bar graphs are not as accurate as data in tables, so we have to estimate carefully when we read graphs.



1. (a) Give the graph a heading. Write the heading in your book. Your heading must tell what the graph is about.

   (b) Estimate the number of rhinos killed each year.

   (c) How many rhinos were killed altogether from 2010 to 2014?

   (d) How many more rhinos were killed in 2014 than in 2013?

   (e) If you look at the graph, how many rhino killings do you expect in 2015? Explain why you say so.

2. Write a short paragraph to tell the story about rhino poaching in South Africa between 2010 and 2014.

7.3 Reading data in pie charts and in bar graphs

Mathematical notes

In pie charts we estimate the bigger fractions as best we can, while we report the very small fractions just as "very small". The comparisons we can make at a glance is the important information. The information about exact numbers is not represented in the pie chart.

Teaching guidelines

Aim to spend about 2 hours on this section.

Prepare the charts/graphs on a poster for use in the class discussion.

Use cut-outs that you can fit onto each other to demonstrate how to compare circle sectors.

Allow learners to use their own words to write answers and to discuss their opinions.

Critical knowledge

Pie charts help us to make comparisons about categories of data. The size of the sector represents the relative proportion of the data. Pie charts do not show the exact numbers. Learners should use their knowledge of fractions to estimate the relative size of the sectors (in later grades learners will use percentages to express the relative size of the sectors).

Notes on questions

In question 1(d) learners are asked to state provinces in which few rhinos were killed. They will look for the narrow sectors. They should also look to see whether any province appears to be missing. For example, the Western Cape does not appear, because the proportion of rhinos killed there was so small.

Answers

1. (a) Most rhinos killed in SA in 2014 were killed in the Kruger National Park.
   (b) Trace and cut out the blue sector and fit it into the yellow sector. Or draw lines. You should notice that the blue sector is about one third of the circle.
   (c) North West, Limpopo, KwaZulu-Natal, Mpumalanga
   (d) Free State, Western Cape, Northern Cape, Gauteng
   (e) It is the largest nature reserve in South Africa and has many animals, including rhinos. If there are more rhinos in the Kruger National Park than in other reserves, we can expect more killings there.
   (f) About half of all rhinos killed in 2014 were killed in Mpumalanga and Limpopo.
   (g) KwaZulu-Natal and Limpopo

7.3 Reading data in pie charts and in bar graphs

1. The pie charts show where rhinos were killed in 2014.

   (a) What does the top pie chart say about rhino killings in South Africa?

   (b) How can you estimate the fraction of the pie chart which is shaded blue?

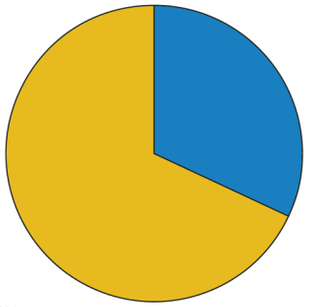
   (c) In which four provinces were most rhinos killed in 2014? Look at the bottom pie chart.

   (d) Four provinces seem to have very few rhino killings. Which provinces are they?

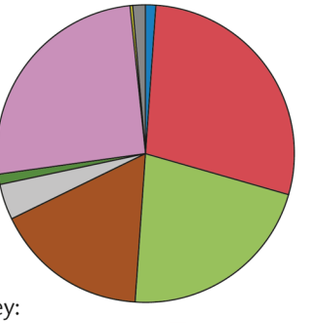
   (e) Why do you think so many rhino killings happened in the Kruger National Park?

   (f) Estimate the fraction of rhino killings that happened in Mpumalanga and Limpopo together.

Where rhinos were killed in SA in 2014



Rhinos killed in the rest of SA in 2014

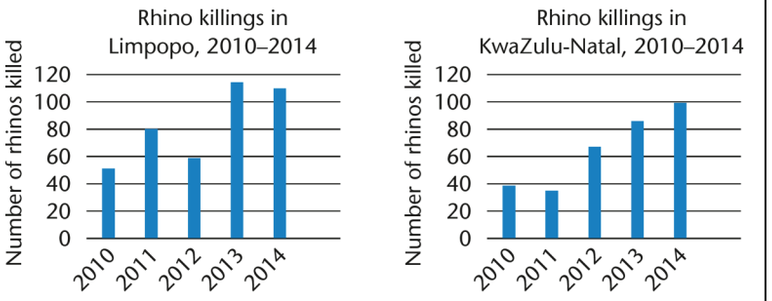


   (g) Which two provinces seem to have the biggest problems with rhino poaching?

Answers

2. (a) Limpopo: more than 40 killed, while in KwaZulu-Natal there were fewer than 40 killed.
   (b) KwaZulu-Natal
   (c) In Limpopo the number of rhinos killed in 2014 was more than double the number in 2010. The highest number of killings occurred in 2013. In 2012 and 2014 there were fewer killings than the year before.
   (d) In KwaZulu-Natal more than twice the number of rhinos were killed in 2014 than in 2010. In 2011 slightly fewer rhinos were killed than in 2010. In all other years the number of rhinos killed increased each year.
   (e) KwaZulu-Natal
   (f) Opinions may differ. Overall it is increasing, although the numbers decreased between 2011 and 2012, and also between 2013 and 2014.

2. Compare the bar graphs of the number of rhinos killed in Limpopo and KwaZulu-Natal from 2010 to 2014.



(a) In which of the two provinces is the number of rhinos killed in 2010 the highest?

(b) In which province did the number of rhinos killed decrease from 2010 to 2011?

When numbers decrease, they become smaller. When numbers increase they become bigger.

(c) How did the number of rhinos killed in Limpopo change from 2010 to 2014?

(d) How did the number of rhinos killed in KwaZulu-Natal change from 2010 to 2014?

(e) In which province do you think the number of rhinos killed is getting more and more as the years go on?

(f) What do you think about rhino poaching in Limpopo? Do you think rhino poaching in Limpopo is decreasing?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
8.1 Patterns in times tables	Diagonal sequences, i.e. patterns with an increasing difference	262 to 264
8.2 Tables, rules and flow diagrams	Families of sequences with a constant difference	265 to 266
8.3 Computer sequences	Equivalence between tables, rules and flow diagrams	267

CAPS time allocation	4 hours
CAPS page references	18 to 19 and 97 to 99

Mathematical background

While providing opportunities to develop understanding of patterns, continuing the sequences or completing the tables according to a pattern also contributes to the development of the Mental Mathematics section of the CAPS.

Numerical patterns, as part of the Content Area "Patterns, Functions and Algebra", should serve as building blocks in developing the basic concepts of algebra in the Senior and FET phases. The study of numeric patterns should develop the concepts of **variable**, **relationships** and **functions**. The function concept is captured in the notion of the triad

Input numbers → Rule → Output numbers

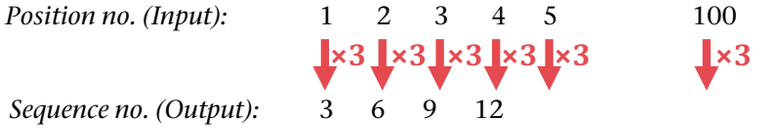
Much of our pattern work focuses on methods to find the calculation plan (rule), because it is so useful to find input and output numbers.

The following two important empowering approaches to pattern work should be emphasised throughout:

- **Recursive ("horizontal") patterns** in **sequences** describing the relationship between any two consecutive numbers in a sequence, and then continuing the sequence. For example:



- **Functional ("vertical") patterns** describing the constant relationship between two sets, and then applying this pattern to calculate further-lying values (e.g. the 100th number). For example:



These two ideas (**recursive** and **functional relationships**) are important horizon knowledge, i.e. important for future mathematical concepts.

8.1 Patterns in times tables

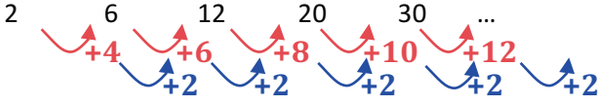
Mathematical notes

The focus of this section is the introduction of a new kind of sequence that is different to the constant difference patterns we have studied so far.

In contrast to a *constant* difference sequence like this:



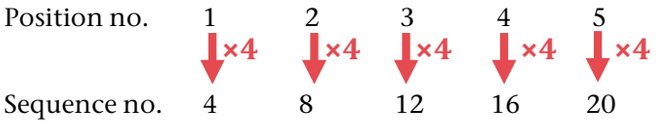
we have *changing* differences as below. Although the differences are changing, there nevertheless is a pattern (a *constant* aspect) in the way that they change. Here, the differences increase with a constant amount:



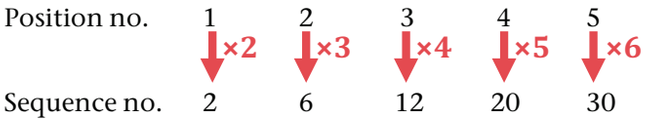
×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16
5	5	10	15	20

We stay with our familiar context of the multiplication table. But whereas we previously studied the horizontal and vertical sequences in the table (all are constant difference, multiples sequences), we now focus on some **diagonal sequences** like 2, 6, 12, 20, ...

The mathematics is interesting. Our previous patterns had a constant horizontal difference and the new patterns have a changing horizontal difference, as shown above. Similarly, our previous multiples sequences in the table had a constant vertical multiplication calculation plan, for example:



We now likewise find that our diagonal sequences have a changing vertical multiplication plan, but that there is a pattern (a *constant* aspect) in the way that it changes, for example:



Teaching guidelines

Instead of lecturing and telling learners about these beautiful and interesting diagonal sequences, we suggest that you should allow learners ample time to find the sequences in the table, to analyse them and to describe the patterns in their own words.

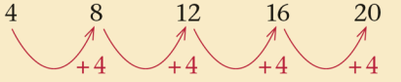
UNIT 8

NUMERIC PATTERNS

8.1 Patterns in times tables

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70

You know that in all horizontal and vertical sequences in the multiplication table, **the same number is added** to get the next number in the sequence. Here is an example:



We say there is a **constant difference** between consecutive numbers. For example:
8 – 4 = 4; 12 – 8 = 4; 16 – 12 = 4; ...

Consecutive numbers are numbers that follow after one another in a pattern.

We will now look at other sequences in the multiplication table, and see what patterns they have.

These sequences are all the same in the sense that they all have a horizontal increasing difference pattern. For example in questions 1 and 2:

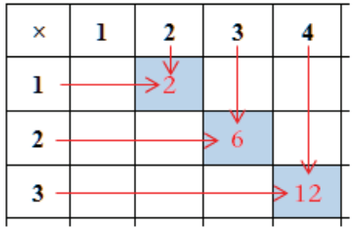
×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

1 → 4 (+3) → 9 (+5) → 16 (+7) → 25 (+9) → 36 (+11) → (+13)

2 → 6 (+4) → 12 (+6) → 20 (+8) → 30 (+10) → 42 (+12) → (+14)

Learners can find the next few numbers by continuing this pattern. But it will not be an efficient method to continue in this way up to 20 or to 100. On the other hand, it may also not at all be easy to find a vertical calculation plan.

You should encourage learners not to try to find patterns in the *numbers*, but in the *structure* of the numbers. In this case, the sequence comes from the multiplication table; the numbers in the multiplication table are formed in a certain way, so we can unravel the structure in the sequences by going back to the multiplication table and understanding its structure. Here is a sample from the table (question 2), showing where the numbers are located, and how they are formed:



Once understood, we can focus on the *structure* and not the *numbers*. Once we see the structure, we can *generalise the structure and not the numbers:*

Pos no.	**1**	**2**	**3**	**4**	**5**	**20**
Seq no.	2	6	12	20	30	?
Structure	**1**×2	**2**×3	**3**×4	**4**×5	**5**×6	**20**×21

Position no. 1 2 3 4 5 20 100

×2 ×3 ×4 ×5 ×6 ×21 ×101

Sequence no. 2 6 12 20 30

Answers

1. (a) There is an increasing difference between consecutive numbers: +3; +5; +7; ...
   (b) ..., 64, 81, 100, 121, 144
   (c) *Sequence no.* = *Position no.* × *Position no.*
   (d) 20th: 20×20 = 400; 100th: 100×100 = 10 000; Vertical is easier than horizontal.

2. (a) There is an increasing difference between consecutive numbers: +4; +6; +8; ...
   (b) ..., 72, 90, 110, 132, 156
   (c) *Sequence no.* = *Position no.* × (*Position no.* + 1)

1. Find this sequence in the multiplication table on page 262.
   1, 4, 9, 16, 25, 36, 49, ...
   (a) How will you describe this sequence? What **horizontal calculation plan** (going from one number to the next) is used to make each next number in the sequence?

   Horizontal means from side to side (in this instance, from left to right). **Vertical** means from top to bottom.

   (b) Continue the sequence for another five numbers.
   (c) Can you find a **vertical calculation plan** describing how the numbers are made?

   Position no.: 1 2 3 4 5 6 ...
   ↓? ↓? ↓? ↓? ↓? ↓?
   Sequence no.: 1 4 9 16 25 36 ...

   (d) Calculate the 20th and the 100th number in the sequence. Did you use a horizontal plan or did you use a vertical plan? Which one is easier?

2. Find this sequence in the multiplication table:
   2, 6, 12, 20, 30, 42, 56, ...
   (a) How will you describe this sequence? What horizontal calculation plan (going from one number to the next) is used to make each next number in the sequence?
   (b) Continue the sequence for another five numbers.
   (c) Can you find a vertical calculation plan describing how the numbers are made?

   Position no.: 1 2 3 4 5 6 ...
   ↓? ↓? ↓? ↓? ↓? ↓?
   Sequence no.: 2 6 12 20 30 42 ...

Answers

2. (d) 20th: 20×21 = 420; 100th: 100×101 = 10 100; Vertical is easier than horizontal.

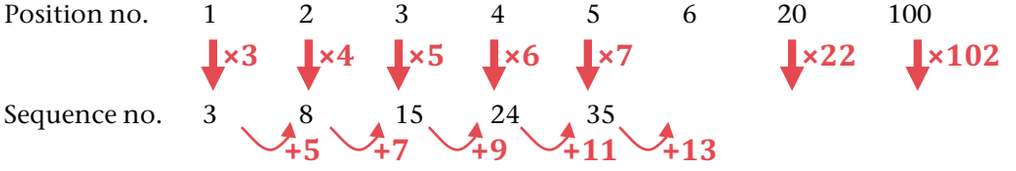
Notes on questions

You should, as always, emphasise that learners should look at these two kinds of patterns:

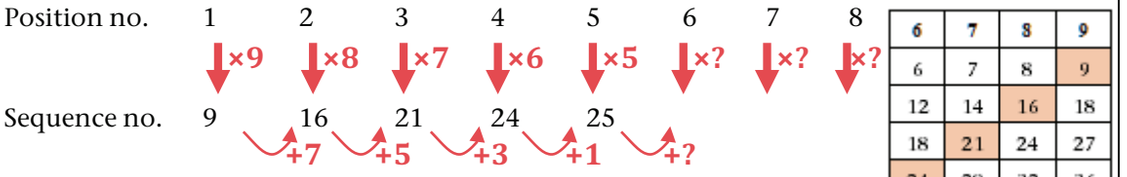
- *Horizontal pattern* of increasing differences: +5; +7; +9; +11; ...
  This pattern is useful to continue the sequence for another few numbers, but not very useful or efficient to continue to the 100th number.
- *Vertical pattern* as the relationship between the position number and the sequence number, leading to a rule that will make it easy to calculate the 100th number.

You should help learners to use the *structure* of the multiplication table on page 262 to find the vertical rule, as explained for question 2 on the previous page. Here are patterns for questions 3 and 4:

Question 3



Question 4



6	7	8	9
6	7	8	9
12	14	16	18
18	21	24	27
24	28	32	36

Answers

3. (a) There is an increasing difference between consecutive numbers: +5; +7; +9; ...
   (b) ..., 80, 99, 120, 143, 168
   (c) *Sequence no.* = *Position no.* × (*Position no.* + 2)
   (d) 20th: 20×22 = 440; 100th: 100×102 = 10 200; Vertical is easier than horizontal.

4. (a) A diagonal sequence from top right to bottom left in the table.
   There is a decreasing difference between consecutive numbers: +7; +5; +3; ...
   (b) The pattern continues like this: +7; +5; +3; +1; –1; –3; ... So ... 24, 21, 16, 9
   Note that the vertical pattern above is easier: ×9, ×8, ×7, ×6, ×5, ×4, ×3, ×2, ×1

(d) Calculate the 20th and the 100th number in the sequence. Did you use a horizontal plan or did you use a vertical plan? Which one is easier?

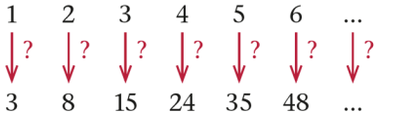
3. Find this sequence in the multiplication table:

   3, 8, 15, 24, 35, 48, 63, ...

   (a) How will you describe this sequence?
   What horizontal calculation plan is used to make each next number in the sequence?

   (b) Continue the sequence for another five numbers.

   (c) Can you find a vertical calculation plan describing how the numbers are made?



   (d) Calculate the 20th and the 100th number in this sequence. Did you use a horizontal plan or did you use a vertical plan? Which one is easier?

CHALLENGE

4. Find this sequence in the multiplication table:

   9, 16, 21, 24, 25, ...

   (a) How will you describe this sequence? What horizontal plan is used to make each next number in the sequence?

   (b) Continue the sequence by writing down the next four numbers in the sequence.

8.2 Tables, rules and flow diagrams

Mathematical notes

We have studied sequences of multiples (the "times tables") and learners should thoroughly know that all the sequences of multiples (tables) are of the same type:

- The multiples of k all have a constant difference of $+k$ between consecutive numbers (the "horizontal" pattern).
- The multiples of k all have a calculation plan of the form $\times k$ (the "vertical" pattern).

Teaching guidelines

We now start to develop the notion of "families of sequences": these sequences are all different, but are nevertheless all the same because they share the property that they all have the same constant difference. For example, this family all have a constant difference of 4:

3, 7, 11, 15, 19, 23, 27, ...
4, 8, 12, 16, 20, 24, 28, ...
6, 10, 14, 18, 22, 26, 30, ...

The activities in this section are designed to develop an understanding of the *relationship between these families of sequences* by comparing their representations in words, flow diagrams, tables and rules. We then identify a relationship between the rules for these families of sequences, which makes it easier for learners to find the rules. For example:

Sequence	Description in words		Flow diagram/Rule
3, 7, 11, 15, 19, 23, 27, ...	one less than multiples of 4		× 4 – 1
4, 8, 12, 16, 20, 24, 28, ...	multiples of 4	*Easy! Start here!*	× 4
6, 10, 14, 18, 22, 26, 30, ...	two more than multiples of 4		× 4 + 2

−1 +2 −1 +2

This provides learners with a very powerful problem-solving strategy: if you have a "difficult" problem, can you first solve an easier related problem? And can you then use the solution of the easier problem to solve the difficult problem?

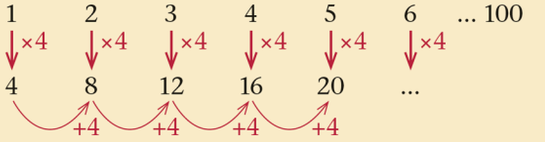
Answers

Due to space limitations, we use the one-line flow diagram notation, for teachers only.

1. 1, 2, 3, 4, 5, 6, 100 → × 4 + 1 → 5, 9, 13, 17, 21, 25, 401 (second table)
   1, 2, 3, 4, 5, 6, 100 → × 4 → 4, 8, 12, 16, 20, 24, 400 (first table)

8.2 Tables, rules and flow diagrams

You know that for the sequence 4, 8, 12, 16, 20, 24, ... we have *horizontal and vertical patterns* with which we can continue the pattern, for example:

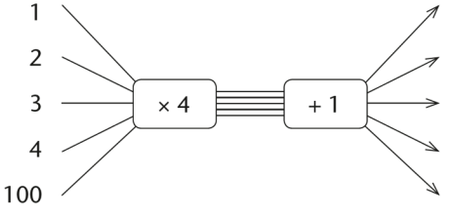


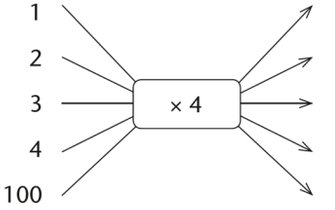
But can you find the 100th number in 5, 9, 13, 17, 21, ...?

1. Below are two tables and two flow diagrams. Which flow diagram is equivalent to which table (gives the same output numbers for the same input numbers)? Calculate all the missing numbers.

Input numbers	1	2	3	4	5	6	100
Output numbers	4	8	12	16	20		

Input numbers	1	2	3	4	5	6	100
Output numbers	5	9	13	17	21		





Teaching guidelines

It is important time wise, but especially conceptually, that learners do not develop the mindset of answering each question as a stand-alone, isolated question.

The idea is that learners will see the relationship between Sequences in A, B, C ..., and therefore the relationship between their flow diagrams and between their rules. If they do, they will have developed a very important and useful problem-solving tool. This will make the work easy, and they can finish quickly.

Answers

Due to space limitations, we use the one-line flow diagram notation, for teachers only.

In the flow diagrams and rules below, we indicate +0 to emphasise the structure and the relationship between the flow diagrams and between the rules for families of sequences. But it is of course not necessary, because of the important additive property of 0, namely that adding 0 leaves the answer unchanged.

2. A (a) 1, 2, 3, ... –[× 4]–[+ 0]→ 4, 8, 12, ... (b) 32, 36, 40, 44, 48 (c) 400
   B (a) 1, 2, 3, ... –[× 4]–[+ 1]→ 5, 9, 13, ... (b) 33, 37, 41, 45, 49 (c) 401
   C (a) 1, 2, 3, ... –[× 4]–[+ 2]→ 6, 10, 14, ... (b) 34, 38, 42, 46, 50 (c) 402
   D (a) 1, 2, 3, ... –[× 4]–[+ 3]→ 7, 11, 15, ... (b) 35, 39, 43, 47, 51 (c) 403
   E (a) 1, 2, 3, ... –[× 4]–[+ 4]→ 8, 12, 16, ... (b) 36, 40, 44, 48, 52 (c) 404

   (d) All the sequences have a difference of 4, but with different starting numbers.
   The flow diagrams all have the same ×4 multiplication operator, but different addition operators.

3. A (a) *Sequence no.* = 5 × *Position no.* + 0 (b) 35, 40, 45, 50, 55 (c) 500
   B (a) *Sequence no.* = 5 × *Position no.* + 1 (b) 36, 41, 46, 51, 56 (c) 501
   C (a) *Sequence no.* = 5 × *Position no.* + 2 (b) 37, 42, 47, 52, 57 (c) 502
   D (a) *Sequence no.* = 5 × *Position no.* + 3 (b) 38, 43, 48, 53, 58 (c) 503
   E (a) *Sequence no.* = 5 × *Position no.* + 7 (b) 42, 47, 52, 57, 62 (c) 507
   F (a) *Sequence no.* = 5 × *Position no.* – 1 (b) 34, 39, 44, 49, 54 (c) 499

   (d) All the sequences have a difference of 5, but with different starting numbers.
   The rules all have the same ×5 multiplication operator, but different addition operators.

2. For each of the Sequences A to E below:
   (a) Write a flow diagram for the sequence.
   (b) Continue the sequence for another five numbers.
   (c) Calculate the 100th number in the sequence.
   (d) How are the sequences the same and different?
   How are the flow diagrams the same and different?

   Sequence A: 4, 8, 12, 16, 20, 24, 28, ...
   Sequence B: 5, 9, 13, 17, 21, 25, 29, ...
   Sequence C: 6, 10, 14, 18, 22, 26, 30, ...
   Sequence D: 7, 11, 15, 19, 23, 27, 31, ...
   Sequence E: 8, 12, 16, 20, 24, 28, 32, ...

3. For each of the Sequences A to F below:
   (a) Write a calculation plan (rule) for the sequence.
   (b) Continue the sequence for another five numbers.
   (c) Calculate the 100th number in the sequence.
   (d) How are the sequences the same and different?
   How are the rules the same and different?

   Sequence A: 5, 10, 15, 20, 25, 30, ...
   Sequence B: 6, 11, 16, 21, 26, 31, ...
   Sequence C: 7, 12, 17, 22, 27, 32, ...
   Sequence D: 8, 13, 18, 23, 28, 33, ...
   Sequence E: 12, 17, 22, 27, 32, 37, ...
   Sequence F: 4, 9, 14, 19, 24, 29, ...

8.3 Computer sequences

Note on matching the tables and rules

To identify which rule goes with which table, learners must connect the properties of the rules with the properties of the tables.

Although this is not knowledge that Grade 4 learners must know and remember (it is in the Senior Phase curriculum), you may know, and some learners may have noticed that:

- in the case of *multiples*, the constant horizontal difference in the sequence and the multiplication operator in the rule are the same, for example:
  3, 6, 9, 12, ... has a constant difference of 3, and the rule is *Output no.* = 3 × *Input no.*
- in the case of *families of sequences* with a constant difference, the constant difference is the same as the multiplication operator in the rule, for example:
  7, 10, 13, ... has a constant difference of 3, and the rule is *Output no.* = 3 × *Input no.* + 4

Alternately, learners can substitute the input numbers 1, 2, 3, ... into each rule (make their own table for the rule) and compare it to P, Q, R and S.

Note on finding input values (solving equations)

To find missing input numbers in the table, learners may use a trial-and-improvement strategy by using a **forward strategy**. For example, to find the input value for 57 in P, learners can try, say 8, giving 8×5 + 2 = 42, which is too small. They should then try bigger numbers until they find the correct one: 11×5 + 2 = 57.

Or, if they can write the rule as a flow diagram (question 3), they can use a **reverse strategy**, i.e. use inverse operations in reverse, e.g. for P: (57 – 2) ÷ 5 = 11.

Answers

1. A → S B → R C → Q D → P

2.

P	**Input no.**	1	2	3	4	5	6	11	100
	Output no.	7	12	17	22	27	32	57	502
Q	**Input no.**	1	2	3	4	5	6	26	100
	Output no.	7	9	11	13	15	17	57	205
R	**Input no.**	1	2	3	4	5	6	13	100
	Output no.	7	11	15	19	23	27	55	403
S	**Input no.**	1	2	3	4	5	6	18	100
	Output no.	7	10	13	16	19	22	58	304

3. A: 1, 2, 3, 4, 5, 6, 18, 100 → [× 3] → [+ 4] → 7, 10, 13, 16, 19, 22, 58, 304
   B: 1, 2, 3, 4, 5, 6, 13, 100 → [× 4] → [+ 3] → 7, 11, 15, 19, 23, 27, 55, 403
   C: 1, 2, 3, 4, 5, 6, 26, 100 → [× 2] → [+ 5] → 7, 9, 11, 13, 15, 17, 57, 205
   D: 1, 2, 3, 4, 5, 6, 11, 100 → [× 5] → [+ 2] → 7, 12, 17, 22, 27, 32, 57, 502

8.3 Computer sequences

To make Tables P, Q, R and S, a computer used Rules A, B, C and D given below.

Rules (calculation plans)

A *Output number* = 3 × *Input number* + 4

B *Output number* = 4 × *Input number* + 3

C *Output number* = 2 × *Input number* + 5

D *Output number* = 5 × *Input number* + 2

Tables

P

Input numbers	1	2	3	4	5	6		100
Output numbers	7	12	17	22	27		57	

Q

Input numbers	1	2	3	4	5	6		100
Output numbers	7	9	11	13	15		57	

R

Input numbers	1	2	3	4	5	6		100
Output numbers	7	11	15	19	23		55	

S

Input numbers	1	2	3	4	5	6		100
Output numbers	7	10	13	16	19		58	

1. Which rule did the computer use for which table? Match the rules and the tables. Describe how you know and how you can be sure that you are right.
2. Complete the tables for the missing input and output numbers.
3. Draw a flow diagram for each of the four rules. Insert the operators, input and output numbers.

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
9.1 Add and subtract distances	Revising addition and subtraction in an extended investigation	268 to 270
9.2 Be smart with addition and subtraction	Replacing calculations with equivalent but simpler calculations	270 to 273
9.3 Add and subtract to find information	Problem solving	273 to 274

CAPS time allocation	4 hours
CAPS page references	14 to 15, 69 to 71 and 100

Mathematical background

Learners continue to make new number facts from number facts they know or are given. They work through examples where they apply this strategy when adding and subtracting.

Learners continue to engage with three methods of addition in Grade 4:

A. Adding by filling up multiples of 10 and 100.
B. Adding by breaking down one number (the smaller one) into place value parts and adding the parts one by one.
C. Adding by breaking down both numbers, rearranging and adding up the answers of the parts.

Learners also engage with two methods of subtraction in Grade 4:

A. Subtracting by filling up multiples of 10 and 100 (subtraction by addition).
B. Subtracting by breaking down both numbers, rearranging and adding up the answers of the parts.

Learners can only apply these methods effectively if they know the addition and subtraction bonds for units and for multiples of ten and hundred well, or are able to quickly reconstruct these facts. This is practised in this unit.

Learners should also continue to use estimation to check whether their answers are reasonable.

9.1 Add and subtract distances

Mathematical notes

Section 9.1 is an investigation. Learners are asked to predict who will win the race (question 1). The situation involves speed, distance and time. This provides an extended context to practise addition and subtraction. As learners analyse the data and do more and more of the addition and subtraction calculations, they continuously are in a better position to predict who will win the race, so they are asked again in question 5 to make the prediction.

Teaching guidelines

Aim to cover this section in 1 hour and 40 minutes. One possibility is to use

- questions 1 to 8 for classwork, and
- questions 9 to 12 for additional practice.

Learners may use different methods of adding and subtracting. If a learner's method differs from the one you expect, do not penalise the learner if his/her calculations are appropriate.

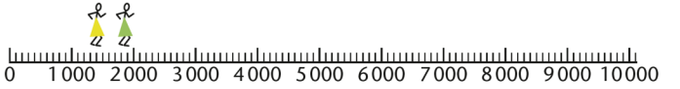
Answers

1. Learners' own responses. Note that they have to provide a reason for their answer. Some learners may say Annie will win because she has run faster than Ellen during the first and second 10-minute intervals. Other learners may feel that they cannot say as Annie may slow down later in the race and Ellen may maintain her speed.
2. (a) 3 702 m
   (b) 6 298 m
   (c) (1 867 + 1 835) – (1 768 + 1 778) = 156 m
   Learners can work out how far Ellen ran in 20 minutes and then subtract it from how far Annie ran in 20 minutes: 3 702 m – 3 546 m = 156 m
   Learners can subtract Ellen's first distance from Annie's first distance, and Ellen's second distance from Annie's second distance, and add the result: 99 m + 57 m = 156 m
3. (a) 5 504 m – 3 702 m = 1 802 m
   (b) 5 345 m – 3 546 m = 1 799 m
   (c) 3 m
4. 50 minutes (Ellen covered 5 345 m in 30 minutes, so she could double that distance in 60 minutes. She could therefore probably cover 10 000 m in 50 minutes.)
5. Learners' own responses. Note that they have to provide a reason for their answer.

UNIT 9 WHOLE NUMBERS: ADDITION AND SUBTRACTION

9.1 Add and subtract distances

Annie and Ellen compete in a race over 10 000 m.



During the first 10 minutes of the race,
Annie runs 1 867 m and Ellen runs 1 768 m.

During the second 10 minutes of the race,
Annie runs 1 835 m and Ellen runs 1 778 m.

1. Who do you expect will win the race? Why do you think so?

2. (a) How far from the starting point is Annie after 20 minutes?
   (b) How far from the endpoint is Annie after 20 minutes?
   (c) How far is the gap between Annie and Ellen after 20 minutes?

After 30 minutes, Annie has covered 5 504 m of the race, and Ellen has covered 5 345 m.

3. (a) How far has Annie run during the third 10 minutes?
   (b) How far has Ellen run during the third 10 minutes?
   (c) How much further than Ellen has Annie run during the third 10-minute period?

4. Approximately how long, to the nearest 10 minutes, do you think Ellen will need to complete the race?

5. Who do you expect will win the race? Why do you think so?

Teaching guidelines

Tables A, B and C are provided in the Addendum (pages 458 and 459). If you have access to a photocopier, you can photocopy them to save time.

Learners will use results from previous questions in later questions. As learners work through the questions, check their answers, so that they don't use incorrect answers in later questions.

Also stop learners from time to time so that they can discuss their thinking, work and answers with the rest of the class.

Answers

6. and 7. Refer to the tables below.

Table A: The distance covered after different times

Time	0 min	10 min	20 min	30 min	40 min	50 min
Annie	0	1 867	3 702	5 504	7 278	9 014
Ellen	0	1 768	3 546	5 345	7 154	8 952
Gap	0	99	156	159	124	62

Table B: The distances covered in different 10-minute periods

Period	First 10 min	Second 10 min	Third 10 min	Fourth 10 min	Fifth 10 min
Annie	1 867	1 835	1 802	1 774	1 736
Ellen	1 768	1 778	1 799	1 809	1 798
Difference	99	57	3	35	62

8. (a) No
   (b) Refer to tables above.

9. (a) Faster
   (b) 10 m
   (c) 21 m

6. Copy Tables A and B to help you to keep a record of the race.

Table A: The distance covered after different times

Time	0 min	10 min	20 min	30 min	40 min	50 min
Annie	0	1 867		5 504		
Ellen	0	1 768		5 345		
Gap	0	99				

Table B: The distances covered in different 10-minute periods

Period	First 10 min	Second 10 min	Third 10 min	Fourth 10 min	Fifth 10 min
Annie	1 867	1 835			
Ellen	1 768	1 778			
Difference					

7. Complete as much of the tables as you can. Use the information you produced when you did questions 2 and 3. Write your answer for question 3(c) in the row for "difference" in Table B.

During the fourth 10-minute period of the race, Annie runs 1 774 m and Ellen runs 1 809 m.

8. (a) Is Ellen now ahead of Annie in the race?
   (b) Write all the new information into Tables A and B.

9. Think about the distances that Ellen ran over the first, second, third and fourth 10-minute periods.
   (a) Is she running faster or slower as the race progresses?
   (b) How much further does she run in the second 10 minutes, than in the first 10 minutes?
   (c) How much further does she run in the third 10 minutes, than in the second 10 minutes?

Answers

10. (a) to (c)

Table C: Differences between the distances covered in different 10-minutes periods

Period	1st and 2nd periods	2nd and 3rd periods	3rd and 4th periods	4th and 5th periods
Annie	32 m slower	33 m slower	28 m slower	38 m slower
Ellen	10 m faster	21 m faster	10 m faster	11 m slower

11. Annie

12. Refer to Tables A, B and C above.

9.2 Be smart with addition and subtraction

Teaching guidelines

Aim to cover this section in 1 hour and 40 minutes. One possibility is to use:

- questions 1, 2 and 7 for mental mathematics,
- questions 3(d) and the tinted passage on page 271 for concept development,
- questions 3(b), (c) and (e), 4(a) and (c), 5(a), (c), (e), (g), (i) and (k), 8, 9(a), (c) and (e), and 10 for classwork, and
- questions 3(a), (f) and (g), 4(b) and (d), 5(b), (d), (f), (h), (j) and (l), 6, and 9(b), (d) and (f) for additional practice.

Note that it is important for learners to see questions 1 and 2 in their textbooks while they calculate the answers.

Learners should always strive to work smarter, better and faster. Sometimes they can use a previous answer to reduce the amount of calculation needed (see question 7). At other times they can manipulate the calculation to make it easier (see the tinted passage on page 271).

Notes on questions

In question 1, pairs of numbers are equal to 2 000, e.g. 1 001 + 999 = 1 000 + 1 000 = 2 000. Each question has 4 pairs. 4 × 2 000 = 8 000.

In question 2 learners can see that 997 is added 10 times. Some learners may say this is 997 × 10 while other learners may say: (10 × 1 000) – (10 × 3).

Answers

1. (a) 8 000 (b) 8 000 (c) 8 000 (d) 8 000

2. 9 970

10. (a) Make a copy of Table C.

(b) Write your answers for questions 9(b) and (c) in the table.

(c) Complete this table as far as you can with the information you now have.

Table C: The differences between the distances covered in different 10-minute periods

Periods	1st and 2nd periods	2nd and 3rd periods	3rd and 4th periods	4th and 5th periods
Annie				
Ellen				

11. Who do you expect will win the race? Why do you think so?

50 minutes after the start of the race, Annie has completed 9 014 m and Ellen has completed 8 952 m.

12. Complete all the tables as far as you can with the information you now have.

9.2 Be smart with addition and subtraction

1. How much is each of the following?

(a) 1 001 + 999 + 1 001 + 999 + 1 001 + 999 + 1 001 + 999

(b) 1 002 + 998 + 1 002 + 998 + 1 002 + 998 + 1 002 + 998

(c) 1 003 + 997 + 1 003 + 997 + 1 003 + 997 + 1 003 + 997

(d) 1 336 + 664 + 1 336 + 664 + 1 336 + 664 + 1 336 + 664

2. How much is

997 + 997 + 997 + 997 + 997 + 997 + 997 + 997 + 997 + 997?

Teaching guidelines

Check that learners understand the language of addition and subtraction:

- The *sum* is the answer to an addition calculation.
- To find the *difference* between two numbers you have to subtract.

Learners have been working with filling up to the next multiple of 10, 100 or 1 000 since Term 1. Learners have also been breaking numbers down, recombining them and building them up. They have also used *transfer*.

Help learners to see that when they use *transfer* to make an addition calculation easier they transfer part of one number to the other number (so *the total remains the same)*. This is really breaking down one number (but not into place value parts) and recombining the numbers before adding the total, for example 3 784 + 2 737 = 3 784 + (16 + 2 721) = (3 784 + 16) + 2 721 = 3 800 + 2 721, etc.

When learners use transfer for subtraction they are still filling up to the next multiple of 10, 100 or 1 000 (usually it is the number subtracted that is changed). With subtraction the amount used to "fill up" is not transferred from one number to the other. It is added to (or taken away from) both numbers: *the difference between the numbers remains the same*, for example 10 – 5 = 12 – 7 = 16 – 11 or 7 243 – 3 569 = 7 2**74** – 3 600 = 7 **6**74 – 4 000.

Answers

3. (a) 7 982 + 2 648 = 10 630
   (b) 8 002 – 796 = 7 206
   (c) 895 + 6 853 + 9 342 = 17 090
   (d) 7 178 – 3 099 = 4 079; 4 079 + 1 021 = 5 100 or
   (7 178 – 3 099) + 1 021 = 4 079 + 1 021 = 5 100
   (e) 4 050 – 3 505 = 545; 2 999 + 1 878 = 4 877; 4 877 + 545 = 5 422 or
   (4 050 – 3 505) + (2 999 + 1 878) = 545 + 4 877 = 5 422
   (f) 3 784 + 2 737 = 6 521
   (g) 7 243 – 3 569 = 3 674

4. (a) $3\ 403 \overset{3}{+} 2\ 265 = 3\ 400 \overset{400}{+} 2\ 268 = 3\ 000 + 2\ 668 = 5\ 668$

   (b) $7\ 259 \overset{35}{+} 2\ 135 = 7\ 294 \overset{100}{+} 2\ 100 = 7\ 394 + 2\ 000 = 9\ 394$

   (c) $3\ 459 \overset{35}{+} 2\ 265 = 3\ 424 \overset{300}{+} 2\ 300 = 3\ 724 + 2\ 000 = 5\ 724$

   (d) $7\ 259 \overset{25}{+} 1\ 875 = 7\ 234 \overset{100}{+} 1\ 900 = 7\ 134 + 2\ 000 = 9\ 134$

3. Calculate:
   (a) The sum of 7 982 and 2 648
   (b) The difference between 8 002 and 796
   (c) The sum of 895, 6 853 and 9 342
   (d) Add 1 021 to the difference between 7 178 and 3 099.
   (e) Subtract 3 505 from 4 050 and add the difference to the sum of 2 999 and 1 878.
   (f) 3 784 + 2 737
   (g) 7 243 – 3 569

It is often possible to replace a calculation with an easier calculation that will produce the same answer.

For example, if you have to calculate 3 784 + 2 737 you can first transfer 16 from 2 737 to 3 784, to fill it up to 3 800. Then you can transfer 200 from the 2 721 that remains, to fill the 3 800 up to 4 000. Now you can calculate 4 000 + 2 521, which is much easier than calculating 3 784 + 2 737.

$3\ 784 \overset{16}{+} 2\ 737 = 3\ 800 \overset{200}{+} 2\ 721 = 4\ 000 + 2\ 521 = 6\ 521$

If you have to calculate 7 243 – 3 569, you can add 31 to both numbers to get 7 274 – 3 600. You can further add 400 to both numbers to get 7 674 – 4 000. This is much easier to calculate than 7 243 – 3 569, but will produce the same answer.

$7\ 243 \overset{+31}{-} 3\ 569 = 7\ 274 \overset{+400}{-} 3\ 600 = 7\ 674 - 4\ 000 = 3\ 674$

4. In each case, first make transfers so that the calculation is replaced with an easier calculation.
   (a) 3 403 + 2 265
   (b) 7 259 + 2 135
   (c) 3 459 + 2 265
   (d) 7 259 + 1 875

Notes on questions

In question 5 remind learners that when they add, they can transfer one amount from one number to the other to keep the totals the same. But when they subtract, they should add or subtract the same amount to or from both numbers to keep the difference the same.

Both questions 6 and 10 have calculations with brackets. Learners know to calculate what is inside the brackets first. Where the brackets are placed relative to the numbers and operations impacts on the answers.

In question 7 learners should use some answers to get other answers. For example, the answer to 7(b) will help with the answer to 7(c), 7(c) will help you to get 7(d), the answer to 7(f) will help with the answer to 7(g), 7(g) helps you to get 7(h).

Answers

5. (a) 6 145 + 2 975 = 6 120 + 3 000
   (b) 4 509 + 2 793 = 4 302 + 3 000
   (c) 6 978 – 3 123 = 6 855 – 3 000
   (d) 5 346 – 1 218 = 5 128 – 1 000
   (e) 7 966 – 4 663 = 7 303 – 4 000
   (f) 6 243 – 4 185 = 6 058 – 4 000
   (g) 8 396 – 5 579 = 7 817 – 5 000
   (h) 5 532 – 1 873 = 5 659 – 2 000
   (i) 4 008 – 2 399 = 3 609 – 2 000
   (j) 5 399 + 3 006 = 5 405 + 3 000
   (k) 2 305 + 5 032 + 1 019 = 2 356 + 5 000 + 1 000
   (l) 9 098 – 4 105 – 1 199 = 8 993 – 4 000 – 1 199 = 8 794 – 4 000 – 1 000 = 3 794
6. (a) 5 557 (b) 5 557 (c) 2 779 (d) 2 779
7. (a) 3 145 (b) 4 945 (c) 5 145 (d) 5 165
   (e) 2 745 (f) 945 (g) 745 (h) 725
8. 9 500 – (2 341 + 578 + 4 690) = 9 500 – 7 609 = 1 891

5. In each case, replace the given calculation with a different easier calculation that will produce the same answer.
   (a) 6 145 + 2 975 (b) 4 509 + 2 793
   (c) 6 978 – 3 123 (d) 5 346 – 1 218
   (e) 7 966 – 4 663 (f) 6 243 – 4 185
   (g) 8 396 – 5 579 (h) 5 532 – 1 873
   (i) 4 008 – 2 399 (j) 5 399 + 3 006
   (k) 2 305 + 5 032 + 1 019 (l) 9 098 – 4 105 – 1 199

6. Calculate.
   (a) 8 764 – (4 596 – 1 389)
   (b) 8 764 – 4 596 + 1 389
   (c) (8 764 – 4 596) – 1 389
   (d) 8 764 – 4 596 – 1 389

7. What number is:
   (a) 200 more than 2 945?
   (b) 2 000 more than 2 945?
   (c) 2 200 more than 2 945?
   (d) 2 220 more than 2 945?
   (e) 200 less than 2 945?
   (f) 2 000 less than 2 945?
   (g) 2 200 less than 2 945?
   (h) 2 220 less than 2 945?

8. The sum of four numbers is 9 500.
   Three of the numbers are 2 341, 578 and 4 690.
   What is the fourth number?

Answers

9. (a) 6 605 (b) 9 290
   (c) 4 063 (d) 7 488
   (e) 1 045 (f) 895
10. (a) B and C
    (b) A: 6 398 B: 4 392 C: 4 392 D: 388

9.3 Add and subtract to find information

Teaching guidelines

Aim to cover this section in 1 hour and 40 minutes. One possibility is to use

- question 1 for concept development,
- questions 3, 4, 6, 7 and 10 for classwork, and
- questions 2, 5, 8, 9 and 11 for additional practice.

You can read question 1 to learners and ask them to explain the question in their own words. Ask questions like: "*What does this question ask you to do?", "What useful information were you given?", "What will you do now?"* Let learners do the actual calculations.

Possible misconceptions

Some learners may just add the two amounts in question 4 to get an answer of R2 398. If they do this, you can ask them questions like: *"Does the fridge cost more or less than the stove?", "How do you know this?", "What is the cost of the stove?", "What is the cost of the fridge?"* Then ask them to recalculate the total cost.

Notes on questions

In question 4 learners first need to find the cost of the fridge: R2 099 + R299 = R2 398. Then add the cost of the stove, i.e. R2 099 + R2 398 = R4 497.

Draw learners' attention to the fact that question 3 does not ask how many girls there are in the school but "how many more boys than girls" there are.

Answers

1. 493 tickets
2. 3 931 tins
3. 157 more boys than girls
4. R2 398 + R2 099 = R4 497
5. R2 018

9. Calculate.
   (a) 3 878 + 2 727 (b) 5 179 + 4 111
   (c) 7 402 – 3 339 (d) 9 973 – 2 485
   (e) 7 153 – 6 108 (f) 2 804 – 1 909

10. (a) Which of the following do you think will have the same answer, if the calculations in brackets are done first?
    A. (5 395 + 3 005) – 2 002
    B. (5 395 – 3 005) + 2 002
    C. 5 395 – (3 005 – 2 002)
    D. 5 395 – (3 005 + 2 002)
    (b) Do the calculations.

9.3 Add and subtract to find information

1. For a school concert there are 1 178 seats available in the hall. On Monday, 457 tickets were sold and on Tuesday, 228 tickets. How many more tickets need to be sold for all the seats to be filled?
2. During a weekend the corner cafe sold 3 657 tins of cooldrink. On Monday morning the cafe had only 274 tins of cooldrink left. How many tins of cooldrink did the cafe have in stock before the weekend?
3. Of the 1 589 learners in our school 873 are boys. How many more boys than girls are in our school?
4. A stove costs R2 099 and a fridge costs R299 more. How much do a fridge and a stove cost altogether?
5. Mrs Hlope wants to buy a washing machine that costs R3 478. She has saved R1 460. How much more does she need to save?

Mathematical notes

Learners can use a variety of calculation methods to get the answers. This is why no number sentences are provided in the answers. For example, learners could break down numbers into place value parts, recombine and then add up the answer. They could fill up to the nearest multiple of 10, 100 or 1 000. Learners may use adding on where you might have thought of subtraction. Do not penalise learners if they use a different method to the one you expected. Check that their calculations and answers are correct.

Teaching guidelines

Remind learners of the value of estimating before calculating.

Also remind learners that they can check every addition calculation by doing a subtraction calculation, and vice versa.

Possible misconceptions

In question 7 some learners might not notice that the volume of water (1 457 ml) is given in millilitres and the capacity of the tank (5 000 ℓ) in litres. If learners get an answer of 3 543 ℓ, it means that they have not noticed this difference in units. Ask learners: *"How much water is in the tank? Is it more or less than 2 litres? 10 litres? 100 litres? 1 000 litres?"*

Answers

6. (a) R6 053
   (b) R3 456
   (c) R2 934
7. 4 998 ℓ and 543 ml
8. 1 459 pear trees
9. 918 loaves of bread
10. (a) 4 576 m
    (b) 2 678 m – 1 898 m = 780 m
11. (a) R9 137
    (b) R2 119

6. Juanita earns R4 756 per month. Latifa earns R1 297 more than Juanita per month.
   (a) How much does Latifa earn?
   (b) How much will Juanita have left over if she pays R1 300 for her rent?
   (c) How much will Latifa have left over after she pays R230 for her TV licence, R2 400 for rent and R489 for life insurance?
7. A tank can hold 5 000 ℓ of water. At the beginning of the rainy season, it contains 1 457 ml of water. How much more water does the tank need to be full?
8. Bongi has 1 286 pear trees on his farm. Josh has 2 745 pear trees on his farm. How many more pear trees than Bongi does Josh have?
9. Red Ribbon Bakery delivers 1 856 loaves of bread daily and Tangwa Bakery delivers 2 774 loaves daily. What is the difference between the numbers of loaves delivered daily by the two bakeries?
10. A road grader scraped 1 254 m of a gravel road on Tuesday. On Wednesday the grader worked 1 898 m of the road and on Thursday 1 424 m.
    (a) How much of the road was scraped altogether?
    (b) How much more of the road was scraped on Tuesday and Thursday together than on Wednesday?
11. Hannah bought a new sewing machine for R5 628 and a new tumble dryer for R3 509.
    (a) How much did she spend altogether?
    (b) How much more did she spend on the sewing machine than on the tumble dryer?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
10.1 Revision	Revision of multiplication of 2-digit numbers by 2-digit numbers	275 to 277
10.2 Different methods of multiplication	Doubling, rounding off and compensating as methods	278 to 279
10.3 Think and make plans	Multiples, estimation and word problems	279 to 280

CAPS time allocation	5 hours
CAPS page references	13 to 15 and 101

Mathematical background

This unit provides for the knowledge and skills listed below.

- How to multiply by units and multiples of 10.

  Where learners do not know the multiplication facts, they should know calculation strategies that allow them to reconstruct the facts quickly. A multiplication fact is something that we know off by heart, such as 9 × 7 = 63. If learners do not know this multiplication fact, they should have a way to work it out quickly. For example, 10 × 7 = 70, so 9 × 7 = 70 – 9 = 63.
- If you change the order of two numbers when multiplying, it does not change the answer. For example, 7 × 9 = 9 × 7; 50 × 6 = 6 × 50. This is the commutative property of multiplication. Learners are *not* expected to know the name of this property. They only need to know how to use it to make calculations more manageable.
- If you change the way numbers are grouped when you multiply, it does not change the answer. For example, 7 × (8 × 100) = (7 × 8) × 100. A learner might know that 6 × 12 = 72 but might not know what 3 × 24 is. 3 × (2 × 12) = (3 × 2) × 12 = 6 × 12 = 72. This is the associative property of multiplication. Learners are *not* expected to know the name of this property. They only need to know how to use it to make calculations more manageable.
- If a calculation involves multiplication as well as addition and/or subtraction, you do the multiplication first. However, if anything is in brackets, then that is done first. These are mathematical conventions.
- You can break a number down into parts and multiply each part. Mostly we break numbers down into place value parts to multiply. Each part must be multiplied by each other part, for example 53 × 75 = 50 × 70 + 50 × 5 + 3 × 70 + 3 × 5. The aim is to replace a difficult calculation with easier calculations.
- You can break a number down into factors to multiply, for example 23 × 32 = 23 × 2 × 16 = 46 × 2 × 8 = 92 × 2 × 4 = 184 × 2 × 2 = 368 × 2 = 736. The aim is to replace a difficult calculation with easier calculations.
- Learners' approach should be one of linking different aspects of mathematics and trying to make "new" number facts from number facts that they already know.
- Estimation skills.

The aim is to be able to use the knowledge and approaches outlined above to make multiplication more manageable.

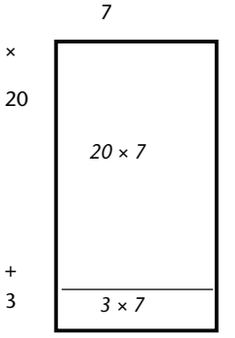
10.1 Revision

Teaching guidelines

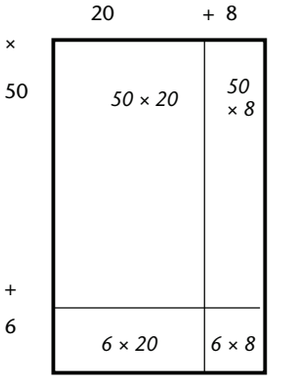
Aim to cover this section in 2 hours. One possibility is to use

- question 7 for mental mathematics,
- questions 2, 5 and 11(a) and (b) for concept development,
- questions 1, 3, 4, 8(a) and (b), 9 and 10 for classwork, and
- questions 6, 8(c) and (d), and 11(c) for additional practice.

For numbers bigger than 10, it is inefficient to use repeated addition to multiply. Begin by asking learners to complete question 1 using any method they know. Let them share their answers. You can show learners how to imagine this calculation in an array of 23 times 7 (23 rows of 7): see Term 1 Section 5.1 in this Teacher Guide (page 65) for how to model this on grid paper. Then you can show how to break this down into 20 rows of 7 and 3 rows of 7.



You can use question 5 to show learners how to use an array to model 2-digit by 2-digit multiplication.



Possible misconceptions

Ben's calculation in question 5 is an example of a common learner error. When learners break down numbers, they sometimes incorrectly only multiply the tens with the tens and the units with the units. This is an overgeneralisation from addition. Drawing a grid to show the number (e.g. 28 × 56) as an array and how this can be broken down into four different, smaller arrays, can help learners to see that breaking numbers down into tens and units to multiply will give them four smaller multiplication calculations to do (see alongside).

Answers

1. 161
2. 20 × 7 + 3 × 7
3. 378
4. ... = 40 × 70 + 40 × 8 + 6 × 70 + 6 × 8 = 2 800 + 320 + 420 + 48 = 3 588
5. No. He should also calculate 20 × 6 + 8 × 50 and add that to 1 000 + 48.

UNIT 10

WHOLE NUMBERS: MULTIPLICATION

10.1 Revision

23 × 7 is 7+7+7+7+7+7+7+7+7+7+7+7+7+7+7+7+7+7+7+7+7 (23 times)

This is

7+7+7+7+7+7+7+7+7+7 + 7+7+7+7+7+7+7+7+7+7 + 7+7+7

10 × 7 + 10 × 7 + 3 × 7

So if you know how much 10 × 7 and 3 × 7 is, you can easily find out how much 23 × 7 is.

1. How much is 23 × 7?
2. Do you think you can calculate 23 × 7 even more quickly if you know how much 20 × 7 is?
   Show how this can be done.
3. Calculate 54 × 7.
4. This is how Raina started to calculate 46 × 78:

   46 × 78 = 40 × 78 + 6 × 78

   Complete Raina's work.
5. This is what Ben did when he tried to calculate 28 × 56:

   28 × 56 = 20 × 50 + 8 × 6 = 1 000 + 48

   Is this right?

   If you think it is not right, describe what Ben should do to get it right.

Critical knowledge

Multiply by units, multiply by 10, multiply by multiples of 10.

You can break numbers down, for example into place value parts, and multiply each part separately, for example 67 × 46 = 60 × 40 + 60 × 6 + 7 × 40 + 7 × 6.

When you break numbers down as above in order to multiply, you must multiply each part by each other part.

If you multiply two or more numbers, you may change the order in which you multiply them. The answer will remain the same.

You can also break numbers down into factors in order to multiply, for example 80 × 70 = 8 × 10 × 7 × 10 = 8 × 7 × 10 × 10 = 8 × 7 × 100 = 56 × 100 = 5 600.

When multiplication occurs in an expression together with addition and/or subtraction, you do the multiplication first.

Teaching guidelines

You can use the tinted passage to show learners one way to set out their work to avoid confusion as to which numbers should be multiplied by which other numbers. You can also encourage learners to show the numbers broken down into place value parts as an array.

×	40	+ 6
60	*60 × 40*	*60 × 6*
+		
7	*7x40*	*7x6*

Answers

6. The last part is wrong. It should be 40 × 60 + 40 × 7 + 6 × 60 + 6 × 7.

7.

×	70	8	30	40	5	60	7	80	9	90
4	280	32	120	160	20	240	28	320	36	360
80	5 600	640	2 400	3 200	400	4 800	560	6 400	720	7 200
7	490	56	210	280	35	420	49	560	63	630
60	4 200	480	1 800	2 400	300	3 600	420	4 800	540	5 400
40	2 800	320	1 200	1 600	200	2 400	280	3 200	360	3 600
8	560	64	240	320	40	480	56	640	72	720
50	3 500	400	1 500	2 000	250	3 000	350	4 000	450	4 500

6. This is what Jaamiah did when she tried to calculate 46 × 67:

   46 × 67 = 40 × 67 + 6 × 67 = 40 × 60 + 40 × 6 + 7 × 40 + 7 × 7

   Where did Jaamiah go wrong?

7. Copy this multiplication table and complete it.

×	70	8	30	40	5	60	7	80	9	90
4										
80										
7										
60					300					
40										
8										
50										

64 × 78 can be calculated like this:

64 is 60 + 4, so 64 × 78 = **60 × 78 + 4 × 78**.

To calculate 60 × 78, we can think of 78 as 70 + 8.

60 × 70 = 4 200 and 60 × 8 = 480, so 60 × 78 = **4 680**.

78 is 70 + 8, so 4 × 78 = 4 × 70 + 4 × 8

which is 280 + 32, so 4 × 78 = **312**.

So 64 × 78 = 4 680 + 312 = 4 992.

This work can be recorded more systematically, for example like this:

64 × 78 = 60 × 78 + 4 × 78
= 60 × 70 + 60 × 8 + 4 × 70 + 4 × 8
= 4 200 + 480 + 280 + 32
= 4 000 + 200 + 400 + 80 + 200 + 80 + 30 + 2
= 4 000 + 800 + 190 + 2
= 4 992

Mathematical notes

When we multiply each part of a number by another number, we say that multiplication is distributed over addition or subtraction. It is the distributive property of multiplication that allows us to say 43 × 38 = 40 × 30 + 40 × 8 + 3 × 30 + 3 × 8. Learners are *not* expected to know these terms, just how to use this property to make multiplication more manageable.

Teaching guidelines

Monitor learners closely when they work on question 8, and provide support where needed.

Learners have already seen that they can break numbers down to multiply, for example 43 × 38 = 40 × 38 + 3 × 38. If we change the order of the numbers multiplied, this gives us 38 × 40 + 38 × 3.

In question 9 learners will see that 4 × 3 bananas + 4 × 5 bananas = 32 bananas. Draw learners' attention to the fact that in question 9, two numbers are multiplied by the same number and the answers (products) added.

In question 10 learners will see that 4 × 8 bananas = 32 bananas. The numbers were first added and then multiplied. The answers to questions 9 and 10 are the same because the same numbers were used.

Explain that this can be used in question 11. All the numbers in question 11 are multiplied by 28. This means that you can add numbers before they are multiplied. Because it is easy to multiply by multiples of 10, you can add numbers to make multiples of 10. So, 28 × 14 + 28 × 6 = 28 × 20 = 560. 28 × 9 + 28 × 11 = 28 × 20 = 560.

28 × 27 + 28 × 3 = 28 × 30 = 840. All the answers can then be added to get 1 960. You could also first add all the numbers, i.e. 28 × (14 + 6 + 9 + 11 + 27 + 3) = 28 × 70 = 1 960.

Answers

8. (a) 1 634 (b) 1 036
   (c) 1 824 (d) 3 648

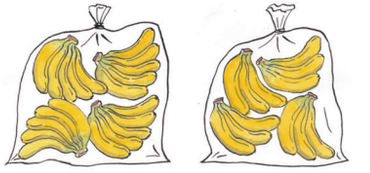
9. 32 bananas

10. 32 bananas

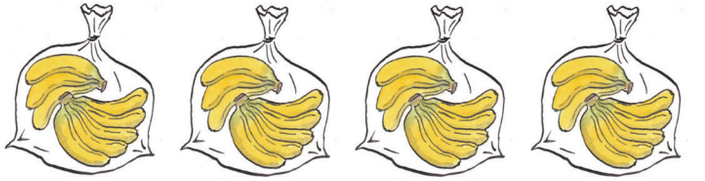
11. (a) Learners may come up with different plans, e.g.
    calculate 14 + 6 and 9 + 11 and 27 + 3 and then 28 × 20 + 28 × 20 + 28 × 30
    or
    calculate 14 + 9 + 27 + 6 + 11 + 3 and then 28 × 70.
    (b) Learners compare their plans.
    (c) 1 960

8. How much is each of the following?
   (a) 43 × 38 (b) 37 × 28
   (c) 32 × 57 (d) 64 × 57

9. Frans bought 4 bunches of bananas with 3 bananas each and 4 bunches of bananas with 5 bananas each. How many bananas did Frans buy?



10. Ahmed bought 4 bags of bananas. In each bag there were one bunch of bananas with 3 bananas and one bunch of bananas with 5 bananas. How many bananas did Ahmed buy?



11. (a) Do not calculate this now:
    28 × 14 + 28 × 9 + 28 × 27 + 28 × 6 + 28 × 11 + 28 × 3

    Think about how you may find out how much it is with as little work as possible.
    Describe your plan in writing.

    (b) Compare your plan with the plans of one or two classmates.

    (c) Find out how much the following is:
    28 × 14 + 28 × 9 + 28 × 27 + 28 × 6 + 28 × 11 + 28 × 3.

10.2 Different methods of multiplication

Mathematical notes

Four different approaches to multiplication are shown here:

- Question 1 shows multiplication by **repeated doubling**.
- Question 2 shows a building-up method based on **multiplying by 10**. This is often used for division.
- Question 3 shows the method of **multiplying by breaking down into place value parts and building up** the answer. This is the method that learners used in Term 2.
- Question 4 shows a **rounding off and compensating** method.

Teaching guidelines

The purpose of this sequence of questions is to promote the ability to read written representations of mathematical thinking. One reason why this is important is that learners have to read and follow calculations written on the board or in the textbook. Allow learners to read each passage by themselves and then apply the method to the suggested calculation. Working through these alternative methods might also help learners to better understand the method of breaking numbers down into place value parts to multiply.

Answers

1. 2 688. This is calculated as follows with Percy's method:

64	+	64	→	128	+	128	→	256	+	256	→	512	+	512	→	1 024
1		1		2		2		4		4		8		8		16

   16 × 64 + 16 × 64 = 32 × 64 = 2 048
   32 × 64 + 8 × 64 = 40 × 64 = 2 048 + 512 = 2 560
   40 × 64 + 2 × 64 = 42 × 64 = 2 560 + 128 = 2 688

2. 1 288. Ensure that learners use Busi's method, as shown on page 278 alongside.

3. 1 036. Ensure that learners use Faiza's method, as shown on page 278 alongside.

4. Accept all reasonable answers. One possible answer is:

   34 × 50 = 1 700

   34 × 4 = 30 × 4 + 4 × 4 = 120 + 16 = 136

   34 × 54 = 1 700 + 136 = 1 836

10.2 Different methods of multiplication

1. This is how Percy calculates 34 × 54:

   54 + 54 → 108 + 108 → 216 + 216 → 432 + 432 → 864
   1 1 2 2 4 4 8 8 16

   16 × 54 = 864
   16 × 54 = 864
   32 × 54 = 1 728
   34 × 54 = 1 728 + 108 = 1 836

   Use Percy's method to calculate 64 × 42.

2. This is how Busi calculates 34 × 54:

   10 × 54 = 540
   10 × 54 = 540
   10 × 54 = 540
   30 × 54 = 1 620
   4 × 54 = 4 × 50 + 4 × 4 = 200 + 16 = 216
   34 × 54 = 1 620 + 216 = 1 836

   Use Busi's method to calculate 46 × 28.

3. This is how Faiza calculates 34 × 54:

   34 × 54 = 30 × 54 + 4 × 54
   = 30 × 50 + 30 × 4 + 4 × 50 + 4 × 4
   = 1 500 + 120 + 200 + 16
   = 1 620 + 216
   = 1 836

   Use Faiza's method to calculate 37 × 28.

4. Siba starts to calculate 34 × 54 like this:
   34 × 100 = 3 400
   Continue as you think Siba may continue.

Notes on questions

In question 5 you might like to help learners to think about which number to round off. In questions 5(a) and (b) it may be easy to see that 98 is close to 100 and 19 is close to 20.
In question 5(c) either 47 can be rounded off to 50 or 17 can be rounded off to 20.
In question 5(d) either 38 can be rounded off to 40 or 23 can be rounded off to 20 (in this case you have to add to compensate).

Answers

5. (a) Rounding off: $44 \times 100 = 4\,400$
Compensating by subtracting 2 lots of 44: $4\,400 - 2 \times 44 = 4\,400 - 88 = 4\,312$

   (b) Rounding off: $34 \times 20 = 680$
Compensating by subtracting 34: $680 - 1 \times 34 = 646$

   (c) Rounding off: $50 \times 17 = 850$
Compensating by subtracting 3 lots of 17: $850 - 3 \times 17 = 850 - 51 = 799$ or
Rounding off: $47 \times 20 = 940$
Compensating by subtracting 3 lots of 47: $940 - 3 \times 47 = 940 - 141 = 799$

   (d) Rounding off: $38 \times 20 = 760$
Compensating by adding 3 lots of 38: $760 + 3 \times 38 = 760 + 114 = 874$ or
Rounding off: $40 \times 23 = 920$
Compensating by subtracting 2 lots of 23: $920 - 2 \times 23 = 920 - 46 = 874$

10.3 Think and make plans

Teaching guidelines

Aim to spend about 2 hours on this section. One possibility is to use

- questions 1(a) and 2(a) for mental mathematics,
- questions 4 and 5(a) and (b) for concept development,
- questions 1(b) to (e), 3, 5(e) to (h), 6(a), (c) and (e), and 7(c) and (d) for classwork, and
- questions 2(b) and (c), 5(c), (d), (i) and (j), 6(b), (d) and (f), and 7(a) and (b) for additional practice.

Question 4 could be a bit of a challenge. Because the original number is not known, learners can represent this with a box: □. Then add 5: □ + 5. Then multiply by 3: (□ + 5) × 3 = 27. Learners can either solve this by trying random numbers or they can ask themselves: *what* × 3 = 27?

In questions 5(a) and (b) the calculations are the same because only the order of the numbers multiplied is swapped. This is also true for 5(e) and (f).

Answers (see next page)

To calculate 33 × 93, Thembi rounded 93 off to 100.
She knows that 33 × 100 = 3 300, but this is 33 × 7 too much.
Thembi now has to **compensate**. That means she has to make a correction. She has to subtract 33 × 7 from 33 × 100.
33 × 7 = 30 × 7 + 3 × 7 = 210 + 21 = 231.
Now Thembi knows that 33 × 93 = 3 300 – 231 = 3 069

5. Round off and compensate, like Thembi did, to calculate each of the following.
   (a) 44 × 98 (b) 34 × 19
   (c) 47 × 17 (d) 38 × 23

10.3 Think and make plans

1. (a) Write the next 10 numbers in this pattern:
   25; 50; 75; . . .
   (b) If this pattern is continued, which of the following numbers will be in it?
   2 115; 1 125; 1 375; 1 040; 2 035; 1 085; 3 050; 1 115
   (c) How did you decide which numbers will be in the pattern?
   (d) What is 14 × 25?
   (e) How many 25s are in 500?

2. (a) Write the next 10 numbers in this pattern:
   15; 30; 45; . . .
   (b) If this pattern is continued, which of the following numbers will be in it?
   300; 1 490; 6 000; 915; 1 800
   (c) Will 145 be in the pattern? Give a reason for your answer.

Answers

1. (a) 100; 125; 150; 175; 200; 225; 250; 275; 300; 325
   (b) 1 125; 1 375; 3 050
   (c) Learners' answers will differ. Learners should recognise that it is a pattern of skip counting in 25s, or a sequence of multiples of 25. Learners may say something like: *thousands and hundreds are multiples of 25, so I looked which tens and units are multiples of 25* (a quick method) or *I divided the numbers by 25* (a long method).
   (d) 350 (e) 20

2. (a) 60; 75; 90; 105; 120; 135; 150; 165; 180; 195
   (b) 300; 6 000; 915; 1 800
   (c) Learners' answers will differ. Some possibilities are:
   15 isn't a multiple of 145; when 145 is divided by 15 there is a remainder, or
   45 is a multiple of 15 but 100 isn't (I can see from above that 90 and 105 are multiples).

3. (a) 12 different outfits

   Learners might work out the answers in different ways. One possibility is:

   Skirt 1  4 blouses Skirt 2  4 blouses Skirt 3  4 blouses

   (b) 12 outfits with jacket 1 and 12 outfits with jacket 2 give 24 outfits.

4. 4

5. Accept any reasonable estimates. The final answers are:
   (a) 1 288 (b) 1 288
   (c) 4 140 (d) 3 375
   (e) 988 (f) 988
   (g) 2 262 (h) 1 218
   (i) 3 648 (j) 6 324

6. (a) R4 902 (b) R3 818 (c) R2 491
   (d) R3 312 (e) R5 696 (f) R6 162

7. (a) R1 872 (b) 1 980c = R19,80
   (c) 24 hours × 7 = 168 hours (d) 60 minutes × 24 = 1 440 minutes

3. Thandi takes 3 skirts and 4 blouses along on her holiday. All of the blouses match all of the skirts.
   (a) How many different outfits does she have to wear? Show how you got your answer.
   (b) She decides to also take two jackets that she can wear with all of the blouses and skirts. From how many different outfits can she now choose?

4. 5 is added to a number and the answer is multiplied by 3. The answer is 27. What is the original number?

5. Estimate the answer first and write it down before you do the multiplications.
   (a) 23 × 56 (b) 56 × 23
   (c) 45 × 92 (d) 75 × 45
   (e) 38 × 26 (f) 26 × 38
   (g) 29 × 78 (h) 42 × 29
   (i) 57 × 64 (j) 68 × 93

When you estimate an answer, you make a very good guess. You can round off the numbers to numbers that are easy to work with, and calculate with them.

6. Determine the total cost of each of the following.
   (a) 86 boxes of cereal at R57 each
   (b) 46 sets of cutlery at R83 for one set
   (c) 53 sets of glasses at R47 for one set
   (d) 72 pairs of socks at R46 for one pair
   (e) 64 T-shirts at R89 for one T-shirt
   (f) 78 caps at R79 for one cap

7. (a) One diary costs R39. How much will 48 diaries cost?
   (b) If one bread roll costs 55c, how much will you pay for 36 bread rolls?
   (c) There are 24 hours in a day. How many hours are there in a week?
   (d) There are 60 minutes in an hour. How many minutes are there in 24 hours?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
11.1 Learn to use number sentences	Solving word problems and linking them with given number sentences	281 to 282
11.2 Use number sentences	Writing own number sentences to solve problems	283 to 284
11.3 Try a number and improve	Solving equations by first trying a random number and then working more systemically to get closer and closer to the answer	284 to 285
11.4 Practice in answering multiple-choice questions	Applying knowledge of numbers and operations to solve number sentences	285

CAPS time allocation	3 hours
CAPS page references	20 and 102 to 103

Mathematical background

It is important that learners are exposed to a range of problem types: these are specified on pages 120 and 121 of the Intermediate Phase Mathematics CAPS. In Sections 11.1 and 11.2 learners work with word problems and translate them into number sentences. There are different situations or problem types associated with each of the operations. Learners work with different problem types associated with *summation*, namely *finding the sum* and *finding the missing parts of a given sum*. They also work with different problem types associated with *increase and decrease,* namely *calculating the result, calculating the change* and *calculating the initial value.* They also work with *comparison by difference* problems (difference models of subtraction), *division as sharing* and *multiplication as repeated addition*.

In Section 11.1 number sentences are provided that learners can link with word problems. This provides a model for the number sentences they are expected to write on their own in Section 11.2. Sometimes writing a number sentence can help learners to clarify what to calculate.

In Section 11.3 learners find numbers that will make the given number sentences true. Initially they guess numbers, but as they find which numbers are too big and which are too small, they begin to work with an increasingly narrow range of numbers until they zone in on the answer.

In Section 11.4 learners apply what they know about numbers and operations as they answer multiple-choice questions.

11.1 Learn to use number sentences

Mathematical notes

This section introduces learners to translating word problems into number sentences. Sometimes learners can see immediately how to solve a problem. At other times it helps to first write a number sentence. This section helps learners to match number sentences with problems. It models for learners how a problem written in words can be translated into a number sentence by providing number sentences that describe each word problem.

Notes on questions

Question 1(b) is probably the easiest sub-question of question 1.

Question 4(a) is similar to question 1(c). They are both examples of the difference model of subtraction. Learners can of course solve them by adding on from the smaller number, for example 60 + *what* = 75.

Number sentences 2(b) and 2(e) are similar and related to number sentences 2(a) and 2(c). Number sentences 2(d), 2(g) and 2(h) are related.

Teaching guidelines

Aim to complete this section in 45 minutes.

Give learners about 10 minutes to answer the sub-questions in question 1 that they find easy. While learners are doing this, you can write the three sub-questions on the board. Then go through each sub-question in question 2 until you can match one with question 1(a). Repeat this with 1(b) and 1(c). Learners can then use another 10 minutes to complete question 1. Now that you have modelled the process for learners, they can work through questions 4 and 5 in a similar way.

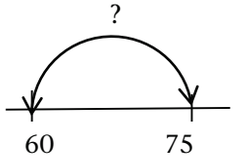
Possible misconceptions

Sometimes learners are too nervous to try to make sense of the language in word problems. They are inclined to just add the numbers they see, or subtract the smaller number from the bigger number. Insist that learners read the questions and talk about what the problem means, or restate the problem in their own words. For example, in question 1(a) you can ask learners: *"How many goats did Gwede start with?", "How many goats did he end with?", "Did he have more goats at the end or in the beginning?", "How did he get the extra goats?", "How many extra goats did he get?"*

Remind learners to always put their answer back in the number sentence to check that it is correct.

Answers

1. (a) 15 goats



UNIT 11 NUMBER SENTENCES

11.1 Learn to use number sentences

Sometimes it is easy to see which calculations you have to do to find information, for example:

Ben has R120 and he pays R50 for food.

How much money does he have left?

Sometimes it is more difficult to see which calculations you must do, for example:

Bettina spent R60 on food and then she had R80 left.

How much money did she have before she bought the food?

In a case like this, it may help to write a number sentence to understand what you must do. We can write:

The money Bettina had – 60 = 80

A number sentence like this will help you to see that in this case you can calculate 80 + 60 to find out how much money Bettina had.

80 + 60 = 140 so she had R140.

To check, you can put your answer into the number sentence:

140 – 60 = 80, so R140 is the right answer.

1. Answer the questions that you find easy. Skip the other questions.

   (a) Gwede has 60 goats. He buys more goats and now he has 75 goats. How many goats did he buy?

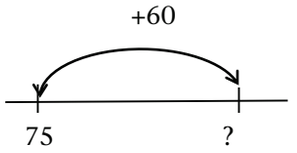


Notes on questions

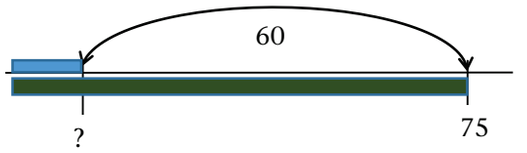
Note that learners are *not* expected to be able to answer all of questions 1 and 4 *immediately*. They will only be able to answer some sub-questions of question 1 after matching number sentences from question 2 to the problems, i.e. when they do question 3. Similarly, some sub-questions of question 4 they will only be able to answer when they do question 5, i.e. when they are asked to match number sentences from question 2 to the problems in question 4 that they found difficult and couldn't solve.

Answers

1. (b) 135 goats



(c) 15 goats



2. (a) 15 (b) 15 (c) 15 (d) 135
   (e) 15 (f) 135 (g) 135 (h) 135

3. Learners try to answer all the parts of question 1 now.
   Note that learners are only expected to match number sentences in question 2 with those problems they could *not* solve in question 1. The complete list is only provided for your information.

Questions	1(a)	1(b)	1(c)
Number sentence	2(c)	2(f)	2(b) or 2(e)

4. (a) 15 goats (b) 15 goats (c) 15 goats
   (d) 15 goats (e) 135 goats (f) 135 goats

5. Learners try to answer all the parts of question 4 now.
   Note that learners are only expected to match number sentences in question 2 with those problems they could *not* solve in question 4. The complete list is only provided for your information.

Questions	4(a)	4(b)	4(c)	4(d)	4(e)	4(f)
Number sentence	2(b) or 2(e)	2(e)	2(a)	2(b)	2(g)	2(d)

(b) Zweli has 75 goats. He buys another 60 goats. How many goats does he have now?

(c) Lerato has 75 goats. This is 60 goats more than Willem has. How many goats does Willem have?

2. Find the missing number in each of these number sentences.
   (a) 75 – . . . = 60 (b) . . . + 60 = 75
   (c) 60 + . . . = 75 (d) 60 + 75 = . . .
   (e) 75 – 60 = . . . (f) 75 + 60 = . . .
   (g) . . . – 60 = 75 (h) . . . – 75 = 60

3. Go back to the parts of question 1 that you skipped because you found them difficult. For each one, find a number sentence in question 2 that can help you. Try to answer all the parts of question 1 now.

4. Answer the questions that you find easy. Skip the other questions.

   (a) Ishmael has 75 goats and Simon has 60 goats. How many more goats does Ishmael have than Simon?

   (b) Pieter had 75 goats. He sold 60 goats. How many goats does he have left?

   (c) Lettie had 75 goats. Some goats were stolen and now she has only 60 goats left. How many goats were stolen?

   (d) Moses buys 60 goats and now he has 75 goats. How many goats did he have before he bought more goats?

   (e) Johan sold 60 goats and now he has 75 goats left. How many goats did he have before he sold some of his goats?

   (f) Mpho has 60 goats and her sister Nellie has 75 goats. How many goats do the two sisters have altogether?

5. Go back to the parts of question 4 that you skipped because you found them difficult. For each one, find a number sentence in question 2 that can help you. Try to answer all the parts of question 4 now.

11.2 Use number sentences

Notes on questions

Questions 1 and 2 have the same structure and are straightforward subtraction calculations.

Questions 4 and 7 are similar to questions 1(c) and 4(a) in Section 11.1: they are "comparison by difference" situations (of difference models of subtraction).

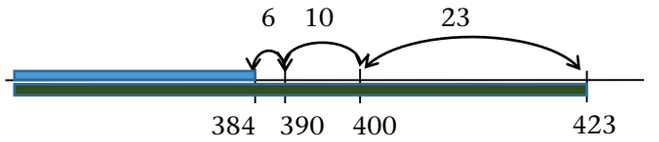
Teaching guidelines

Aim to cover this section in 45 minutes. One possibility is to use

- questions 3 and 5 for concept development,
- questions 1, 2, 4 and 6 for classwork, and
- questions 7 and 8 for additional practice.

Answers

1. 36 learners (because 78 learners – 42 learners = 36 learners)
2. 952 learners (because 2 378 learners – 1 426 learners = 952 learners)
3. 8 721 chickens (because 8 721 chickens – 5 478 chickens = 3 243 chickens)
4. R423 – R384 = R39



5. R3 677 – R2 780 = R897 = cost of 3 chairs
   Cost of one (each) chair = R897 ÷ 3 = R299
6. 300 m
   Gertie walks 184 m – 124 m = 60 m further than Simon each day.
   So Gertie walks 60 m × 5 = 300 m further than Simon in 5 days.
   Learners may also first calculate how far Simon walked in 5 days (620 m) and subtract this from how far Gertie walked in 5 days (920 m): 920 m – 620 m = 300 m.

11.2 Use number sentences

For some of the questions below you will know immediately which calculations to do. For other questions you may need to write a number sentence first.

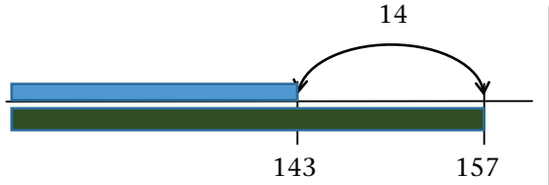
1. There are 78 learners on two buses together.
   There are 42 learners on the one bus.
   How many learners are on the other bus?
2. There are 2 378 learners in two schools together.
   There are 1 426 learners in the one school.
   How many learners are there in the other school?
3. After 5 478 chickens were killed in a storm, Nomvula had 3 243 chickens left. How many chickens did she have before the storm?
4. Jamie paid R384 for a pair of shoes and Vusi paid R423 for his shoes. How much more did Vusi pay than Jamie?
5. Peter bought 3 chairs for his house, all at the same price. He also bought a refrigerator for R2 780. Peter paid R3 677 in total. How much did each of the chairs cost?



6. Gertie walks 184 m to school and back home every day, and Simon walks 124 m to school and back home. How much further than Simon does Gertie walk to school and back, in five days?

Answers

7. 14 cm (because 157 cm – 143 cm = 14 cm)
8. R174 – (3 × R44) = R174 – R132 = R42



11.3 Try a number and improve

Teaching guidelines

Aim to cover this section in 45 minutes. One possibility is to use

- questions 1 and 2 for concept development, and
- two sub-questions of question 3 for classwork.

Learners who like a challenge can try more of question 3 for classwork and additional practice. In these number sentences learners start by trying any number. Advise learners not to start with 0 or 1. However, learners should not continue to try numbers randomly.

In question 1 learners will see that:

- with 6, the left-hand side (LHS) is less than the right-hand side (RHS).
- with 10, the left-hand side (LHS) is more than the right-hand side (RHS).

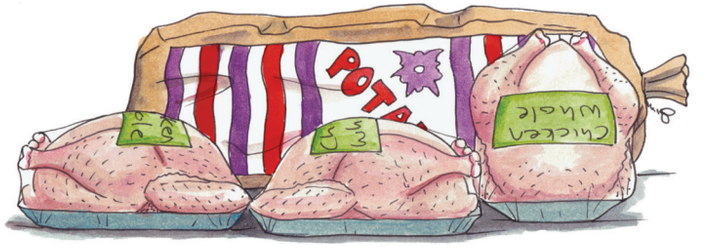
This means that to make the two sides equal, you need a number between 6 and 10. Learners should aim to find a narrower and narrower number range to work with as they zone in on the answer. For example, in question 2 they can start with 5 (and find that the LHS is more than the RHS), then try 20 and 10 (and find that the LHS is less than the RHS). Then they know that the answer is between 5 and 10.

You might like to start with a simpler "Guess my number" game. Let learners decide on a number and write it down where you cannot see it. Each time you guess a number, learners should tell you whether it is too high or too low. Show how you use this to zone in on the answer.

Answers

1. (a) No (because the LHS = 38 and the RHS = 40)
   (b) No (because the LHS = 58 and the RHS = 52)
   (c) Yes (because both sides of the equation = 43)
2. (a) Learners' own responses
   (b) Learners try their chosen number.
   (c) 9 is the correct number, because 6 × 9 + 10 = 64 and 10 × 9 – 26 = 64

7. John is 143 cm tall and Janet is 157 cm tall. How much taller than John is Janet?
8. Ma Minah bought 3 chickens for R44 each. She also bought a bag of potatoes. Ma Minah paid R174 in total. How much did the potatoes cost?



11.3 Try a number and improve

1. (a) Is the number sentence below true if you put 6 in both places where a number is missing?

   5 × *a number* + 8 = 3 × *the same number* + 22

   (b) Is the number sentence true if you put 10 in both places?

   (c) Is it true if you put 7 in both places?

2. (a) Which of these numbers do you think will make the number sentence below true?

   5 20 10 25 9 8 7

   6 × *a number* + 10 = 10 × *the same number* – 26

   (b) Put the number you think will work in the two places and calculate to see if it works. If it does not work, try another number.

   (c) Continue until you find a number that makes the number sentence true.

Answers

3. (a) 3 (b) 10 (c) 15 (d) 13
   (e) 11 (f) 25 (g) 17 (h) 1

11.4 Practice in answering multiple-choice questions

Learners sometimes need to answer multiple-choice questions, for example in the Annual National Assessments. This short section gives learners some practice in applying what they know and answering multiple-choice questions.

Question 3 tests learners' practical understanding of some of the properties of operations.

Teaching guidelines

This is a short section: aim to cover it in 30 minutes.

Question 1 is a simple place value question: learners can do it on their own.

For questions 2, 3 and 4, you could first introduce the simpler examples on pages 102 and 103 of the Intermediate Phase Mathematics CAPS to help orient learners to the form of the questions.

In question 2 learners need to try following the steps of the calculation with each example. It will help learners to write the rule as a number sentence: 3 × *number* + 5. They will see that some of the examples are false as soon as they have multiplied 3 × 3.

In question 3 learners might like to start with statement D. They will see that only the order of the "numbers" in the brackets have been swapped. Learners should know that when adding, the answer will not change if the order of the two numbers added is swapped. So they should be able to see that statement D is true.

In 3C, learners should know that they first calculate what is inside the brackets. This means that it does not matter whether ×3 is before or after the brackets; they will do it once they have calculated what is inside the brackets. So learners should also be able to see that statement C is true.

In 3A and B, learners should try to substitute any numbers (except 0 or 1) into the expressions, for example 3 × (4 + 2) = 3 × 6 = 18, but 3 × 4 + 2 = 12 + 2 = 14. So, option A is not correct.

Answers

1. C 2. C 3. B, C, D 4. C

3. In each case, try different numbers until you find the number that makes the number sentence true:
   (a) 20 × *the number* + 40 = 30 × *the same number* + 10
   (b) 23 × *the number* – 60 = 15 × *the same number* + 20
   (c) 6 × *the number* + 5 = 10 × *the same number* – 55
   (d) 6 × *the number* + 5 = 10 × *the same number* – 47
   (e) 6 × *the number* + 5 = 10 × *the same number* – 39
   (f) 400 – 10 × *the number* = 4 × *the same number* + 50
   (g) 8 × *the number* + 14 = 10 × *the same number* – 20
   (h) 37 × *the number* + 15 = 15 × *the same number* + 37

11.4 Practice in answering multiple-choice questions

Choose the correct answer and write down the letter only.

1. Choose the correct answer for 3 000 + □ + 80 + 4 = 3 684.
   A. 6 B. 60 C. 600 D. 100

2. For which pairs of numbers can you use the rule "multiply the first number by 3 and add 5 to get the second number"?

	First number	Second number
A.	3	5
B.	3	8
C.	3	14
D.	1	6

3. Which of the following statements are true?
   A. 3 × (□ + 2) = 3 × □ + 2 B. 3 × (□ + 2) = 3 × □ + 6
   C. 3 × (□ + 2) = (□ + 2) × 3 D. 3 × (□ + 2) = 3 × (2 + □)

4. How much more is 17 × 37 than 14 × 37?
   A. 3 B. 20 C. 3 × 37 D. 37

Grade 4 Term 3 Unit 12 Transformations

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
12.1 Combining figures	Building up composite 2-D figures using a collection of figures with simple shapes	286 to 287
12.2 Using figures to make pictures	Using a few figures with curved shapes to make many different pictures	288

CAPS time allocation	3 hours
CAPS page references	23 and 103

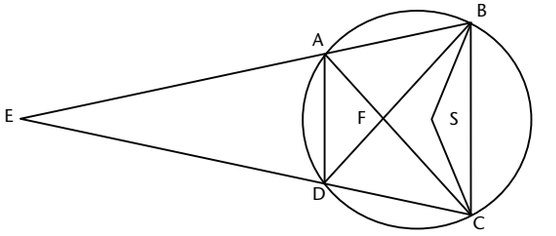
Mathematical background

Transformation means change. We can transform figures by changing the shape of the figure and/or by changing its size. This unit does not look at this kind of transformation.

We can also transform figures by changing their position, but keeping the size of the figure the same. This is called isometric transformation.

We can change the position of figures by turning them, flipping them over, or sliding them into a new position. It is possible to make many different figures by using just a few figures and applying different transformations to the figures. In this unit we combine single figures with other single figures to make new composite figures or pictures.

In Senior Phase and FET geometry many learners struggle to interpret complex geometric figures like the one shown alongside. Learners need to see figures within figures and they also need to recognise shapes, no matter what direction they face. Work done on isometric transformations in the Intermediate Phase helps learners to do this. Working with transformation helps them to develop a bank of images in their mind, which is useful to draw on in more formal geometry in later years.



Although this unit is short, it is very rewarding mathematically speaking. Transformations are at the heart of many important mathematical ideas. This section is an introduction to transformations and is *informal*. It is important that learners have the cut-out figures to experiment with and move around. Learners should not expect to see the answers immediately. They need to try to find them by moving the pieces in relation to each other. Learners enjoy puzzling out the solutions.

In Section 12.1 learners work with a tangram. This is a Chinese geometrical puzzle consisting of *seven* figures with straight sides (five triangles, a square and a parallelogram). In Section 12.2 learners work with *nine* figures. All the figures have some curved and some straight sides.

Resources

This topic involves a number of practical activities for which various resources are needed. These resources are listed in the various sections.

12.1 Combining figures

Resources

A tangram set for each learner or pair of learners. (Learners can make these as shown on page 286. If you have access to a photocopier, you can photocopy the template provided in the Addendum on page 460.)

A large demonstration tangram set that you can stick to the board when discussing solutions.

Scissors.

Teaching guidelines

Aim to spend about 2 hours on this section.

Most learners really enjoy the challenge of solving puzzle questions using tangram pieces. Questions 3, 4 and 7 can be a bit daunting at first. You can help to build learners' confidence by first working with a smaller number of pieces, as in questions 2 and 6.

Another way to develop learners' confidence is to build one or two of the figures on the board, and to ask learners to build a copy of these on their desks. You could use some of the examples in question 7 for this. This is more complex than it sounds. It can help learners to learn that some of the figures, especially the parallelogram, can look quite different if they face different directions.

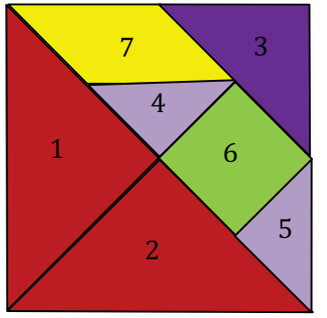
After completing questions 2 and 6, and copying some figures from question 7, you might like learners to try Section 12.2. The questions in Section 12.2 are easier than questions 3, 4 and 7 of Section 12.1. There are two reasons for this: all the internal lines are shown (the figures that make up each picture are shown), and there are only *three different* kinds of shapes amongst these figures.

Do not expect learners to work fast, or to be able to complete all the questions in the allotted time. You can choose only one or two examples in questions 3, 4 and 7.

In this unit learners should draw all their solutions. This will help them to keep track of the different possibilities. Learners can work independently on this and then join up in groups to compare their results. Encourage them to check for possible repetitions.

Answers

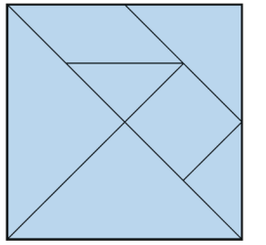
1. Learners make their own tangram set.



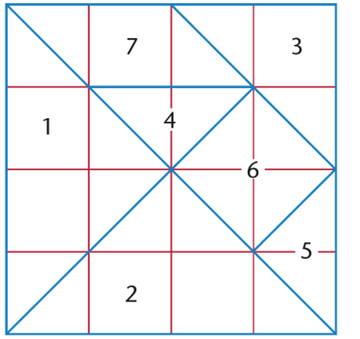
UNIT 12 TRANSFORMATIONS

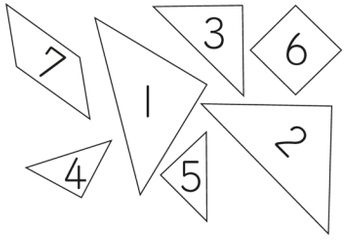
12.1 Combining figures

A Chinese **tangram** is a pattern of seven figures that form a square.

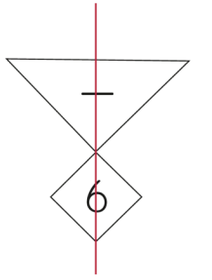


1. Make your own tangram by following these steps:
   - Draw a square with sides 16 cm long on a piece of thick paper or cardboard.
   - Divide the square into 16 smaller squares as shown by the red lines on the right. Then draw the blue lines to divide the square into the seven pieces.
   - Number the pieces as shown.
   - Cut out the pieces.





Pieces 1 and 6 can be placed together to form a symmetrical figure:



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Notes on questions

Often figures with a more broken outline (for example 3(a)) are easier to build than figures like triangles, squares, rectangles, etc.

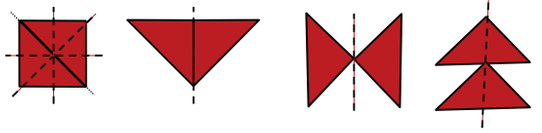
Check the shape of the trees that learners build in question 3(a). There are many similar but slightly different trees that can be built with a tangram set.

There is more than one solution (different ways to arrange the pieces) for some of these questions, especially in question 7.

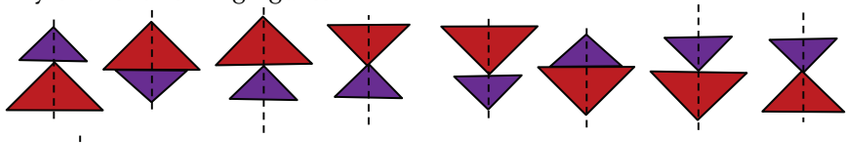
Answers

Note that in all the answers below the compound figures could face in any direction.

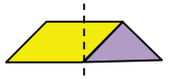
2. (a) and (b)
   Any two of the following figures. Others may also be possible.



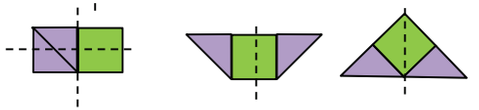
   (c) Any of the following figures.



   (d)



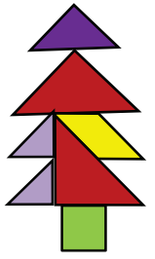
   (e)



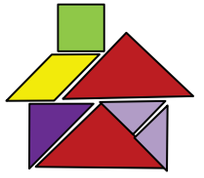
   (f) Learners' own work: too many examples to illustrate.

3. In these solutions gaps have been left between the pieces to make it easier to see which pieces are used where. Learners should not leave a gap between pieces.

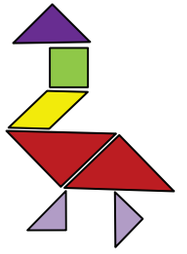
   (a)



   (b)



   (c)



2. Make a symmetrical figure with each combination of tangram pieces given below. Also draw each figure you make, and its line or lines of symmetry.
   (a) Pieces 1 and 2
   (b) Pieces 1 and 2 differently
   (c) Pieces 1 and 3
   (d) Pieces 7 and 5
   (e) Pieces 4, 5 and 6
   (f) Any pieces you like

3. Use all seven tangram pieces each time to make each of the following figures. The pieces may not overlap.

   (a)

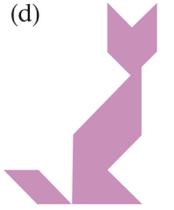


   (b)

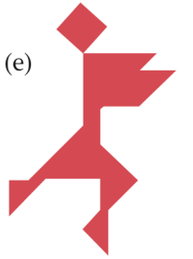
   (c)



   (d)



   (e)



   (f)

4. Use all seven tangram pieces each time to make the following symmetrical diagrams. The pieces may not overlap. Then draw each diagram and its line of symmetry.

   (a)

   (b)

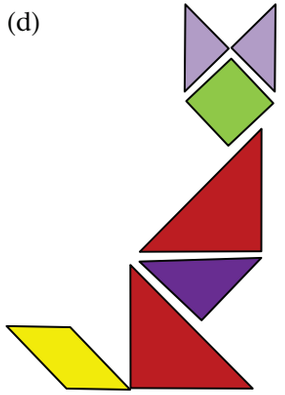
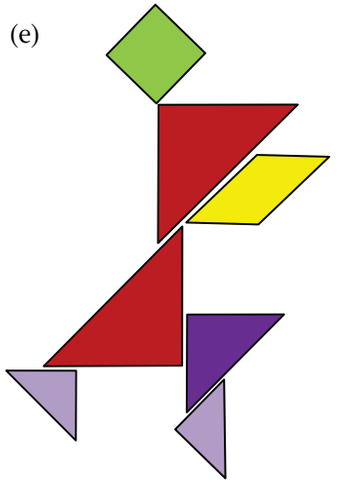
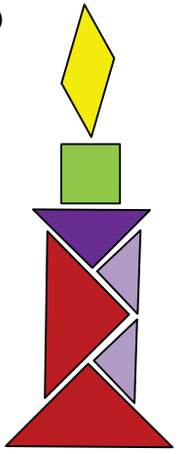


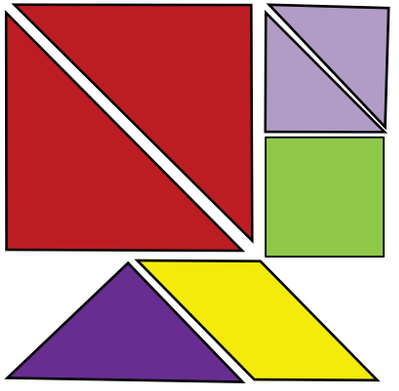
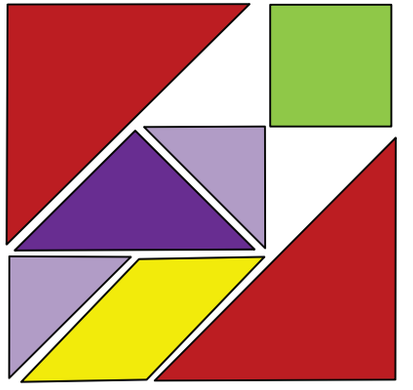
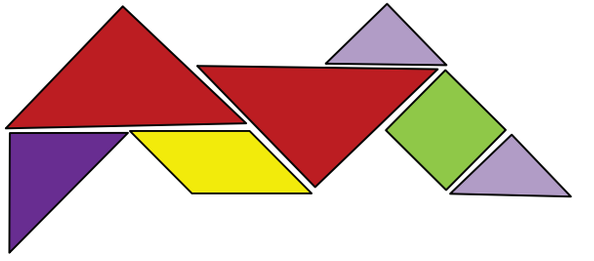
   (c)



5. Explain to someone else how to make tangram figures. What makes it easy?

Answers *(continued)*

3. (d)  (e)  (f) 

4. (a)  (b)  (c) 

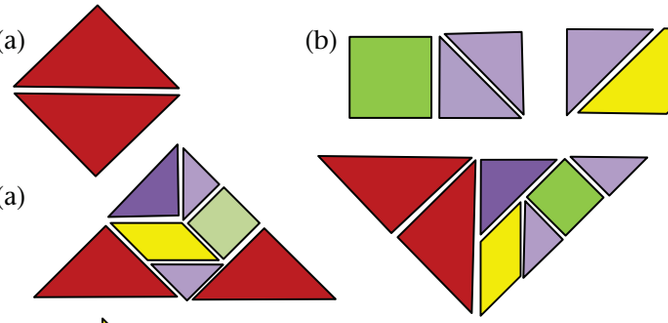
5. Learners' own explanations

Answers *(for Learner Book page 288)*

The direction in which the composite figures face does not matter. Check all learners' work, as other combinations are possible.

6.

(a) (b)

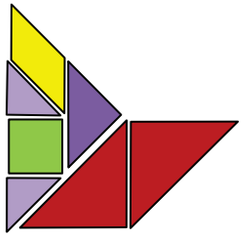


7. (a)

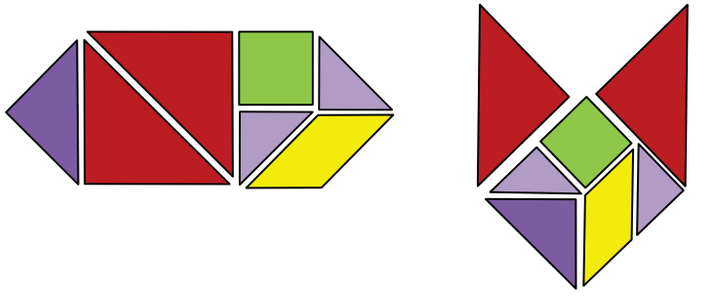
(b)



(c)



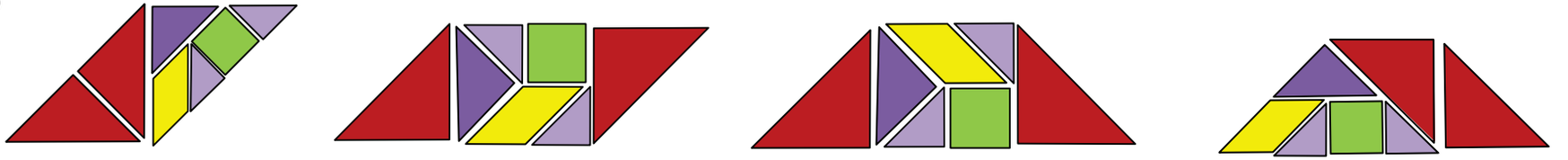
(d)



(e)



(f)



12.2 Using figures to make pictures

Mathematical notes

Puzzles in which learners join simple figures to make composite figures and pictures are called dissection puzzles. There are many, many different kinds of dissection puzzles. Only two are used in this unit.

Thousands of different figures can be built by using the seven tangram pieces. Many different pictures can also be made by the dissection puzzle used in this section.

The objectives here are the same as in the previous section, only a very different set of figures with different shapes is being used. These figures have some curved sides and some straight sides.

Resources

Each learner should make a set of the nine pieces, as shown on page 288. If you have access to a photocopier, you can photocopy the template provided in the Addendum (page 461).

Make a large version of the figures for sticking on the board for demonstration purposes.

Teaching guidelines

Aim to spend about 1 hour on this section.

Learners could work in pairs and build three of these figures. They could then demonstrate to the rest of the class how they built them. Learners can build the rest of the figures for extra practice or at home.

Notes on questions

There is a bit of a "cheat" in the fish second from the left since the tiny quarter circles forming the dorsal fin have straight edges, but are placed against the curved surface of the larger pieces making up the body of the fish. In other words, there is not a clean fit between the dorsal fins and the fish body.

Answers

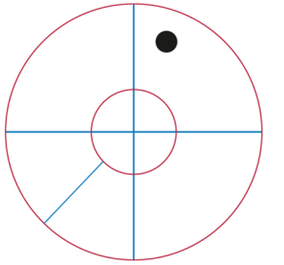
1. Learners' own work
2. Learners' own work
3. Learners' own work

6. Make drawings to show all the different ways in which
   (a) two tangram pieces can be used to make a square, and
   (b) three tangram pieces can be used to make a rectangle.

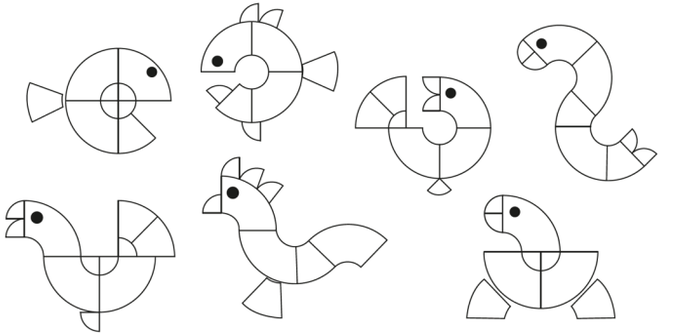
7. Make drawings to show how to make each of the following figures by using all seven tangram pieces each time:
   (a) a triangle (b) a square
   (c) a pentagon (d) a hexagon
   (e) a rectangle that is not a square
   (f) a quadrilateral that is not a rectangle

12.2 Using figures to make pictures

1. Use two round objects to draw two circles with the same centre, about three times as big as this.



2. Draw the blue lines to divide the figure into nine pieces. Draw the black dot in the right place. Then cut out the nine pieces.
3. Use the pieces to make these pictures of animals.



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Grade 4 Term 4 Unit 1 Whole numbers

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
1.1 Refresh your knowledge of numbers	Counting, ordering and comparing numbers in the thousands number range	291
1.2 Numbers on number lines	Identifying numbers on number lines in the thousands number range	292

CAPS time allocation	1 hour
CAPS page references	13 to 14, 67 and 106

Mathematical background

While providing opportunities to develop understanding of whole numbers, questions 1, 2 and 3 in Section 1.1 also address the content specified in the Mental Mathematics section of the CAPS.

Counting in multiples of large numbers helps learners to develop a sense of larger numbers. Once learners have a feel for larger numbers, it is easy for them to order and compare these numbers. The work on ordering and comparing numbers in this unit is based on the work done in previous terms on developing learners' sense of numbers into the thousands, and their understanding of the structuring of these numbers.

1.1 Refresh your knowledge of numbers

Teaching guidelines

Try to spend only 30 minutes on this section. One possible way to organise the section is to use

- questions 1, 2 and 3 for mental mathematics,
- questions 5(a), (e) and (f), and 6(c), (e) and (f) for classwork, and
- questions 4, 5(b) to (d), and 6(a), (b) and (d) for additional practice.

Learners are working in the thousands number range.

In questions 2 and 3 all numbers are between 1 000 and 9 000. Help learners to see and understand that in each of the two questions there is only one example of a number in each thousands band, i.e. one number between 1 000 and 2 000, one number between 2 000 and 3 000, etc. This means that learners only have to look at the thousands value of each number in order to sequence the numbers. In questions 5(a), (c) and (f) it is important that learners look beyond the thousands value before they decide which number is bigger and which is smaller.

Possible misconceptions

If learners think of numbers as a string of separate digits, for example if they think of the numbers in question 5(e) as one-nine-nine-nine and two-oh-oh-one, they may be tempted to say that 1 999 is greater than 2 001. This is because they are not considering the value of the parts of the number. If learners make this mistake, ask them to build the numbers with their place value cards and to show the place value parts of the number, or to write the numbers in expanded notation.

Answers

1. 6 500 6 750 7 000 7 250 7 500 7 750 8 000 8 250 8 500 8 750
2. 1 628 2 006 3 769 4 123 5 599 7 309
3. 8 901 7 803 6 351 5 182 3 736 2 853
4. (a) 4 000 4 500 5 000 5 500 6 000 6 500 7 000 7 500 8 000 8 500 9 000
   (b) 1 000 2 500 4 000 5 500 7 000 8 500 10 000
5. (a) 1 492 < 1 942 (b) 3 678 < 6 873 (c) 2 892 < 2 929
   (d) 8 506 > 7 505 (e) 1 999 < 2 001 (f) 4 089 < 4 890
6. (a) 2 000 (b) 8 000 (c) 9 250
   (d) 3 750 (e) 2 900 (f) 8 985

UNIT 1 WHOLE NUMBERS

1.1 Refresh your knowledge of numbers

1. Count in 250s from 6 500 until you pass 8 500. Write down the numbers as you go along. Start like this:

   6 500 6 750 7 000 . . . . .

2. Arrange these numbers from smallest to biggest.

   5 599 3 769 2 006 7 309 4 123 1 628

3. Arrange these numbers from biggest to smallest.

   7 803 5 182 8 901 2 853 6 351 3 736

4. Write down the numbers as you go along.

   (a) Count in 500s from 4 000 until you reach 9 000.

   (b) Count in 1 500s from 1 000 until you reach 10 000.

5. In each case decide which is the bigger of the two numbers. Use the < and > signs. The open part of the sign is always towards the bigger number.

   Examples: 2 000 < 2 500 7 000 > 3 465

   (a) 1 492 and 1 942 (b) 3 678 and 6 873
   (c) 2 892 and 2 929 (d) 8 506 and 7 505
   (e) 1 999 and 2 001 (f) 4 089 and 4 890

6. Write the numbers.
   (a) 500 smaller than 2 500 (b) 500 bigger than 7 500
   (c) 150 smaller than 9 400 (d) 250 bigger than 3 500
   (e) 2 800 bigger than 100 (f) 15 smaller than 9 000

1.2 Numbers on number lines

Teaching guidelines

Try to spend only 30 minutes on this section. One possible way to organise the section is to use

- the first two number lines in question 1, the first number line in question 2, and the first number line in question 3 for classwork, and
- the third number line in question 1, the second number line in question 2, and the second number line in question 3 for additional practice.

In all the number lines in questions 1 and 2 there are 10 unnumbered intervals between every two numbered intervals. In the first number line in question 1, the numbered intervals increase in 10s. In the other number lines in questions 1 and 2, the numbered intervals increase by different amounts, such as 100, 1 000, 20 and 50. Here learners need to know that the value of each unnumbered interval is $\frac{1}{10}$ of the value of the numbered interval. For example, in the first number line in question 2, each unnumbered interval has a value of 20 ÷ 10 = 2; in the second number line in question 2, each unnumbered interval has a value of 50 ÷ 10 = 5.

Possible misconceptions

Learners may assume that all unnumbered intervals have a value of 1 unit.

Notes on questions

In question 3, the subdivisions into units are not provided. Learners need to *estimate* these distances. They need to imagine 10 divisions, and at which tenth the arrow is positioned. Accept all answers reasonably close to the answers provided for question 3 below.

Answers

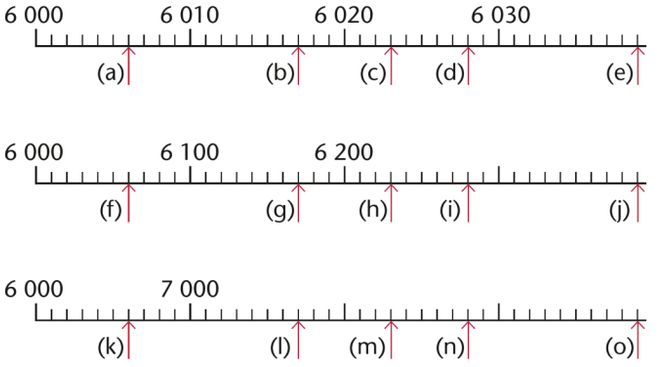
1. (a) 6 006 (b) 6 017 (c) 6 023 (d) 6 028 (e) 6 039
   (f) 6 060 (g) 6 170 (h) 6 230 (i) 6 280 (j) 6 390
   (k) 6 600 (l) 7 700 (m) 8 300 (n) 8 800 (o) 9 900

2. (a) 6 012 (b) 6 034 (c) 6 046 (d) 6 056 (e) 6 078
   (f) 6 030 (g) 6 085 (h) 6 115 (i) 6 140 (j) 6 195

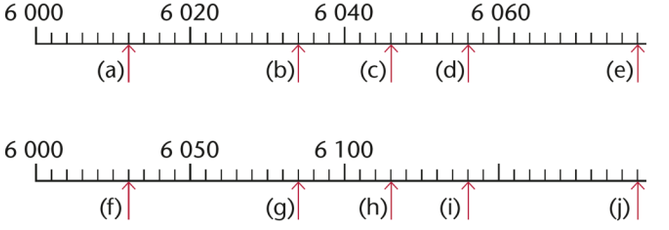
3. Accept all answers reasonably close to the answers provided below.
   (a) 3 504 (b) 3 513 (c) 3 518 (d) 3 529 (e) 3 532
   (f) 3 503 (g) 3 511 (h) 3 523 (i) 3 526 (j) 3 538

1.2 Numbers on number lines

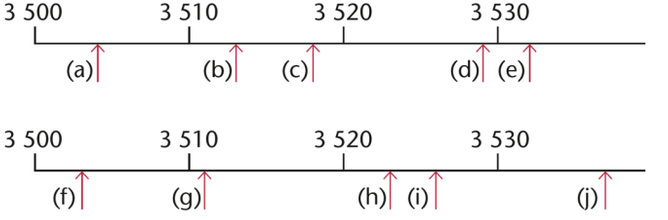
1. Write the numbers shown by the arrows.



2. Write the numbers shown by the arrows.



3. Write the numbers shown by the arrows. Estimate as well as you can.



Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 Practice	Revision of addition and subtraction including solving problems	293 to 294
2.2 Increases, decreases and differences	Solving problems about capacity and volume	295 to 296

CAPS time allocation	4 hours
CAPS page references	14 to 15, 69 to 71 and 107

Mathematical background

Learners practise addition and subtraction bonds for multiples of ten and multiples of hundred. They need to know these number facts, or how to reconstruct them quickly, if they are to successfully use the recommended methods of addition and subtraction.

Learners also estimate answers.

Learners solve a variety of problems. They can choose any method for adding and subtracting.

2.1 Practice

Teaching guidelines

Aim to cover this section in 2 hours. One possibility is to use

- question 1 for mental mathematics,
- questions 10 and 13 for concept development,
- questions 2, 3, 4, 5, 6, 7 and 12 for classwork, and
- questions 8, 9 and 11 for additional practice.

In question 1(a) learners can change the order of the numbers to add pairs of numbers that make 100, i.e. 80 + 20 and 30 + 70, and then add the rest of the numbers.

Remind learners always to think: "*What have I done before that can help me here?*" Ask them: "*What is the same about questions 1(a) and (b)?", "What is the same about questions 1(c), (d), (e), (f) and (g)?"* Learners should be able to use their answers to 1(a) and (c) to vastly reduce the amount of calculating they are required to do.

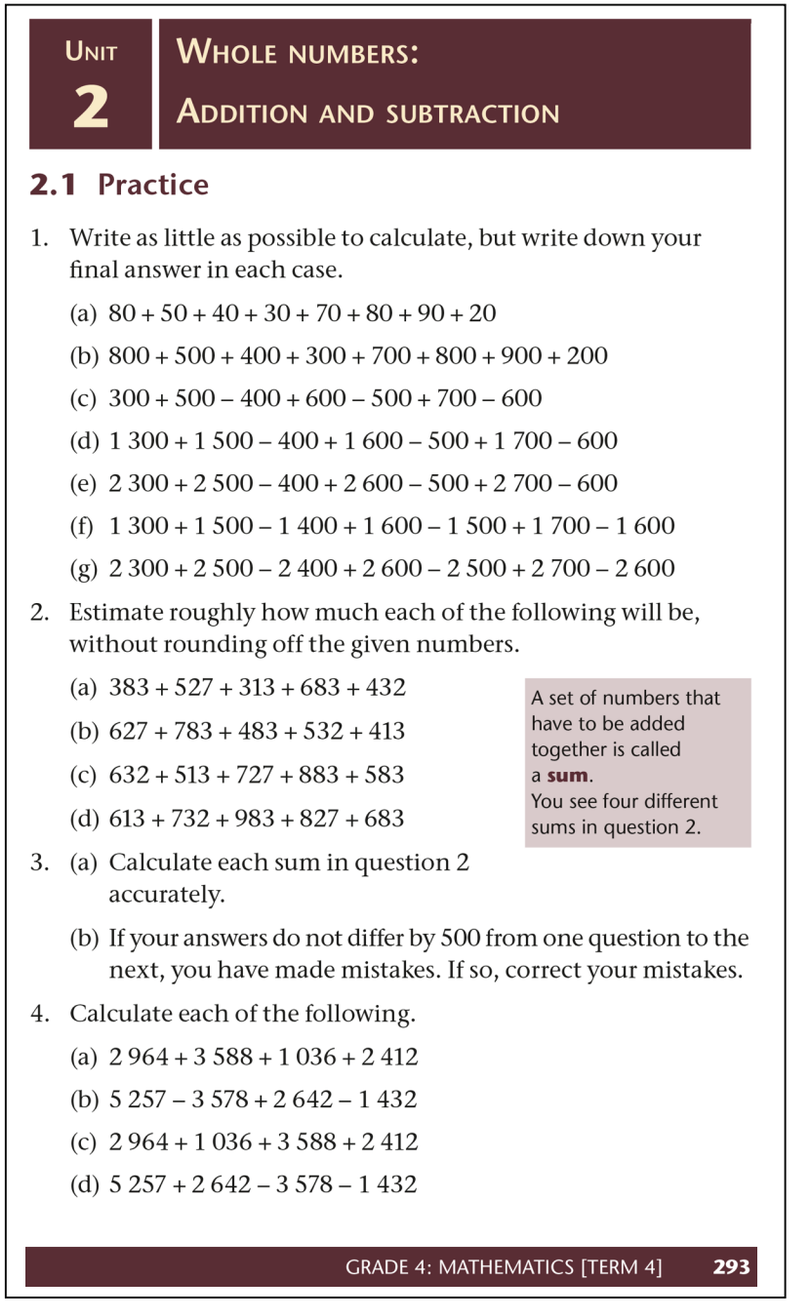
Ask learners to estimate the answers to questions 4(a) and (c). Then ask them: "*What is the same about questions 4(a) and (c)?", "What is the same about questions 4(b) and (d)?"* Learners can use their answers to questions 4(a) and (b) in questions 4(c) and (d).

Aim to complete questions 1 to 7 in the first hour and questions 8 to 13 in the second hour.

When you use questions 10 and 13 for concept development, it does *not* mean writing the calculations on the board as worked examples. Rather read the questions out loud and ask learners to explain the question in their own words. Ask questions such as: "*What does this question ask you to do?", "What useful information have you been given?", "What will you do now?"* Let learners do the actual calculations.

Answers

1. (a) 460 (b) 4 600 (c) 600 (d) 4 600
   (e) 8 600 (f) 1 600 (g) 2 600
2. Learners' estimates will differ.
3. (a) 2 338 (b) 2 838 (c) 3 338 (d) 3 838
4. (a) 10 000 (b) 2 889 (c) 10 000 (d) 2 889



UNIT 2 WHOLE NUMBERS: ADDITION AND SUBTRACTION

2.1 Practice

1. Write as little as possible to calculate, but write down your final answer in each case.
   (a) 80 + 50 + 40 + 30 + 70 + 80 + 90 + 20
   (b) 800 + 500 + 400 + 300 + 700 + 800 + 900 + 200
   (c) 300 + 500 – 400 + 600 – 500 + 700 – 600
   (d) 1 300 + 1 500 – 400 + 1 600 – 500 + 1 700 – 600
   (e) 2 300 + 2 500 – 400 + 2 600 – 500 + 2 700 – 600
   (f) 1 300 + 1 500 – 1 400 + 1 600 – 1 500 + 1 700 – 1 600
   (g) 2 300 + 2 500 – 2 400 + 2 600 – 2 500 + 2 700 – 2 600
2. Estimate roughly how much each of the following will be, without rounding off the given numbers.
   (a) 383 + 527 + 313 + 683 + 432
   (b) 627 + 783 + 483 + 532 + 413
   (c) 632 + 513 + 727 + 883 + 583
   (d) 613 + 732 + 983 + 827 + 683

   A set of numbers that have to be added together is called a **sum**. You see four different sums in question 2.
3. (a) Calculate each sum in question 2 accurately.
   (b) If your answers do not differ by 500 from one question to the next, you have made mistakes. If so, correct your mistakes.
4. Calculate each of the following.
   (a) 2 964 + 3 588 + 1 036 + 2 412
   (b) 5 257 – 3 578 + 2 642 – 1 432
   (c) 2 964 + 1 036 + 3 588 + 2 412
   (d) 5 257 + 2 642 – 3 578 – 1 432

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Possible misconceptions

In questions 8 and 9 learners subtract the smaller amount from the larger amount. They might assume that they should do this in question 10 as well. However, here the context and structure of the problem is different. If about 1 500 elephants have already died and there are about 6 500 elephants left, this means that there were more elephants to begin with. Learners need to *add* the two amounts to find out how many elephants there were before the disease killed some of them.

Notes on questions

In question 13, check that learners notice that Mr Nhlapo has three children in high school but four children in primary school.

Answers

5. Learners check their answers and make corrections if necessary.

6. (a) 4 000 + 6 000 = 10 000
   (b) 7 899 – 2 146 = 5 753
   (c) 7 899 – 5 010 = 2 889

7. Learners will respond using their own language, but they should notice that the answer to question 6(a) is the same as the answers to questions 4(a) and (c). (This is because if you are only doing addition you can change the order in which you add the numbers. It does not affect the answer.) The answer to question 6(c) is the same as the answers to questions 4(b) and (d).

8. 1 289 km

9. R3 023

10. 8 345 elephants

11. 1 696 voters

12. By 381 votes

13. R3 858 + R3 496 = R7 354

5. Your answers for questions 4(a) and 4(c) should be the same, and your answers for questions 4(b) and 4(d) should be the same. If this is not the case, you have made mistakes.

   Go back to what you have written when you did the calculations and find your mistakes. Correct them.

6. Do the calculations inside the brackets first when you do these calculations.

   (a) (2 964 + 1 036) + (3 588 + 2 412)

   (b) (5 257 + 2 642) – (3 578 – 1 432)

   (c) (5 257 + 2 642) – (3 578 + 1 432)

7. Compare your answers for question 6 with your correct answers for question 4. Write down what you notice.

8. A cyclist has covered 1 456 km of a 2 745 km journey. How far does he still have to go?

9. Jerry paid off some of his study debt. He now owes R6 735 of an amount of R9 758. How much did he pay off?

10. 1 768 elephants in a game reserve died of disease, and now there are 6 577 elephants left. How many elephants were there before the disease started?

11. There are 9 283 registered voters in a certain area. 7 587 votes were cast during a municipal election. How many registered voters did not cast their votes?

12. Candidate A received 8 843 votes during an election, and Candidate B received 9 224 votes. By how many votes did Candidate B win?

13. Mr Nhlapo has to pay R1 286 school fees for each of his three children in high school. He also has to pay R874 for each of his four children in primary school. How much is that in total?

2.2 Increases, decreases and differences

Mathematical notes

This section provides practice in addition and subtraction in the context of volume and capacity. Questions 1, 2 and 3 involve calculating in millilitres. Questions 4, 5 and 6 involve calculating in litres.

Teaching guidelines

Aim to cover this section in 2 hours. One possibility is to allocate 1 hour to questions 1 to 3. Let learners work through the questions, share their answers and then explain and discuss how they got the answers. Then spend the second hour on questions 4 to 6.

Learners can work through questions 1, 2 and 3 on their own.

It is best for learners to complete question 5 before completing question 4(e).

Question 5 involves doing a lot of very similar subtraction calculations. To save time and ensure that learners complete this question, you can divide the class into six groups. Each group can calculate the total water outflow (water lost) for one tank:

- Groups 1 and 3 can calculate the outflow for Tank A.
- Groups 2 and 5 can calculate the outflow for Tank B.
- Groups 3 and 6 can calculate the outflow for Tank C.

Each group can write their answers on the board. All learners can then individually calculate the total outflow from each tank and from all three tanks together (question 4(e)). Templates of the three tables for question 5 are provided in the Addendum (page 462). If you have access to a photocopier, you can photocopy them for learners to save time.

Note, however, that there is another way to calculate the answer to question 4(e). You can calculate the total amount of water in Tanks A to C at 10:00 and subtract the total amount of water in Tanks A to C at 15:00 from it:

$(6\,392 + 8\,654 + 7\,358) - (2\,530 + 5\,052 + 3\,333) = 22\,404 - 10\,915 = 11\,489\ \ell$

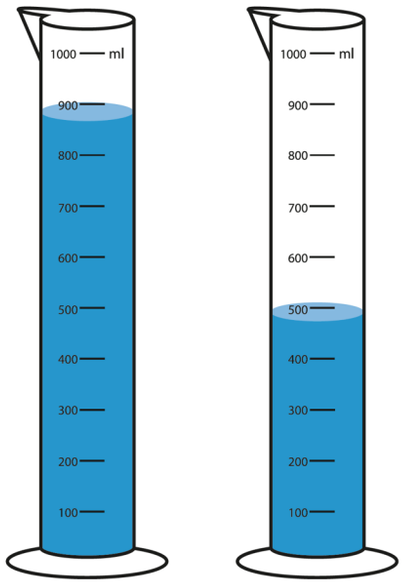
Answers

1. 581 ml
2. 132 ml
3. (a) 396 ml
   (b) 1 340 ml
   (c) 716 ml

2.2 Increases, decreases and differences

1. There is 868 ml of water in the measuring cylinder on the left. If Merrill pours 287 ml water out of the cylinder, how much water is left?

2. If there is 868 ml of water in the cylinder on the left, how much more water is needed to fill it up to the 1 000 ml mark?



Increase means by how much an amount or quantity has become more.
Decrease means by how much an amount or quantity has become less.
Difference means how much more one amount or quantity is than another.

3. The water level in the cylinder on the right is 472 ml.
   (a) How much less water is there in this cylinder than in the cylinder on the left?
   (b) How much water is there in the two cylinders together?
   (c) What will the volume of water in the cylinder on the right be if 244 ml is added?

Notes on questions

Question 6 is a challenging question. Learners need to estimate or calculate the average outflow from each tank for each hour. For example, for Tank A learners could estimate that the average outflow per hour is a bit less than 800 ℓ. They then need to work out how long the water will last at the average rate of outflow. There are 2 530 ℓ left in Tank A at 15:00. 3 × 800 litres = 2 400 litres, so Tank A will be empty in about 3 to 4 hours after 3 p.m., i.e. between 18:00 and 19:00.

Answers

4. (a) 6 611 ℓ
   (b) 5 743 ℓ – 4 163 ℓ = 1 580 ℓ
   (c) Tank A lost 782 ℓ Tank B lost 676 ℓ Tank C lost 828 ℓ
   Tank C lost the most water.
   (d) Tank A lost 778 ℓ Tank B lost 811 ℓ Tank C lost 830 ℓ
   Tank C lost the most water.
   (e) 11 489 ℓ

5. Tank A

Period of time	10:00 to 11:00	11:00 to 12:00	12:00 to 13:00	13:00 to 14:00	14:00 to 15:00
Water lost (in ℓ)	765	782	773	764	778

Tank B

Period of time	10:00 to 11:00	11:00 to 12:00	12:00 to 13:00	13:00 to 14:00	14:00 to 15:00
Water lost (in ℓ)	663	676	704	748	811

Tank C

Period of time	10:00 to 11:00	11:00 to 12:00	12:00 to 13:00	13:00 to 14:00	14:00 to 15:00
Water lost (in ℓ)	787	828	801	779	830

6. Tank A: Between 18:00 and 19:00
   Tank B: At approximately 22:00
   Tank C: At approximately 19:00

Mr Naidoo has three 10 000-litre water tanks in his vegetable garden. While watering his plants one day, he measures the water levels in the tanks each hour. His readings in litres are shown in the tables below.

Tank A

Time of the day	10:00	11:00	12:00	13:00	14:00	15:00
Water level (in ℓ)	6 392	5 627	4 845	4 072	3 308	2 530

Tank B

Time of the day	10:00	11:00	12:00	13:00	14:00	15:00
Water level (in ℓ)	8 654	7 991	7 315	6 611	5 863	5 052

Tank C

Time of the day	10:00	11:00	12:00	13:00	14:00	15:00
Water level (in ℓ)	7 358	6 571	5 743	4 942	4 163	3 333

4. (a) How much water was there in Tank B at 13:00?
   (b) How much water flowed out of Tank C between 12:00 and 14:00?
   (c) Which tank lost the most water in the period 11:00 to 12:00?
   (d) Which tank lost the most water in the period 14:00 to 15:00?
   (e) How much water, in total, flowed out of the three tanks between 10:00 and 15:00?

5. Complete a table like this for each of the three tanks.

Period of time	10:00 – 11:00	11:00 – 12:00	12:00 – 13:00	13:00 – 14:00	14:00 – 15:00
Water lost (in ℓ)					

6. Look at the numbers in the tables that you completed in question 5. If Mr Naidoo continues to water his plants for the next 24 hours, at what times do you think the different tanks will be empty?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Heavy and light	When comparing different substances, mass is not necessarily directly proportional to volume	297 to 298
3.2 Using a balance to compare mass	Using a balance to compare the mass of objects	298 to 299
3.3 Estimating and measuring mass in kilograms	Developing a feel for how heavy 1 kg is	299 to 300
3.4 Reading bathroom scales	Reading bathroom scales and understanding that we need to choose scales with appropriate units when measuring mass	300 to 301
3.5 Estimating and calculating in grams and kilograms	Estimating and calculating in grams and kilograms, including converting between units	302 to 303
3.6 Measuring in grams and kilograms	Reading kitchen scales in grams and kilograms	303 to 304
3.7 Solving mass problems	Solving problems using mass as a context	304 to 305

CAPS time allocation	6 hours
CAPS page references	26 and 108 to 110

Mathematical background

Learners go through four stages when learning to measure:

1. Identifying and understanding the property they are measuring.
2. Comparing and ordering examples of a particular measure.

   Learners hold one of two objects in each hand to compare their relative mass (see question 2 in Section 3.1).
3. Using informal or non-standard units to measure.

   When measuring with non-standard units we normally count the number of non-standard units used. Learners can use a balance (see Section 3.2) and a range of non-standard units such as blocks, bottle tops, peach pips (seeds), stones, nails, etc. They can count how many of a unit balances the mass of their objects. Eventually learners will realise that comparing measures done with non-standard units can be problematic.
4. Using formal or standard units to measure.

   A difficulty with standard units is that the instruments are often complex to read (see Sections 3.4 to 3.6).

Resources

This topic involves a number of practical activities for which various resources are needed. These resources are listed in the various sections.

3.1 Heavy and light

Mathematical notes

Length, area, capacity and volume are called spatial or geometric measures. We are often able to see these properties of an object: we can often see how long something is, how much area it covers, how much space it takes up.

Mass is not a spatial measure. It is called a physical measure. Mass – the heaviness or lightness of something – is a measure of the amount of matter in an object. We cannot always see how heavy something is. Weight and mass are different measures. Weight is the force on an object due to gravity. In everyday language we often use the word weight when we mean mass.

Young learners should first develop a sound sense of mass, as a property of an object, before they begin to read scales or to do calculations with mass. In this unit learners compare objects and describe them as heavier or lighter.

Possible misconceptions

Young learners often assume that the bigger something is, the heavier it must be. This is sometimes true, for example a big apple will be heavier than a small apple. If we compare the mass of the same kinds of things, then more of that kind of matter has more mass. This is not always true if we compare different objects or substances, for example a large ball of cotton wool might be lighter than a small apple. This is the focus of Section 3.1.

Resources

A roll of toilet paper, a pillow, a 5 kg packet of mealie meal, a brick, a loaf of bread, a rolled up pair of socks, a stone (that is smaller than the ball of socks), a 450 g tin of jam, waste paper, plastic shopping bags.

Teaching guidelines

Ask learners to talk about what they see in the top picture. Use question 1 to guide the discussion. You could let learners act out the scene by giving them one packet full of light things and another with a few heavy things in it.

For question 2, let learners work physically with their school bags when they order them. Discourage learners from just looking at the bags to decide on how to order them according to mass. Allow learners to compare masses by holding one of a pair of objects in each hand as they decide which object in each pair has the greater mass.

Answers

1. The packet in his right hand is heavier. You can see that he is leaning to the right.
2. There is a range of possible explanations. These include comparing how heavy the school bags are by holding one in each hand, until all the bags have been compared.
3. 6 empty bottles

UNIT 3 MASS

3.1 Heavy and light



1. Which packet is heavier: the packet in Tebogo's left hand or the packet in Tebogo's right hand?

   Explain your answer.

2. Work with five classmates. Order your school bags from lightest to heaviest.

   Explain what you did to order the bags.

3. Which is easier to carry: six empty bottles or six full bottles?

We say: The **mass** of six full bottles is greater than the mass of six empty bottles. The mass of six empty bottles is less than the mass of six full bottles.

4. Look at the pictures below and on the next page. Compare each pair of objects: A and B; C and D; E and F; G and H.

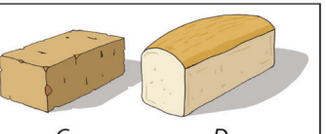
   (a) Which object is bigger?

   (b) Which object has the greater mass?



A *A pillow*

B *5 kg mealie meal*



C *A brick*

D *A loaf of bread*

Answers (*continued*)

4. (a)
   The pillow (A) is bigger.
   The loaf of bread (D) is bigger.
   The pair of socks (E) is bigger.
   The toilet paper (H) is bigger.

   (b)
   The packet of mealie meal (B) has the greater mass.
   The brick (C) has the greater mass
   The stone (F) has the greater mass.
   The tin of jam (G) has the greater mass.

5. The Mathematics textbook is heavier. Once learners have made the balance in Section 3.2 they can check their answer.

6. (a) The Mathematics textbook has a mass of about 630 g. Their exercise book will be lighter than this.
   (b) Learners' answers will differ. Almost all furniture will be heavier. Their school bags may be heavier. A litre or even 750 ml of water or juice will have a greater mass.
   (c) Learners' answers will differ. Almost all stationery will have less mass than their Mathematics textbooks. Most fruit will have less mass.

3.2 Using a balance to compare mass

Teaching guidelines

You can find out how to make better balances with coat hangers on page 192 of COUNT. (Rev. ed.) (2013). *Guidelines for Teaching Numeracy in the Foundation Phase*. Johannesburg. This is freely available on the following websites:

http://www.saide.org.za/resources/Reports/Count.pdf

http://www.saide.org.za/resource/guidelines-teaching-numeracy-foundation-phase

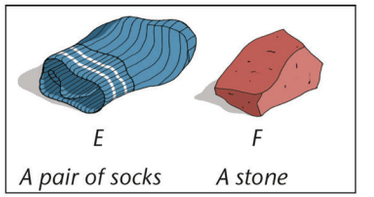
Make a balance for each pair of learners. Let them do this activity practically in pairs.

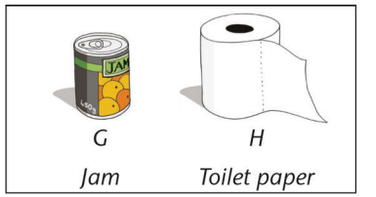
Notes on questions

In questions 1 and 2 learners' answers will differ depending on the kind and size of the objects that they use. In question 2 the kind of bottle top used will also impact on the answers. The differences in learners' answers may help to motivate the need for both standard units of measurement and standardised scales.

Answers

1. (a) Learners' answers will differ depending on the size of their erasers and pencils.
   (b) Learners' answers will differ depending on the type of rulers and sock they use.
   (c) Three rulers will probably be heavier than 20 bottle tops. You can also ask learners how many extra bottle tops they will need to balance the rulers.





5. Crumple waste paper into balls. Fill a plastic bag with these crumpled paper balls. Hold your Mathematics textbook in one hand and the bag of paper balls in the other. Which is heavier?

6. (a) Which has less mass: your Mathematics exercise book or your Mathematics textbook?
   (b) Find three things with a mass that is greater than the mass of your Mathematics textbook.
   (c) Find three things with a mass that is less than the mass of your Mathematics textbook.

3.2 Using a balance to compare mass

We often decide which of two objects is heavier just by estimating how difficult it is to lift the objects.

We can also compare mass by using a **balance**.

You can make your own balance using a clothes hanger and two shopping bags.



1. Use your balance to find which has less mass:
   (a) your eraser or your pencil (b) 3 rulers or a sock
   (c) 3 rulers or 20 bottle tops

Answers

2. Learners' answers will differ as they have different types of rulers, sharpeners and erasers.
3. (a) Probably not (b) Probably not
   (c) Probably not (d) Maybe or maybe not

3.3 Estimating and measuring mass in kilograms

Mathematical notes

For the rest of this unit learners work with standard units. In this section learners develop a feel for how heavy 1 kg is. They use a litre bottle of water as their unit.

Possible misconceptions

A litre of water does not always have a mass of 1 kg. The mass of 1 ℓ of pure water at a temperature of 4 °C at sea level is 1 kg. The bottle also has mass, so this unit (a litre bottle of water) will be a little more than 1 kg. Although all measurements are approximate, they are close enough for learners to get a sense of how heavy 1 kg is. Because their unit is not exactly 1 kg, it is important to always say "about 1 kg".

Resources

Ask each learner to bring a 1 ℓ plastic bottle filled with water (if possible) to class.
You will also need some sand and a few cups (see question 3).

Teaching guidelines

In question 1 learners can first judge the mass by holding the litre of water in one hand and the other object in the other hand. Then they can work in pairs and use the balances they made in Section 3.2.

Answers

1. Learners' answers will differ, as they are estimating.
2. (a) Heavier than 1 kg:
   (a) the heaviest book in the class (c) a desk
   (d) a chair (e) your school bag

   (b) Lighter than 1 kg:
   (b) a shoe (f) a pencil

2. Use your balance to find out how many bottle tops have the same mass as:
   (a) your ruler (b) your sharpener (c) your eraser
   Order your ruler, sharpener and eraser from lightest to heaviest.
3. Compare your answers in questions 1 and 2 with the answers of some of your classmates.
   (a) Did you get the same answers?
   (b) Is everyone's pencil the same size?
   (c) Did everyone use the same kind of rulers?
   (d) Did everyone use the same kind of bottle tops?

If we want to measure how heavy something is, we need a **unit**. The unit you used in question 2 was bottle tops.

If you used different kinds of bottle tops than your classmates, you cannot compare your measurements because your units were different.

3.3 Estimating and measuring mass in kilograms

When mass measurements need to be accurate, we have to agree to compare the mass of all objects to the standard unit. The standard for mass measurement is **one kilogram (kg)**.

The mass of 1 ℓ of water is about 1 kg.

1. Fill a 1 ℓ plastic bottle with water and use it to estimate the mass of the following objects:
   (a) the heaviest book in the class (b) a shoe
   (c) a desk (d) a chair
   (e) your school bag (f) a pencil

Answers

2. See previous page.
3. (a) The shoe is probably lighter than 1 kg.
   (b) Number of cups of sand will differ depending on the type of sand and the size of the cup, but answers may be between 2 and 3 cups.
   (c) 30 rulers are lighter than 1 kg (unless very heavy wooden rulers are used).

3.4 Reading bathroom scales

Mathematical notes

Measurement provides a very useful context for understanding estimation and rounding off. Questions that ask "is it closer to ..." help learners to understand rounding off.

Analogue bathroom scales are usually numbered every 10 kilograms and usually have 10 unnumbered spaces between the numbered lines. Learners find these relatively easy to read, as they are similar to the way rulers are marked.

Resources

Many learners do not have bathroom scales at home. Try to bring at least one analogue bathroom scale to class. Also try to bring the objects shown in question 3 (roll of tape, hammer and bottle of soap).

Teaching guidelines

Learners should first measure the mass of several objects heavier than 1 kg on a bathroom scale. *If they measure their own mass, try to ensure that this does not become known to the whole class. When learners find out each other's mass, it can lead to bullying.* Once learners have worked with a bathroom scale, they can answer the questions in the book.

Answers

1. (a) kilogram (b) 1 kg
   (c) They mostly go up to 120 kg; some stop at 100 kg and some go up to 150 kg.
2. See next page.

Possible misconceptions

All objects have mass. There is nothing that has no mass. When a scale shows a mass of zero it simply means that scale is not sensitive enough to show the mass – the units on the scale are too large to register the mass. The unit of measurement shown on the bathroom scale, namely kilogram, is not small enough to show the mass of the objects in the pictures in question 3.

2. Think again about your estimates in question 1.
   (a) Which of the objects are heavier than 1 kg? Say how much heavier you estimate them to be.
   (b) Which objects are lighter than 1 kg? Say how much lighter you estimate them to be.
3. Place your litre bottle of water in your balance.
   (a) Check if your shoe is heavier or lighter than 1 kg.
   (b) How many cups of sand have a mass of 1 kg?
   (c) Are 30 rulers heavier or lighter than 1 kg?

3.4 Reading bathroom scales

1. (a) What unit of measurement is used on a bathroom scale?
   (b) What is the smallest mass that a bathroom scale can measure?
   (c) What is the biggest mass that a bathroom scale can measure?

A **scale** is an instrument used to measure mass.

2. Look at the pictures and answer the questions on the next page.



Teaching guidelines

In question 3 learners should estimate the mass of each object by holding the object in one hand and a litre of water in the other hand. They should then measure the mass of each of these objects on a bathroom scale.

Question 3 motivates the need for a different kind of scale, and a smaller unit (learners will use kitchen scales in Section 3.6). Learners will know that a hammer is heavier than a roll of tape, so it will be obvious to them that the objects do have mass. You may use the tinted passage on page 301 to explain to learners why the objects all look like they have no mass, and that this simply means we need another kind of scale and a smaller unit of measurement.

Answers

2. (a) The girl (on the scale on the left)
   (b) 38 kg
   (c) 34 kg
   (d) The boy's mass (on the right)
   (e) The girl's mass (on the left)

3. The masses are all less than 1 kg.
   Learners cannot meaningfully estimate the mass of the objects from the pictures.
   If learners are able to hold the real objects, they will be able to estimate their mass.

(a) Who weighs more: the girl on the scale on the left or the boy on the scale on the right?

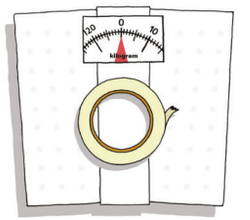
(b) What is the mass of the girl on the left?

(c) What is the mass of the boy on the right?

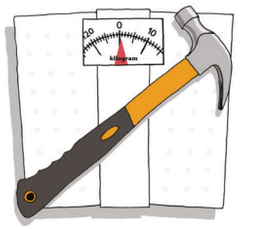
(d) Whose mass is closer to 30 kg?

(e) Whose mass is closer to 40 kg?

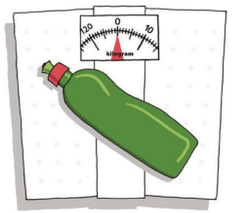
3. The bathroom scale is as accurate as the smallest mass it can measure. Estimate and write down the mass of the following objects.



(a) a roll of tape



(b) a hammer



(c) a bottle of soap

On the bathroom scale, the hammer, the roll of tape and the bottle of soap look as if they have the same mass. It looks as if their mass is nothing. Their mass is less than 1 kg. Their mass is too small to show on the bathroom scale. We need to use another instrument and another unit of measurement to find their mass.

3.5 Estimating and calculating in grams and kilograms

Mathematical notes

In the Intermediate Phase learners only work with grams and kilograms.

1 kg = 1 000 g, so 1 g = $\frac{1}{1\,000}$ kg. Learners can learn these conversion factors off by heart. However, as with everything learnt off by heart, learners will sometimes forget them and use an incorrect conversion factor. It may be better for learners to understand how the relationship between metric units works in general. See Term 2 Unit 4 Sections 4.2 and 4.4 for more about the relationship between units in the metric place value system.

Teaching guidelines

You can help learners to practically consolidate the relationship between grams and kilograms by doing the practical activity described below.

Let learners place two 500 g items, for example packets of beans, sugar, rice or salt, on one side of a balance and a 1 kg item, for example a packet of sugar, on the other side of the balance. If you have access to a kitchen or bathroom scale, learners could place two 500 g packets on the scale and see that together they make up 1 kg. This will show them that 500 g + 500 g = 1 000 g = 1 kg.

Answers

1. (a) 1 000 g (b) 2 000 g (c) 500 g (d) 250 g
2. (a) 1 kg (b) $\frac{1}{2}$ kg (c) $\frac{1}{4}$ kg (d) 3 kg
   (e) $\frac{3}{4}$ kg (f) $5\frac{1}{2}$ kg or 5,5 kg
3. (a) 4 kg and 125 g = 4 125 g
   (b) 2 350 g = 2 kg and 350 g
   (c) 250 g + 250 g → 500 g + 250 g → 750 g + 250 g → 1 000 g
   (d) 200 g + 200 g → 400 g + 200 g → 600 g + 200 g → 800 g
4. (a) 1 kg and 500 g + 250 g → 1 kg and 750 g + 250 g →
   2 kg + 0 g + 250 g → 2 kg + 250 g + 250 g →
   2 kg + 500 g + 250 g → 2 kg + 750 g + 250 g →
   3 kg
   (b) 2 kg and 800 g + 200 g → 3 kg and 0 g + 200 g →
   3 kg + 200 g + 200 g → 3 kg + 400 g + 200 g →
   3 kg + 600 g + 200 g → 3 kg and 800 g + 200 g →
   4 kg

3.5 Estimating and calculating in grams and kilograms

Many objects are lighter than 1 kg. We can measure lighter objects in grams.

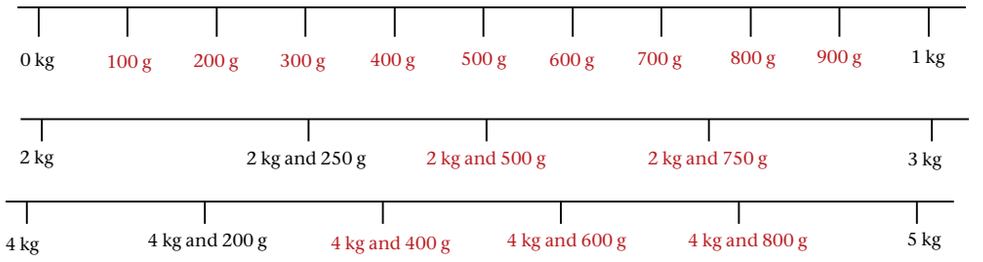
1 kg = 1 000 g

2,5 kg = $2\frac{1}{2}$ kg = 2 kg and 500 g = 2 500 g

1. Write the mass in grams.
   (a) 1 kg (b) 2 kg
   (c) $\frac{1}{2}$ kg (d) $\frac{1}{4}$ kg
2. Write the mass in kilograms or fractions of kilograms.
   (a) 1 000 g (b) 500 g
   (c) 250 g (d) 3 000 g
   (e) 750 g (f) 5 500 g
3. Fill in the missing numbers to make the sentences true.
   (a) 4 kg and 125 g = ___ g
   (b) 2 350 g = ___ kg and 350 g
   (c) 250 g + 250 g → ___ g + 250 g → ___ g + 250 g → ___ g
   (d) 200 g + 200 g → ___ g + 200 g → ___ g + 200 g → ___ g
4. Count in kilograms and grams.
   (a) 1 kg and 500 g + 250 g → 1 kg and ___ g + 250 g →
   ___ kg + ___ g + 250 g → ___ kg + ___ g + 250 g →
   ___ kg + ___ g + 250 g → ___ kg + ___ g + 250 g →
   ___ kg
   (b) 2 kg and 800 g + 200 g → ___ kg and ___ g + 200 g →
   ___ kg + ___ g + 200 g → ___ kg + ___ g + 200 g →
   ___ kg + ___ g + 200 g → ___ kg and ___ g + 200 g →
   ___ kg

Answers

5.



6. Estimates will depend on objects collected.

3.6 Measuring in grams and kilograms

Mathematical notes

Learners are familiar with reading measurements off rulers. On a ruler and most measuring tapes, there are 10 unnumbered intervals between numbered intervals. There are usually also 10 unnumbered intervals between the numbered intervals on a bathroom scale. This is not always the case on a kitchen scale.

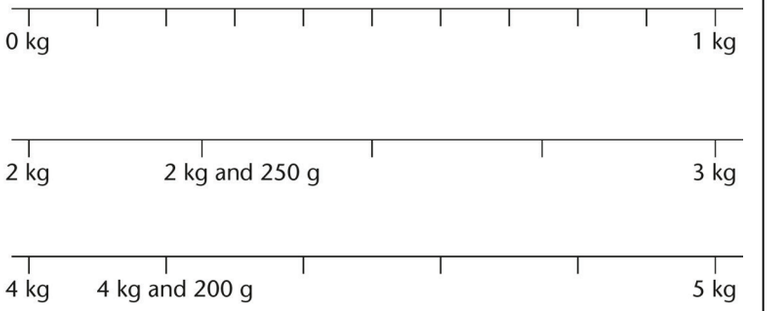
Possible misconceptions

Learners often assume that there are 10 unnumbered intervals between each numbered interval on a kitchen scale. Some kitchen scales have 4 or 5 unnumbered intervals between numbered intervals: this is the case with the examples given on page 304. Teach learners to check how many unnumbered intervals there are between each numbered interval.

Resources

Many learners do not have kitchen scales at home. Try to bring at least one analogue kitchen scale to class, preferably one that does not have 10 unnumbered intervals between each numbered interval. Let learners measure a range of items using the scale. You can use the objects shown in Section 3.4, question 3.

5. Copy the number lines below. Fill in the grams, or kilograms and grams, on your number lines.



We can hardly feel the mass of 1 g. It is very difficult to estimate the mass of very light objects by holding them in your hand.

6. Collect objects that have a small mass that would be measured in grams, for example a pencil, a paper clip, an eraser, a sheet of paper or a coin.
Estimate the mass of each in grams.

If we work with estimates of measurements, our answers must always say "*about* so much". We say this is the **approximate** measurement.

3.6 Measuring in grams and kilograms

We can use kitchen scales to measure in grams and kilograms.

Rulers and bathroom scales have 10 spaces between each numbered line.

Some kitchen scales have 10 spaces between each numbered line. Other kitchen scales have fewer spaces between each numbered line.

Teaching guidelines

In Section 3.5, question 4 (skip counting in 250 g and 200 g intervals) and question 5 (filling in the unnumbered intervals on a number line) are designed to help learners with reading scales.

Learners can think about the scale (i.e. the kg and g markings) on kitchen scales as number lines. They can use the following steps to find out the mass at the dial/needle/pointer.

- **To find the value of the unnumbered intervals**
  Count the number of unnumbered intervals (not lines) between numbered intervals.
  In example 2 (the scale with the potatoes) there are 5 unnumbered intervals.
  Divide this number into the value of the numbered intervals.
  In example 2, this is 1 kg ÷ 5 = $\frac{1}{5}$ kg, or 1 000 g ÷ 5 = 200 g.

- **To find the value of the unnumbered interval at the pointer**
  Count on (or back) in the intervals calculated above.
  In example 2, this is 4 kg, $4\frac{1}{5}$ kg, $4\frac{2}{5}$ kg, $4\frac{3}{5}$ kg or
  4 kg, 4 kg and 200 g, 4 kg and 400 g, 4 kg and 600 g.

Answers

1. $2\frac{1}{4}$ kg or 2 kg and 250 g
2. $4\frac{3}{5}$ kg or 4 kg and 600 g

3.7 Solving mass problems

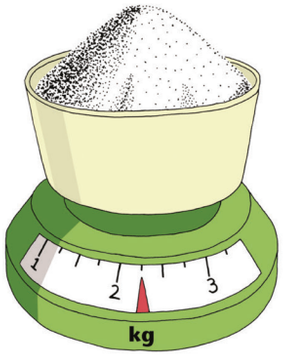
Teaching guidelines

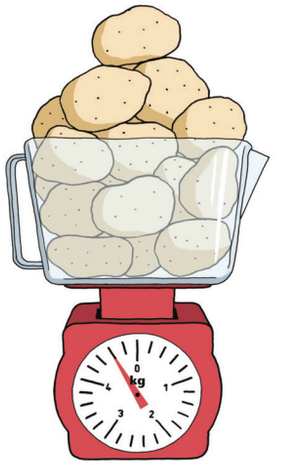
Questions 1(a) and (b) are difference models of subtraction. If learners struggle to understand what to do, you can let them hold a light object in one hand and a heavy object in the other hand. Ask them how they could find the difference in their masses. You can suggest that they use a number line to estimate the difference.

Answers

1. (a) 824 g – 126 g = 698 g (b) 126 g – 69 g = 57 g
   (c) 20 boxes of matches: 160 g
   1 000 ÷ 160 = 6 $\frac{1}{4}$ packs
   (d) 227 g × 6 = 1 362 g = 1 kg and 362 g

To work out the values in grams at the unnumbered lines, you first count the spaces and divide this number into 1 000. Use your answer to count on or back from the nearest numbered line.





1. What is the mass of the sugar?
2. What is the mass of the potatoes?

3.7 Solving mass problems

1. (a) A hammer weighs 824 g and a roll of tape weighs 126 g. How much heavier is the hammer than the tape?

   If an answer is bigger than 1 000 g, give the answer in kilograms and grams.

   (b) A bottle of salt weighs 69 g and a small bottle of soap weighs 126 g. How much lighter is the bottle of salt than the bottle of soap?

   (c) The mass of a box of matches is 8 g. What is the mass of a pack of 20 boxes of matches? How many packs have a mass of 1 kg?

   (d) The mass of a bar of soap is 227 g. What is the mass of a pack that has 6 bars of soap?

Answers

1. (e) 4 × 318 g = 1 272 g = 1 kg and 272 g
   (f) 1 000 ÷ 200 = 5 blocks
2. Yes, because 500 g = $\frac{1}{2}$ kg
3. (a) 2 packets (2 × $2\frac{1}{2}$ kg = 5 kg)
   (b) 5 packets (5 × 500 g = 2 kg and 500 g = $2\frac{1}{2}$ kg)
   (c) 10 packets (10 × $2\frac{1}{2}$ kg = 25 kg)
   (d) 20 packets (20 × 250 g = 5 000 g = 5 kg)
   (e) 10 packets (10 × 100 g = 1 000 g = 1 kg)
4. (a) 5 g (1 000 ml has a mass of 1 000 g, so 5 ml has a mass of 5 g)
   (b) 50 litres has a mass of 50 kg. 50 kg + 4,5 kg = 54,5 kg, or $54\frac{1}{2}$ kg

(e) The mass of a box with 6 eggs is 318 g. What is the mass of 24 eggs packed in the same way?

(f) How many 200 g blocks can you cut from a 1 kg brick of margarine?

2. Zolani has 2 kg and 500 g of sugar. Can Zolani say that she has $2\frac{1}{2}$ kg of sugar? Explain your answer.

3. (a) How many $2\frac{1}{2}$ kg packets are there in a 5 kg box of milk powder?
   (b) How many 500 g packets can you fill with $2\frac{1}{2}$ kg of sugar?
   (c) How many $2\frac{1}{2}$ kg bags can you fill with 25 kg of flour?
   (d) How many 250 g packets can you make from 5 kg of dog food?
   (e) How many 100 g packets can you make from 1 kg of washing powder?

4. The mass of 1 ℓ of water is about 1 kg.
   (a) Estimate the mass of a teaspoonful (5 ml) of water.
   (b) Estimate the mass of a drumful of water. The drum takes about 50 ℓ of water. The drum itself weighs about 4,5 kg.

4,5 kg = $4\frac{1}{2}$ kg

Grade 4 Term 4 Unit 4 Properties of three-dimensional objects

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Making objects from cut-out 2-D shapes	Focusing on the two-dimensional faces of three-dimensional objects	306 to 308
4.2 Identifying the shapes of objects	Seeing geometrical three-dimensional objects in everyday objects	308 to 309

CAPS time allocation	4 hours
CAPS page references	22, 78 to 79 and 111

Mathematical background

This unit builds on the ideas in Term 2 Unit 6: *Properties of three-dimensional objects*, in which three-dimensional objects and the properties of their faces were discussed. It also builds on Term 3 Unit 5: *Viewing objects*, where the idea of viewing a three-dimensional object from different positions was explored. There is a strong focus on the shapes of the faces of three-dimensional objects. This helps learners to identify and classify three-dimensional objects.

All three-dimensional objects with flat faces can be built from sheets of paper or cardboard (an idea also introduced in Term 2 Unit 6). Many three-dimensional objects with curved surfaces (for example cones and cylinders) can also be built from paper or cardboard. In this unit learners will build three-dimensional objects from paper or cardboard. This focuses their attention on the features or properties of the objects; in particular the faces of the objects. Once they have built the objects, they can use them as learning aids. It is easier for learners to learn about the properties of objects if they can refer to real physical examples. It is more difficult to learn from written texts and/or pictures.

Resources

Four sheets of stiff paper or cardboard (can be scrap cardboard) for each learner.

Scissors; glue; sticky tape.

Models of geometric objects or examples of everyday objects in the shape of spheres, cones, rectangular prisms, triangular prisms, cylinders and triangular pyramids, for example balls, oranges, marbles, boxes, bricks, cardboard tubes, cylindrical piping, cans, caps of some spray bottles, glue sticks, shoe polish tins, snuff or mentholated ointment tins, etc.

4.1 Making objects from cut-out 2-D shapes

Notes on questions

In question 1 learners need to imagine how they could use the 2-D shapes to make the faces of a 3-D object. This requires a good knowledge of rectangular prisms and square-based pyramids. It also requires sound visualising skills. This question is the most abstract and most difficult question in the unit. You could leave it until after learners have built the objects.

In questions 2, 3, 4 and 5 learners have to redraw the faces, cut them out and stick them together. If you have access to a photocopier you can copy the templates in the Addendum (pages 463 to 466). If possible, copy them onto cardboard.

These questions are more practical and help learners to focus on and develop a better knowledge of the flat faces and curved surfaces of three-dimensional geometric objects.

Teaching guidelines

Aim to spend up to 3 hours on this section.

You might like to start by helping learners to revise the three-dimensional geometric objects they learnt about in Term 2: spheres, cones, rectangular prisms, triangular pyramids, cylinders. Let learners have examples of some of these objects on their desks. Ask them to find the objects in turn, and to explain how they know it is the required object.

Answers

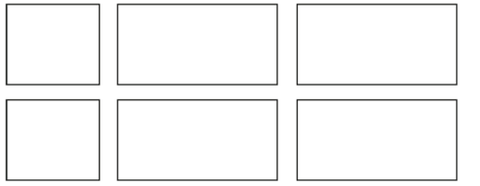
1. (a) Rectangular prism
   (b) Square-based pyramid
2. Learners make their own models of rectangular prisms.

UNIT 4 PROPERTIES OF THREE-DIMENSIONAL OBJECTS

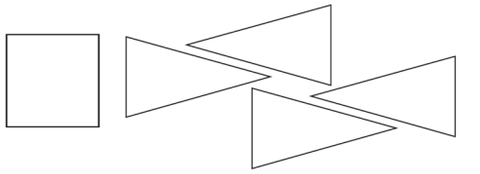
4.1 Making objects from cut-out 2-D shapes

1. Name the 3-D objects that have faces as shown in each case below.

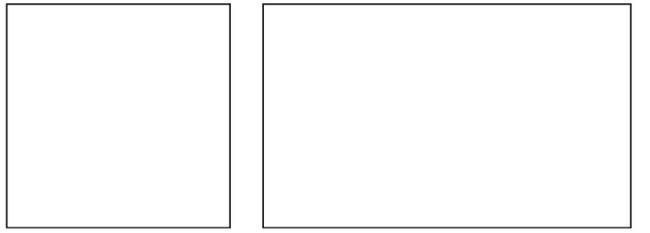
   (a)



   (b)



2. Cut out two small and four large rectangles like those shown below, from stiff paper or cardboard. Use sticky tape or clay to join them to form a rectangular prism.



306 UNIT 4: PROPERTIES OF THREE-DIMENSIONAL OBJECTS

Teaching guidelines

Assist learners at the start of each activity by asking them questions like: "*What three-dimensional objects did we learn about in Term 2?", "Could you put these shapes together to form a perfect sphere?", "How do you know?", "What is a face of a geometric object?", "Could you use only these faces to make a cylinder?", "Why or why not?", "Could you use only these faces to make a pyramid?"*

You can also refer learners to the picture of the yellow, blue and green faces of the rectangular prisms on page 174.

Let learners work in pairs (it is easier for one learner to hold the shapes in place and the other learner to stick them together). If you do not have access to a photocopier, learners can trace the stipulated number of each shape on pages 306 to 308. They can then stick their shapes onto card. Ask learners to draw a different face (e.g. ☺, 😐, ☹, etc.) on each of the shapes that will be flat surfaces on the 3-D objects. This will help them to remember that we call the flat surfaces *faces*.

Learners may find it challenging to put the faces together to form the objects. As you walk around while they are making their constructions, you may need to ask them questions to prompt them.

Once each pair of learners has built all the objects, ask them to name each object, and talk about the kinds of surfaces and the shapes of the faces on each object.

Possible misconceptions

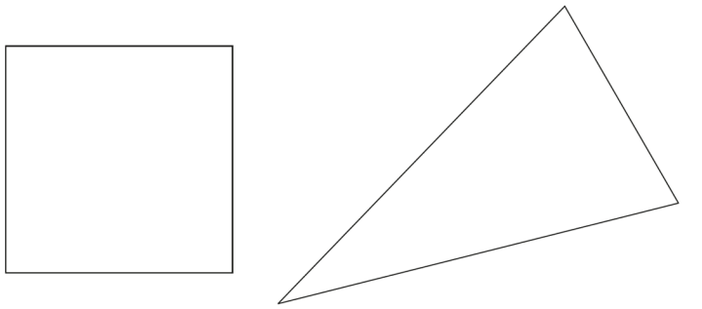
Some learners may miss the point of the "thinking" tasks in question 1. Be sure to help them understand that they need to *imagine* cutting out the flat figures and joining them along their sides to form three-dimensional objects. *What objects will result?*

Learners who struggle with this and are not completely successful *must* be engaged in a discussion about this after they have drawn, cut out and joined the actual faces. Without going back, they will not learn as much as they could have. You may choose to give some (or all) learners a second set of similar thinking tasks to reinforce what they have learnt. You could model this on question 1 on page 306, or question 4 on page 180.

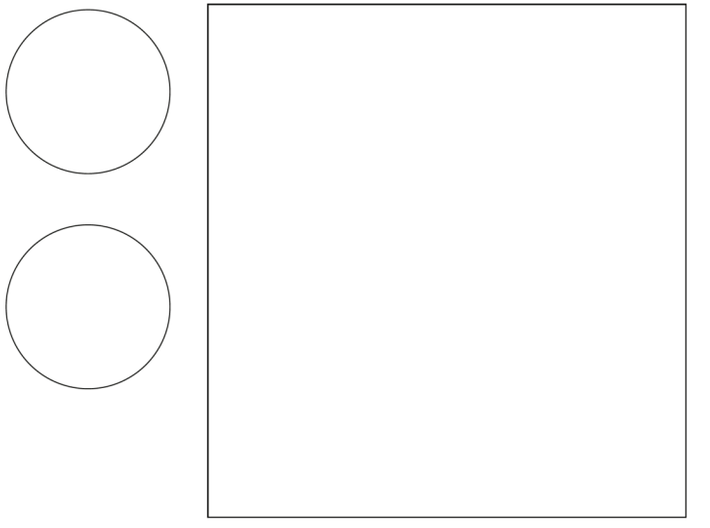
Answers

3. Learners make their own square-based pyramids.
4. Learners make their own cylinders.

3. Cut out one square and four triangles like those shown below, from stiff paper or cardboard. Use sticky tape or clay to join them to form a square-based pyramid.



4. Trace these figures on a loose sheet. Cut the figures out. Roll the rectangle into a tube. Join the tube and the two circles with tape or clay to make a cylinder.



Answers

5. Learners make their own cones.

4.2 Identifying the shapes of objects

Mathematical notes

As was the case in Term 2 Unit 6, the purpose here is to reinforce the fact that the simple shapes of three-dimensional objects that we study in Mathematics can be seen in everyday objects. The ability to see the mathematics that is embedded in the world around us is a very important skill and lies at the heart of any successful application of mathematics.

Teaching guidelines

Aim to spend 1 hour on this section.

In the previous section learners built and worked with real objects. Here they apply what they have learnt and interpret drawings of three-dimensional objects. This is more abstract. Let learners work directly from the textbook and write their answers in their exercise books. This will prepare them for answering questions in tests and exams.

Notes on questions

Question 1 is about the three-dimensional objects and the names of the shapes of their faces (flat surfaces). Although it is difficult to see what kind of pyramid is shown in question 1(d), learners have only learnt about square-based pyramids in Grades 3 and 4. Therefore assume it is a square-based pyramid.

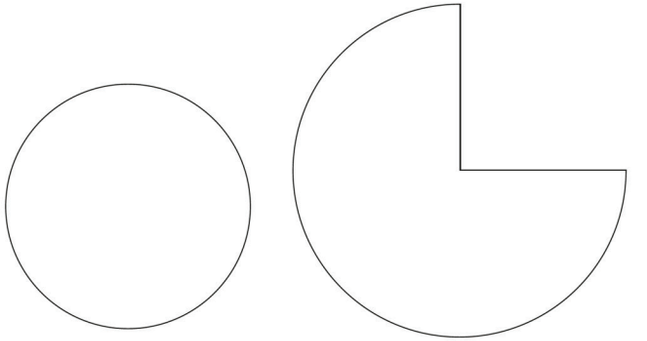
In question 2(b) learners should ignore the handle of the cup.

Answers

1.

	Name of 3-D objects	Shape(s) of their faces (flat surfaces)
(a)	cylinder	circles
(b)	cone	circle
(c)	rectangular prism	rectangles
(d)	pyramid	triangles and square

5. Trace these figures on a loose sheet. Cut the figures out. Roll the figure on the right into an open cone. Join the cone and the circle with tape or clay to make a closed cone.



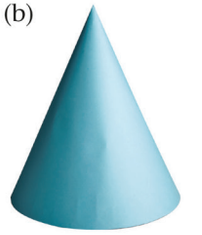
4.2 Identifying the shapes of objects

1. Name the 3-D objects that you see below, and write the names of the 2-D shapes of the faces of each object.

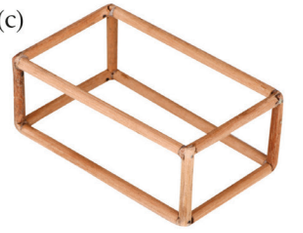
(a)



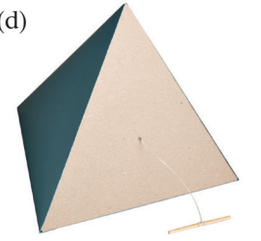
(b)



(c)



(d)



Mathematical notes

A square is a special kind of rectangle because it satisfies all the conditions of a rectangle, and in addition all of its sides are the same length. Similarly, a cube is a special kind of rectangular prism in which all the faces are squares.

Notes on questions

Question 3 requires learners to be able to read with understanding a lot of terminology about three-dimensional geometric objects. You may choose to read the questions to learners and use the opportunity to consolidate the language.

Question 3 requires learners to match particular holes in the box with particular faces on some of the objects. You may need to explain this to learners. It is important that learners name the objects each time rather than saying "this object" or "that one".

Learners were exposed to triangular prisms on page 174. Note that while they can work with these objects is it *not* expected that they know them for assessment purposes.

As mentioned above, a cube is a kind of prism (a special kind of rectangular prism).

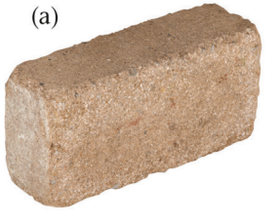
Answers

2. (a) Rectangular prism
   (b) Cylinder (if the handle is ignored)
   (c) Cylinder and cone
   (d) Cone

3. (a) Cube
   (b) Cube
   (c) Cylinder
   (d) Triangular prism

2. Name the 3-D objects that you can see in these pictures.

(a)



(b)



(c)



(d)



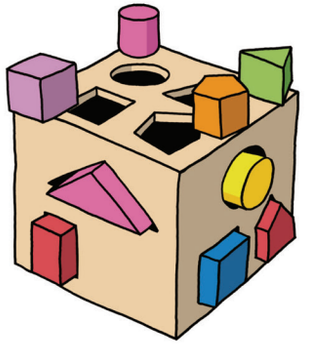
3. Zinzi's uncle makes wooden toys. As you can see, the toy below consists of a number of different 3-D objects.

   (a) What kind of prism is the box into which the small objects are put?

   (b) What kind of prism can go through the square hole at the top of the box?

   (c) What do we call the curved object that can go through the circular hole?

   (d) What do we call the object that goes through the triangular hole?



GRADE 4: MATHEMATICS [TERM 4] **309**

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 Grouping and sharing into fraction parts	Fractions of collections and fractions on a number line	310 to 311
5.2 Problem solving with fractions	Solving problems with fractions. Fractions on a number line	312

CAPS time allocation	5 hours
CAPS page references	16 and 112

Mathematical background

A fraction is a **number** of equal parts of the same object or measurement unit, for example "7 hundredths of a metre".

Fractions can be used in different ways:

- To describe a part of a whole, for example 3 eighths of a cake.
- To describe **parts of collections**, for example 3 eighths of the learners in a school or 63 hundredths of the available marks. In a case like the latter, the percentage notation (% for hundredths) is normally used.
- To describe units of measurement smaller than the standard (reference) unit, for example a centimetre ("centi" means "hundredth of").

Mathematically, the fraction concept is critical to the understanding of decimals because the place value parts after the decimal comma are fractions. The expanded notation for the number 23,47 is $20 + 3 + \frac{4}{10} + \frac{7}{100}$ or 2 tens + 3 units + 4 tenths + 7 hundredths. A good understanding of common fractions will lay a sound basis for decimal fractions when learners work with them in Grade 6. Decimal fractions are not a topic in Grade 4. Grade 4 learners only work with decimals in the context of measurement.

Learners' concept of fractions is often undermined by the idea that a fraction consists of or is made up of two numbers, for example that "3 fifths" is made up of the numbers 3 and 5, without understanding the entirely different roles of the two numbers. This misconception is often supported by misleading language, such as referring to $\frac{3}{5}$ as "three over five" instead of "3 fifths". The use of the proper fraction name, for example "3 fifths", supports understanding of what the denominator of a fraction is.

Resources

Rulers; string.

5.1 Grouping and sharing into fraction parts

Mathematical notes

This section is a revisit, on a higher level, of the relationship between fractions and division.

Notes on questions

The questions in this section focus on different kinds of fractions.

In questions 1 and 3 a single whole is divided into equal parts.

In questions 2, 4, 5 and 6 learners find fractions of collections of objects. In questions 7 to 11 fractions are represented as images of lines (string) and on number lines. In these questions fractions are either measures or numbers.

Note that although learners have worked with equivalent fractions, there is no expectation that their answers must be in the "simplest" form. Learners are not expected to know procedures for "simplifying" fractions.

To develop a sound fraction concept it is useful to work with a range of fractions. However, learners are only required to know up to eighths for assessment purposes.

Answers

1. Learners need to understand that each piece of cake is $\frac{1}{8}$. They can then simply count up the pieces of the cake to get 6 eighths ($\frac{6}{8}$).

2. (a) 8 bags (b) $\frac{1}{8}$ of the apples

3. (a) one third (b) R210 ÷ 3 = R70

4. Accept any of the following answers:
   Learners could count columns of beads to get $\frac{10}{12}$ of the beads are purple and $\frac{2}{12}$ are yellow. They could count all the beads to get $\frac{50}{60}$ of the beads are purple and $\frac{10}{60}$ are yellow. Some learners may say $\frac{5}{6}$ of the beads are purple and $\frac{1}{6}$ of the beads are yellow.

5. Purple beads: $\frac{12}{60}$ Yellow beads: $\frac{48}{60}$

6. Purple beads: $\frac{36}{60}$ Yellow beads: $\frac{24}{60}$

UNIT 5 COMMON FRACTIONS

5.1 Grouping and sharing into fraction parts

1. A cake is cut into eight equal slices. Katie eats 2 slices and Farida eats 1 slice and Ben eats 3 slices. What fraction of the cake did they eat?

2. Musi puts 40 apples in bags. He puts 5 apples into each bag.
   (a) How many bags will he fill?
   (b) What fraction of all the apples is in one bag?

3. Three children are helping old Mr Botha in his garden. He gives them R210 for helping him. The children share the money equally.
   (a) What fraction of the money does each child get?
   (b) How much money does each child get?

4. What fraction of the beads in this bead mat is purple, and what fraction is yellow?



5. What fraction of the beads in this bead mat is purple, and what fraction is yellow?



6. What fraction of the beads in this bead mat is purple, and what fraction is yellow?



Notes on questions

In questions 7 and 8 learners are told how long the piece of string is and into how many equal parts to divide it. They have to find the length of the fraction parts.

In questions 9 and 10 learners are also asked to subdivide the string, but here they are told how long to make each piece. They have to find the number of equal parts and hence the size of the fraction that each part constitutes.

In question 11 learners work with fractions as numbers on a number line. This develops their understanding of fractions as numbers.

In question 11, ask learners first to focus on the section of the number lines between 0 and 1. Then ask how many parts each line is divided into, and what fraction each part of the line is. Learners can then count up in fraction parts, for example 1 fifth, 2 fifths, 3 fifths, 4 fifths, 1.

Answers

7. 12 cm ÷ 4 = 3 cm

8. 10 cm ÷ 5 = 2 cm

9. 12 cm ÷ 2 cm = 6. There are 6 pieces of 2 cm: each piece is 1 sixth of the 12 cm string.

10. 20 cm ÷ 4 cm = 5. There are 5 pieces of 4 cm: each piece is 1 fifth of the 20 cm string.

11. (a) A = $\frac{3}{5}$ B = $\frac{4}{5}$ C = $1\frac{2}{5}$ D = $1\frac{4}{5}$

    (b) A = $\frac{1}{3}$ B = $\frac{2}{3}$ C = $1\frac{1}{3}$ D = $1\frac{2}{3}$

    (c) A = $\frac{4}{10}$ B = $\frac{9}{10}$ C = $1\frac{2}{10}$ D = $1\frac{5}{10}$ E = $1\frac{7}{10}$

    (d) A = $\frac{1}{8}$ B = $\frac{3}{8}$ C = $\frac{6}{8}$ D = $1\frac{2}{8}$ E = $1\frac{6}{8}$

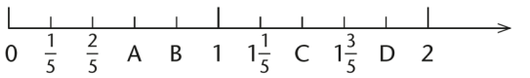
This piece of string is 12 cm long.



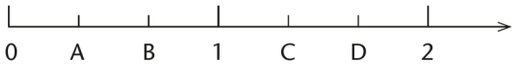
The marks show how the string can be divided into four equal parts, or quarters.

7. The piece of string is 12 cm long. It is divided into quarters. How long is each quarter?
8. A piece of string is 10 cm long. It is divided into fifths. How long is each fifth?
9. 12 cm of string is divided into equal pieces of 2 cm each. What fraction part of 12 cm is each of these pieces?
10. 20 cm of string is divided into equal pieces of 4 cm each. What fraction part of 20 cm is each of these pieces?
11. The numbers 0, 1 and 2 are shown on each of the number lines below. Write down the fractions that can replace the letters on each number line.

(a)



(b)



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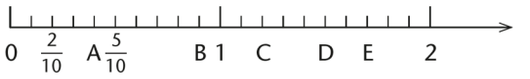
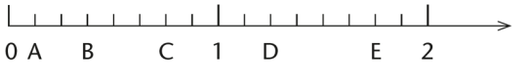


(d)



5.2 Problem solving with fractions

Teaching guidelines

Learners can use any method they like to solve the problems. Drawing pictures might help.

Grade 4 learners are *not* expected to learn off by heart any rules or procedures for finding fractions of wholes (see question 4). They can use what they know about fractions and division to find unit fractions, for example $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{10}$ of wholes. To find other fractions of wholes they simply multiply the unit fraction by the number required, for example 4 fifths = 4 × 1 fifth.

Possible misconceptions

When learners do not have a good fraction concept, they sometimes add both numerator and denominator. For example, in question 6(a) a learner may mistakenly write $\frac{1}{5} + \frac{3}{5} = \frac{4}{10}$.

To prevent this, learners could change the symbols to words and add 1 fifth plus 3 fifths to see that it gives 4 fifths. They can also draw bars of fifths, or number lines showing fifths, and show the fifths being added on these.

Notes on questions

Learners can use repeated addition for question 3. It may be easier for learners to rewrite fractions before they add them: to change from the symbol form to using a number and a word. For example, instead of $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ they can write 3 quarters + 3 quarters + 3 quarters + 3 quarters + 3 quarters. They can then count up in groups: 3, 6, 9, 12, 15 quarters.

Sometimes it is easier to split fractions to make a whole, for example $\frac{3}{4} + \frac{3}{4} = (\frac{3}{4} + \frac{1}{4}) + \frac{2}{4}$. Learners can group the fractions to make wholes and count up the remaining fraction parts.

Grade 4 learners are *not* expected to know any rules for multiplying fractions in question 4.

Answers

See next page.

5.2 Problem solving with fractions

1. 34 loaves of bread are shared equally among 8 families.
   How much bread does each family get?
2. 12 loaves of bread are equally shared by 5 families.
   How much bread does each family get?
3. Jacob bakes cakes. For one cake he needs:

   $\frac{1}{5}$ cup margarine $\frac{3}{4}$ cup sugar 1 egg

   $2\frac{1}{3}$ cups flour $\frac{1}{4}$ teaspoon salt $\frac{3}{5}$ cup milk

   $2\frac{1}{2}$ teaspoons baking powder

   How much of each ingredient does he need for 5 cakes?
4. A quarter of an hour is 15 minutes.
   How many minutes are each of the following?

   (a) $\frac{1}{5}$ of an hour (b) $\frac{3}{5}$ of an hour

   (c) $\frac{2}{3}$ of an hour (d) $2\frac{1}{2}$ hours

   (e) $\frac{1}{6}$ of an hour (f) $\frac{5}{6}$ of an hour
5. Write as single fractions.

   (a) 1 fifth + 3 fifths (b) $\frac{2}{8} + \frac{6}{8}$

   (c) 5 eighths + 7 eighths (d) $\frac{2}{7} + \frac{2}{7}$
6. How many *millilitres* are each of the following?

   (a) $\frac{1}{5}\ \ell + \frac{3}{5}\ \ell$ (b) $\frac{2}{8}\ \ell + \frac{6}{8}\ \ell$

   (c) $\frac{5}{8}\ \ell + \frac{7}{8}\ \ell$ (d) $\frac{2}{10}\ \ell + \frac{3}{10}\ \ell$

Answers

1. $8 \times 4 = 32$. So each family will get 4 whole loaves. $34 - 32 = 2$. The 2 remaining loaves are shared by 8 families (divided by 8). So each family will get another $\frac{1}{4}$ loaf. Each family therefore gets $4\frac{1}{4}$ loaves.

2. $2\frac{2}{5}$. Learners can use a similar method as in question 1, or they could use drawings.

3. *Margarine:* $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{5}{5} = 1$. He needs 1 cup of margarine.

   Learners could also count 1 fifth of a cup, 2 fifths of a cup, 3 fifths of a cup, 4 fifths of a cup, 5 fifths of a cup to get 1 cup of margarine.

   Sugar: 3 quarters + 3 quarters + 3 quarters + 3 quarters + 3 quarters = 15 quarters
   $= 3\frac{3}{4}$ cups of sugar

   Learners might also think of splitting and grouping the 3 quarters into groups of 4 quarters, i.e. 3 quarters + 3 quarters + 3 quarters + 3 quarters + 3 quarters
   = 3 quarters + (1 quarter + 1 quarter + 1 quarter) + 3 quarters + 3 quarters + 3 quarters
   = (3 quarters + 1 quarter) + (3 quarters + 1 quarter) + (3 quarters + 1 quarter) + 3 quarters
   $= 3\frac{3}{4}$ cups of sugar

   Eggs: 5 eggs

   Flour: 5 × 2 cups of flour = 10 cups of flour *and*

   5 × 1 third cup of flour = 1 third + 1 third + 1 third + 1 third + 1 third
   = 5 thirds = $1\frac{2}{3}$ cups. So, 10 cups of flour + $1\frac{2}{3}$ cups of flour = $11\frac{2}{3}$ cups of flour

   Or: $2\frac{1}{3} + 2\frac{1}{3} \rightarrow 4\frac{2}{3} + 2\frac{1}{3} \rightarrow 7 + 2\frac{1}{3} \rightarrow 9\frac{1}{3} + 2\frac{1}{3} \rightarrow 11\frac{2}{3}$ cups of flour

   Salt: $(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4})+\frac{1}{4}=1\frac{1}{4}$ teaspoon salt

   Milk: 3 fifths cup + 3 fifths cup + 3 fifths cup + 3 fifths cup + 3 fifths cup
   = 15 fifths cups = 3 cups of milk

   Baking powder: 5 × 2 teaspoons = 10 teaspoons + $5 \times \frac{1}{2}$ teaspoon = $2\frac{1}{2}$ teaspoons
   = 10 teaspoons + $2\frac{1}{2}$ teaspoons = $12\frac{1}{2}$ teaspoons baking powder

5.2 Problem solving with fractions

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2. 12 loaves of bread are equally shared by 5 families. How much bread does each family get?
3. Jacob bakes cakes. For one cake he needs:

   $\frac{1}{5}$ cup margarine | $\frac{3}{4}$ cup sugar | 1 egg
   $2\frac{1}{3}$ cups flour | $\frac{1}{4}$ teaspoon salt | $\frac{3}{5}$ cup milk
   $2\frac{1}{2}$ teaspoons baking powder

   How much of each ingredient does he need for 5 cakes?
4. A quarter of an hour is 15 minutes. How many minutes are each of the following?

   (a) $\frac{1}{5}$ of an hour
   (b) $\frac{3}{5}$ of an hour
   (c) $\frac{2}{3}$ of an hour
   (d) $2\frac{1}{2}$ hours
   (e) $\frac{1}{6}$ of an hour
   (f) $\frac{5}{6}$ of an hour

5. Write as single fractions.

   (a) 1 fifth + 3 fifths
   (b) $\frac{2}{8}+\frac{6}{8}$
   (c) 5 eighths + 7 eighths
   (d) $\frac{2}{7}+\frac{2}{7}$

6. How many *millilitres* are each of the following?

   (a) $\frac{1}{5}\ \ell + \frac{3}{5}\ \ell$
   (b) $\frac{2}{8}\ \ell + \frac{6}{8}\ \ell$
   (c) $\frac{5}{8}\ \ell + \frac{7}{8}\ \ell$
   (d) $\frac{2}{10}\ \ell + \frac{3}{10}\ \ell$

Answers *(continued)*

4. (a) 60 minutes ÷ 5 = 12 minutes
   (b) 3 × 12 minutes = 36 minutes
   (c) $\frac{1}{3}$ of an hour = 60 minutes ÷ 3 = 20 minutes, so $\frac{2}{3}$ of an hour = 2 × 20 minutes = 40 minutes
   (d) 2 hours = 120 minutes
   $\frac{1}{2}$ hour = 30 minutes
   $2\frac{1}{2}$ hours = 120 + 30 = 150 minutes
   (e) 60 minutes ÷ 6 = 10 minutes
   (f) 5 × 10 minutes = 50 minutes

5. (a) $\frac{4}{5}$ or 4 fifths
   (b) $\frac{8}{8} = 1$
   (c) $\frac{12}{8}$ or 12 eighths
   (d) $\frac{4}{7}$ or 4 sevenths

6. (a) 1 ℓ = 1 000 ml
   $\frac{1}{5}$ of 1 000 ml = 1 000 ÷ 5 = 200 ml. So, $\frac{3}{5}$ of 1 000 ml = 3 × 200 ml = 600 ml
   So, $\frac{1}{5}$ of 1 000 ml + $\frac{3}{5}$ of 1 000 ml = 200 ml + 600 ml = 800 ml
   (b) $\frac{2}{8} + \frac{6}{8} = \frac{8}{8} = 1$. So, $\frac{2}{8} + \frac{6}{8}$ of 1 000 ml = 1 000 ml
   (c) $\frac{5}{8} + \frac{7}{8} = \frac{5}{8} + \frac{3}{8} + \frac{4}{8} = 1 + \frac{4}{8} = 1\frac{1}{2}$. So, $1\frac{1}{2}$ × 1 000 ml = 1 500 ml
   (d) $\frac{1}{10}$ of 1 000 ml = 1 000 ml ÷ 10 = 100 ml
   $\frac{2}{10} + \frac{3}{10} = \frac{5}{10}$
   $\frac{5}{10}$ of 1 000 ml = 5 × $\frac{1}{10}$ of 1 000 = 5 × 100 ml = 500 ml
   Some learners might recognise that 5 tenths is equal to 1 half and say half of 1 000 ml = 500 ml.

5.2 Problem solving with fractions

1. 34 loaves of bread are shared equally among 8 families.
   How much bread does each family get?
2. 12 loaves of bread are equally shared by 5 families.
   How much bread does each family get?
3. Jacob bakes cakes. For one cake he needs:

   $\frac{1}{5}$ cup margarine | $\frac{3}{4}$ cup sugar | 1 egg
   $2\frac{1}{3}$ cups flour | $\frac{1}{4}$ teaspoon salt | $\frac{3}{5}$ cup milk
   $2\frac{1}{2}$ teaspoons baking powder

   How much of each ingredient does he need for 5 cakes?
4. A quarter of an hour is 15 minutes.
   How many minutes are each of the following?
   (a) $\frac{1}{5}$ of an hour
   (b) $\frac{3}{5}$ of an hour
   (c) $\frac{2}{3}$ of an hour
   (d) $2\frac{1}{2}$ hours
   (e) $\frac{1}{6}$ of an hour
   (f) $\frac{5}{6}$ of an hour
5. Write as single fractions.
   (a) 1 fifth + 3 fifths
   (b) $\frac{2}{8} + \frac{6}{8}$
   (c) 5 eighths + 7 eighths
   (d) $\frac{2}{7} + \frac{2}{7}$
6. How many *millilitres* are each of the following?
   (a) $\frac{1}{5}$ ℓ + $\frac{3}{5}$ ℓ
   (b) $\frac{2}{8}$ ℓ + $\frac{6}{8}$ ℓ
   (c) $\frac{5}{8}$ ℓ + $\frac{7}{8}$ ℓ
   (d) $\frac{2}{10}$ ℓ + $\frac{3}{10}$ ℓ

Grade 4 Term 4 Unit 6 Whole numbers: Division

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
6.1 Multiply so that you can divide	Using multiplication to divide	313 and 314
6.2 Equal parts in different situations	Different situations that represent division	315 and 316
6.3 Practice	Revision and consolidation of Grade 4 division	317

CAPS time allocation	3 hours
CAPS page references	13 to 15, 84 to 85 and 113

Mathematical background

Division is applicable to three different kinds of situations:

- Situations in which a quantity is shared (divided) into a **given number of parts** of unknown equal size, like situation B on page 315. Thus, a situation in which the number of equal parts is known but the size of each part is unknown. Situations like these are called **sharing** situations.
- Situations in which a quantity is shared (divided) into an unknown number of **parts of given equal size**, like situation A on page 315. Thus, a situation in which the number of equal parts is unknown but the size of each part is known. Situations like these are called **grouping** situations.
- Scaling situations where two quantities of the same kind are compared in terms of their **ratio**, not the difference between the two quantities. Situation C on page 316 is an example of a **ratio situation** that requires division.

To perform division, for example to calculate 906 ÷ 3, it is useful to ask what the divisor must be multiplied with to produce the given total: 3 × ? = 906. Stated differently, the fact that multiplication is the inverse of division can be utilised to perform division. In an example like 906 ÷ 3 = 302, 906 is called the dividend, 3 is called the divisor, and 302 is called the quotient.

6.1 Multiply so that you can divide

Teaching guidelines

Remind learners to think about what they already know that can help them to do any calculation or solve any problem. This includes using known multiplication facts, or multiplication facts that can easily be derived, to solve division calculations.

It also includes seeing whether what they do in one question can help them with another question. Learners should use the answers in question 1 to get the answers in question 2. In question 2 learners only need to multiply 3 × 80 (that is just 3 × 8 × 10; they already worked out 3 × 8 in 1(g)), and 3 × 6 (they worked out 3 × 60 in 1(d)). For the rest, they only need to use the answers from question 1 and add.

Try to move quickly through this section: aim to cover it in 1 hour. One possibility is to use

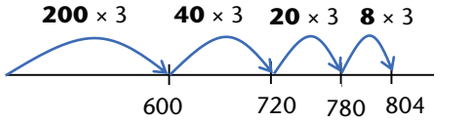
- questions 3(a) and (b), and 6(a) and (b) for concept development,
- questions 1, 2, 3(c), (d) and (h), 5 and 6(c) and (d) for classwork, and
- questions 3(e) to (g), 4 and 6(e) and (f) for additional practice.

Remind learners how to make clue boards of easy multiples. Numbers that are easy to multiply by are 1, 2 and 10 (zero is also an easy number to multiply by, but it does not help with division). Learners can first write out 1×, and 10×. By repeated doubling they can find 2× and 20×; 4× and 40×; 8× and 80×. They can find 5× by halving 10×. They can use these values and an empty number line to calculate the answer.

Learners can use this method for questions 3 and 6. Below we have done this for 3(a).

1 × 3 = 3	10 × 3 = 30	100 × 3 = 300
2 × 3 = 6	20 × 3 = 60	200 × 3 = 600
4 × 3 = 12	40 × 3 = 120	400 × 3 = 1 200
8 × 3 = 24	80 × 3 = 240	
5 × 3 = 15	50 × 3 = 150	

Using this for question 3(a):



804 ÷ 3 = 200 + 40 + 20 + 8 = 268

Multiplying to check: 200 × 3 + 60 × 3 + 8 × 3 = 600 + 180 + 24 = 804

Answers

1. (a) 600 (b) 900 (c) 150 (d) 180
   (e) 270 (f) 210 (g) 24 (h) 12

2. (a) 600 + 180 + 24 = 804 (b) 300 + 240 + 12 = 552
   (c) 300 + 210 +18 = 528

UNIT 6

WHOLE NUMBERS: DIVISION

Multiplication can be used to work out the answers for division questions like the following.

How many sweets will each child get if 120 sweets are shared equally between 5 children?

If you know that 5 × 24 = 120, you know that each child will get 24 sweets and that no sweets will be left over.

We can also say that 120 is divided into 5 equal parts.
We write this as 120 ÷ 5 = 24.

Multiplication can also be used to answer division questions like the following:

How many children can each get 5 sweets from a packet of 180 sweets?

If you know that 36 × 5 = 180, you know that 36 children can each get 5 sweets.

We can also say that 180 is divided into 36 equal parts of 5 each.
We write this as 180 ÷ 5 = 36.

6.1 Multiply so that you can divide

1. How much is each of the following?
   (a) 3 × 200 (b) 3 × 300 (c) 3 × 50 (d) 3 × 60
   (e) 3 × 90 (f) 3 × 70 (g) 3 × 8 (h) 3 × 4

2. How much is each of the following?
   Do the calculations in brackets first.
   (a) (3 × 200) + (3 × 60) + (3 × 8)
   (b) (3 × 100) + (3 × 80) + (3 × 4)
   (c) (3 × 100) + (3 × 70) + (3 × 6)

Your answers for questions 1 and 2 can serve as a **clue board** when you do question 3.

Teaching guidelines

1 × 3 = 3	10 × 3 = 30	100 × 3 = 300
2 × 3 = 6	20 × 3 = 60	200 × 3 = 600
4 × 3 = 12	40 × 3 = 120	400 × 3 = 1 200
8 × 3 = 24	80 × 3 = 240	
5 × 3 = 15	50 × 3 = 150	

Learners should always estimate answers before calculating. They can use the clue board to do this. For example, in question 3(a) you can ask learners whether the answer will be:

- more or less than 200. They can see from the clue board that it will be more than 200.
- more or less than 400. They can see from the clue board that it will be less than 400.
- more or less than 300. They can add 100 × 3 and 200 × 3 from the clue board to find out that the answer will be between 200 and 300.
- more or less than 250. They can add 200 × 3 and 50 × 3 from the clue board to see that the answer will be close to 250 (250 × 3 = 650) but a bit more than 250.

Encourage them to multiply to check all their answers. If they do not get what they started with as the answer, they should check their calculations to see where they made a mistake.

Show learners how they can use some previous answers, or parts of previous answers, in a current calculation. For example in 3(a): 804 ÷ 3 = 268 means that you do not have to calculate 3(b) 806 ÷ 3. You can see that 806 is 2 more than 804 and they are both divided by 3. So, 806 ÷ 3 = 268 remainder 2. Similarly, the answer for 3(g) can be read off the answer to 3(f). Question 3(e) can help you to see the answer for 4(b). In question 6 the answer to 6(b) is obtained from 6(a), 6(d) from 6(c), and 6(f) from 6(e).

Answers

3.	(a)	268	(b)	268 remainder 2	(c) 260
	(d)	208	(e)	184	(f) 176
	(g)	176 remainder 2	(h)	190	
4.	(a)	303 remainder 2	(b)	185	
	(c)	209 remainder 2	(d)	93 remainder 1	
5.	(a)	700 + 210 + 56 = 966	(b)	350 + 42 = 392	
	(c)	350 + 210 + 28 = 588	(d)	350 + 70 + 49 = 469	
6.	(a)	138	(b)	138 remainder 4	(c) 56
	(d)	57 remainder 1	(e)	67	(f) 67 remainder 6

3. How much is each of the following?
   (a) 804 ÷ 3 (b) 806 ÷ 3
   (c) 780 ÷ 3 (d) 624 ÷ 3
   (e) 552 ÷ 3 (f) 528 ÷ 3
   (g) 530 ÷ 3 (h) 570 ÷ 3

4. Calculate the following.
   (a) 911 ÷ 3 (b) 555 ÷ 3
   (c) 629 ÷ 3 (d) 280 ÷ 3

5. How much is each of the following?
   (a) (7 × 100) + (7 × 30) + (7 × 8)
   (b) (7 × 50) + (7 × 6)
   (c) (7 × 50) + (7 × 30) + (7 × 4)
   (d) (7 × 50) + (7 × 10) + (7 × 7)

6. Calculate the following.
   (a) 966 ÷ 7 (b) 970 ÷ 7
   (c) 392 ÷ 7 (d) 400 ÷ 7
   (e) 469 ÷ 7 (f) 475 ÷ 7

The mathematical statement 970 = 138 × 7 + 4 tells us that 970 ÷ 7 = 138 remainder 4.

Division is called the **inverse** of multiplication.

The mathematical statement 970 ÷ 7 = 138 remainder 4 tells us that 970 = 138 × 7 + 4.

Multiplication is called the **inverse** of division.

To **invert** means to go the opposite way.

6.2 Equal parts in different situations

Mathematical notes

There are different types of situations that can be represented by division.

In Section 6.2:

- Situation A is a **grouping problem** (the number of groups must be calculated).
- Situation B is a **sharing problem** (the size of each group must be calculated).
- Situation C is a **ratio problem** (the size of one of the parts must be calculated).
- Situations D and E are **rates problems** (the number of booklets is related to the time to print them. In D, the number of booklets printed per hour must be calculated. In E, the time to print all the booklets must be calculated).

Teaching guidelines

This is a short section. Allow about 45 minutes for it.

Read Situation A to the class. Ask them what needs to be calculated. Ask them to write a number sentence to describe the problem. Learners may write 720 ÷ 8 = □. They may also write □ × 8 = 720. Both of these are valid.

Repeat this for Situation B. Let learners read and answer question 1.

You can also read situations C, D and E on the next page and ask learners what needs to be calculated each time, and to write a number sentence to describe the problem.

Answers

1. (a) True. 90 groups of 8 litres make 720. Therefore 90 households can each get a "group" of 8 litres.
   (b) True. 720 litres are shared equally among 8 households. So each household will get 90 litres.

6.2 Equal parts in different situations

Read the story below. Think about the two situations and the questions that are asked. Try to see how they differ.

During a water shortage, drinking water is supplied to households from a water truck.

Situation A

Each household gets 8 litres of water. The water truck carries 720 litres of water. How many households can be supplied with water?

In Situation A, the question is **how many equal parts** of 8 each can be made from 720.

Situation B

720 litres of water is available to be shared equally between 8 households. How much water should each household get?

In Situation B, the question is **how big is each part** if 720 is arranged into 8 equal parts.

In Situation A, the size of each part is given, and the number of parts is unknown. Situations like these are sometimes called **grouping** situations.

In Situation B, the number of parts is given, and the size of each part is unknown. Situations like these are sometimes called **sharing** situations.

1. Now read the questions in Situation A and Situation B again. Then check whether the two statements below are true. Write good reasons for your answers.
   (a) In Situation A, 90 households can be provided with 8 litres of water each.
   (b) In Situation B, each household should get 90 litres of water.

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Answers

2. (a) True
   (b) True
   (c) True
3. All of them

Here are three more situations in which the numbers 720 and 8 appear. Read them carefully and think about the questions that are asked.

Situation C

Simon prepares drinks for a big soccer game, by adding concentrated fruit juice to water.
For every 8 litres of water, he uses 1 litre of concentrated fruit juice.
How many litres of concentrated fruit juice should he add to 720 litres of water?

Situation D

A printing machine works at the same pace all the time. It takes 8 hours to print 720 booklets on the machine.
How many booklets are printed each hour?

Situation E

720 booklets have to be printed on another machine, which prints 8 booklets each minute.
How long will it take to print all 720 of these booklets?

2. Now check whether the statements below are true. Write good reasons for your answers.
   (a) In Situation C, Simon should add 90 litres of concentrated fruit juice to 720 litres of water.
   (b) In Situation D, 90 booklets are printed each hour.
   (c) In Situation E, it will take 90 minutes to print the 720 booklets.
3. Which of Situations A to E can be described with the number sentence 720 ÷ 8 = □?

6.3 Practice

Teaching guidelines

This section revises what has been done with division up until now. Aim to complete this section in $1\frac{1}{4}$ hour. One possibility is to use

- questions 2, 3 and 7 for concept development,
- questions 1(a) to (c), 5, 6 and 8 for classwork, and
- question 1(d) to (f), 4 and 9 for additional practice.

Help learners to make sense of division firstly by estimating. For example: *"Will the answer to question 2(b) be about 100 mm? About 50 mm? About 200 mm? About 20 mm? About 1 000 mm?"* Learners can use their clue board of easy multiples to answer.

Then ask learners to use their estimate (and calculation if they have completed it) to estimate the answer to question 2(a). *"Are there more or fewer pieces in 2(a) than in 2(b)? Will the pieces be longer or shorter than in 2(b)? How much shorter? About 7 times shorter? About 4 times shorter? About 2 times shorter, in other words about half as long? How can you tell? About how long will each piece be: 200 mm? 100 mm? 1 000 mm? 50 mm? 80 mm?"*

Similarly with question 3(b): *"Will the answer be about 100 bags? About 50 bags? About 20 bags? About 25 bags?"* Ask learners whether Taro will fill more or fewer bags in question 3(a). Ask them to use the estimate from 3(b) (or the answer to 3(b) if they have worked it out) to say whether the answer to 3(a) will be about 50, about 100, about 25 or about 20.

Remind learners to check all solutions by multiplying out their answers. They should find and correct any errors.

Answers

1. (a) 82 (b) 57 (c) 93
   (d) 72 remainder 8 (e) 142 remainder 4 (f) 63
2. (a) 161 mm (b) 92 mm
3. (a) 18 bags (b) 27 bags
4. R57
5. (a) 10 kilometres (b) 15 kilometres
6. 100 full cups with a capacity of 100 ml
7. R38
8. 22 sheets of paper for each learner and 2 will remain unshared. If these two sheets are shared out each learner will get an extra $\frac{2}{9}$ of a sheet of paper.
9. 79 bags can be filled with 8 onions each.

6.3 Practice

Check all your answers by doing multiplication.

1. Calculate.
   (a) 738 ÷ 9 (b) 399 ÷ 7 (c) 744 ÷ 8
   (d) 656 ÷ 9 (e) 856 ÷ 6 (f) 378 ÷ 6
2. (a) What is the length of each piece if 644 mm of lace is cut into 4 equal pieces?
   (b) What is the length of each piece if 644 mm of lace is cut into 7 equal pieces?
3. Taro baked 108 bread rolls.
   (a) If he puts 6 bread rolls in a bag, how many bags can he fill?
   (b) How many bags can he fill with 4 bread rolls each?
4. Jamal pays R456 for 8 kg of dried fruit. What is the cost of 1 kg?
5. Willem takes nine minutes to walk one kilometre.
   (a) How far can he walk in 90 minutes if he does not get tired?
   (b) How far can he walk in 135 minutes if he does not get tired?
6. A watering can has a capacity of 10 ℓ. A small cup has a capacity of 100 ml. How many full cups will fill the watering can?
7. Naledi bought 7 tickets for a show. She paid R266 altogether for the tickets. What was the price of one ticket?
8. 200 sheets of paper must be shared equally between 9 learners. How many sheets will each learner get, and how many sheets will remain?
9. Mike sells onions in bags. He puts 8 onions in one bag. He bought 635 onions from a farmer. How many bags can he fill?

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 Perimeter, area and capacity	Distinguishing between perimeter, area and volume as different quantitative properties of objects	318 to 322
7.2 Calculate perimeter	Calculating perimeters when lengths in metric units are stated on diagrams	323
7.3 Perimeter and area of curved figures	Using a square grid to approximate the area of a curved figure, and straight lines to approximate the perimeter of circles	324 to 325
7.4 Capacity and volume	Informal measurement of capacity and volume using blocks or very small boxes	326 to 328

CAPS time allocation	7 hours
CAPS page references	28 and 114 to 115

Mathematical background

Learners go through four stages when learning to measure:

1. Identifying and understanding the property they are measuring.
   In Grade 3 learners have measured perimeters and lots of lengths. However, they have not measured volume, and only touched on measuring area. For this reason it is important that Grade 4 learners get the chance to explore measuring three-dimensional objects and to develop an understanding of how measuring perimeter is different to measuring area, which are both different to measuring volume. It is important not to skip this first stage of measuring when learners are exposed to properties that are new to them. (See pages 318 and 319 in Section 7.1.)
2. Comparing and ordering examples of a particular measure. (See Section 7.1, questions 4(a) and (b), and 6(a).)
3. Using informal or non-standard units to measure.

   Most of this unit deals with informal measurement of area and volume using non-standard units. Learners can learn all the principles of measuring using non-standard units. They can also develop a sound conceptual understanding of the property they are measuring because they do not get involved in the complexity of reading instruments calibrated in standard units, or in converting between units. When learners use non-standard measurement it always involves counting, for example counting the number of grid squares covered, or the number of blocks or small boxes used.

   > "Area and volume are only measured informally in the Intermediate Phase.
   > Learners are not required to know or apply formulae for the perimeter, area or volume of any shapes or objects." (CAPS, page 114)
4. Using formal or standard units to measure.

   Only perimeter is measured formally using metric units (see Section 7.2). This is because learners already have an understanding of measuring lengths and perimeters.

Resources

This topic involves a number of practical activities for which various resources are needed. These resources are listed in the various sections.

7.1 Perimeter, area and capacity

Mathematical notes

There are different ways to measure three-dimensional objects. We can find the distance around the shape. We can find the surface area. We can find the capacity. In Grade 4 learners work with the perimeter and area of two-dimensional shapes and the capacity/volume of three-dimensional objects.

Resources

Boxes with different shapes, preferably examples that hold similar amounts. If you give learners a matchbox, a shoebox, a box that held photocopying paper and an apple box, it will be easy to order these from biggest to smallest simply by looking at them. If, however, you give learners a large shoebox and the lid of a box that held photocopying paper, it is not so obvious which one is bigger.

Square grid paper. If you have access to a photocopier you can photocopy the square grid paper provided in the Addendum (page 433). If not, ask learners to draw square grid paper as shown on page 102.

Teaching guidelines

Aim to spend about $2\frac{1}{2}$ hours on this section.

If possible, let each group of learners have two or three boxes of a similar size but with different shapes. It should not be immediately obvious which box is the biggest and which the smallest. In the illustration on page 318 it is not immediately obvious which box is the biggest: the red box is the tallest, but it has the smallest base; the yellow box appears to have the biggest base, but it is the shallowest. Learners can work from the illustrations, but it may make more sense to them if they work with actual boxes.

Ask learners which box is the biggest. They should motivate their answers and also talk about how they would measure the size of the boxes. The important issues here are the reasons that learners give for estimating the size of the boxes, and also their suggestions for how to measure the size of the boxes. Focus discussion on these issues (see the "Teaching guidelines" on the next page).

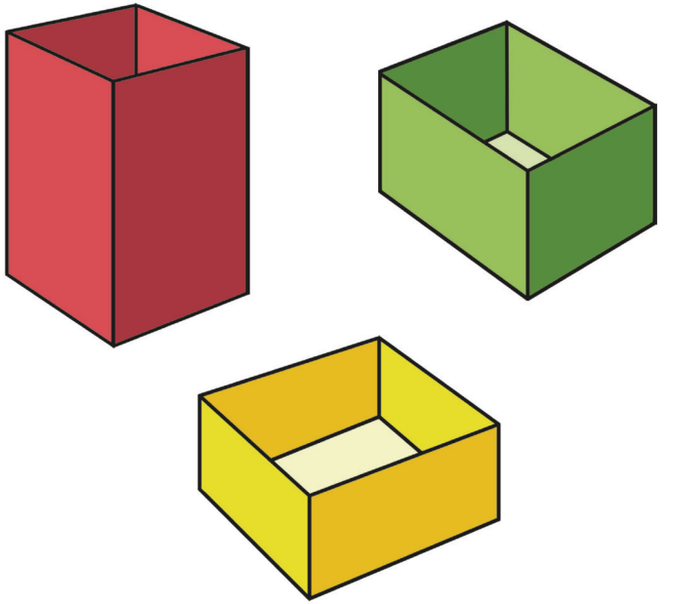
Answers

1. Learners estimate to get possible answers.

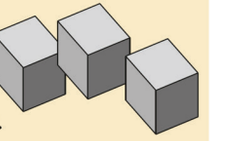
UNIT 7 PERIMETER, AREA AND VOLUME

7.1 Perimeter, area and capacity

How can we know which box is the biggest?



We can find out how many cubes like these can be packed into each box.



The maximum number of cubes that the box can hold is called the **capacity** of the box.

There are some other ways too of saying how big the boxes are.

1. How many of the grey cubes do you think can be packed into each of the three boxes?

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Teaching guidelines

From learners' discussions, focus their attention on the following measurements:

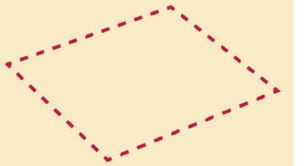
- perimeter – the distance around a shape
- area – the size of the surface
- volume – the amount of space that something takes up.

You can use the tinted passage on page 319 to guide you. You can also talk with learners about dimensions.

The rest of this section and Sections 7.2 and 7.3 focus on perimeter and area. Capacity and volume is the focus of Section 7.4.

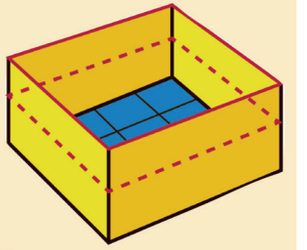
If your classroom has a tiled floor, you can ask learners to use the tiles to describe the area of the floor, for example the floor is 400 tiles big. Some schools have tiled or paved passages, or paved areas outside the classrooms. All of these could be used to describe the size of the areas. Also measure the perimeters of these spaces using builder's tape measures, measuring tapes or strides. The aim is to help learners distinguish between the distance around a shape or space, which is the perimeter, and the size of the surface, which is the area.

We can measure the total length, right around the top edge of each box. This length is called the **perimeter**.



The perimeter can be measured using a measuring tape or a ruler.

You can also put a piece of string right around the box or the edge of the box, and then measure the length of the string.

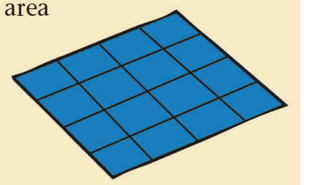


Here is another way to say how big a box is:

You may say how many square stickers or tiles are needed to cover the bottom or the "floor" of the box.

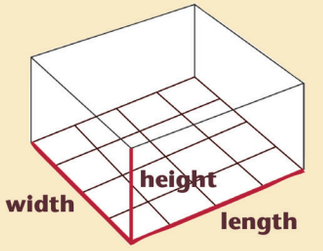
This is called the floor **area** of the box.

When people describe the size of a house or piece of land, they normally use the area rather than the perimeter.



You may also describe the size of a box by saying how wide, how long and how high it is.

The length and the width and the height are called the **dimensions** of a box or a room.



GRADE 4: MATHEMATICS [TERM 4] 319

Mathematical notes

Shapes can have the same perimeter but different areas. Shapes can also have the same area but different perimeters.

Teaching guidelines

In question 2 learners work out the area of the shapes by counting grid squares. They work out the perimeter by counting the number of sides of grid squares, or measure it with rulers.

You can start by asking learners to check whether the area of the green shape is 21 grid squares big (i.e. does it cover 21 grid squares) and whether the perimeter is 20 cm. Learners can then continue with shapes (a) and (c) to (f).

Let learners report back on their answers. Discuss how all the shapes have the same perimeter, but the areas vary from 9 grid squares big to 25 grid squares big. Be explicit about the fact that in general there is no fixed relationship between perimeters and areas of shapes.

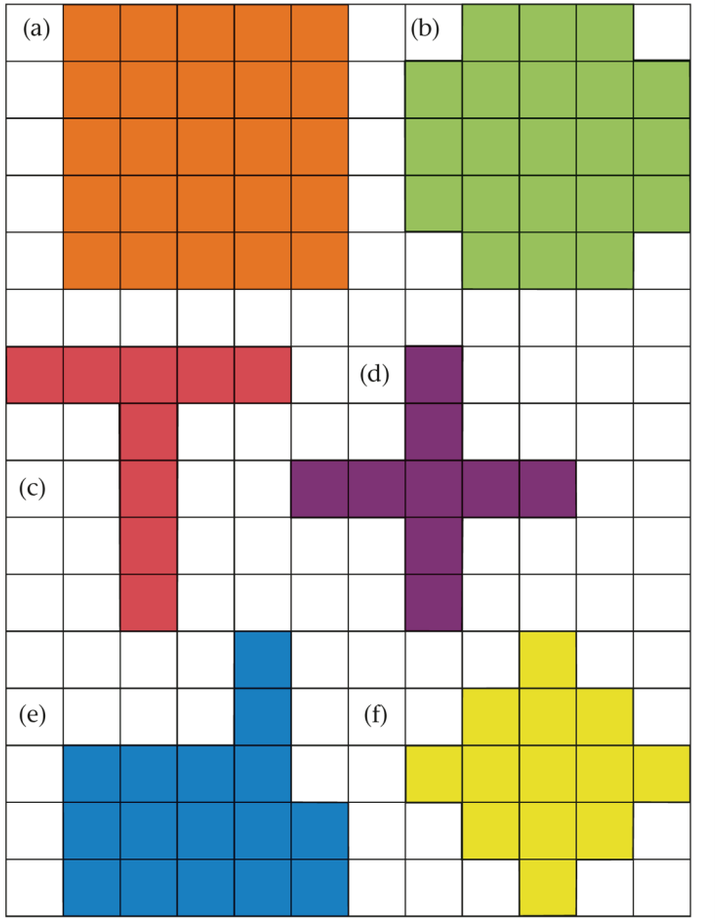
If you have access to a photocopier, you can photocopy the square grid paper provided for this unit in the Addendum (page 433). If not, ask learners to draw square grid paper as shown on page 102. Ask learners to draw three different looking shapes, all with an area of 12 grid squares. Ask learners to count the perimeter of each shape. Learners should compare shapes, check that they all have an area of 12 grid squares, and compare perimeters. This will show that shapes can have the same area but different perimeters.

Answers

2. (a) A: 25 grid squares P: 20 cm (A = Area; P = Perimeter)
   (b) A: 21 grid squares P: 20 cm
   (c) A: 9 grid squares P: 20 cm
   (d) A: 9 grid squares P: 20 cm
   (e) A: 16 grid squares P: 20 cm
   (f) A: 13 grid squares P: 20 cm

Each side of the squares in the grid below is 1 cm long.
The area of the green figure is 21 grid squares.
The perimeter of the green figure is 20 cm.

2. Find the area and perimeter of each figure.



Teaching guidelines

When learners work through the questions on this page and on page 322, it is useful for them to have both pages open. One learner can have page 321 of the Learner Book open while the learner next to him/her can have page 322 open.

Learners will find that the rectangles in 3(a) and (b) have the same size: both the perimeters and areas are the same. These two rectangles are identical: only their colours and the directions they face are different, but these aspects do not impact on size.

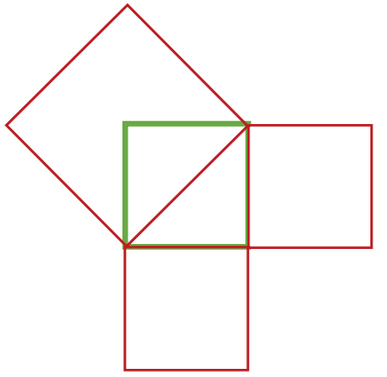
Questions 4 and 5(a) are intended to prevent the possible misconception that shapes have to be covered neatly with full squares.

Question 5 is another reminder that equal areas does not imply equal perimeters.

Possible misconceptions

You may need to remind learners that a square is a special kind of rectangle. So the square with the green edge can be called a rectangle.

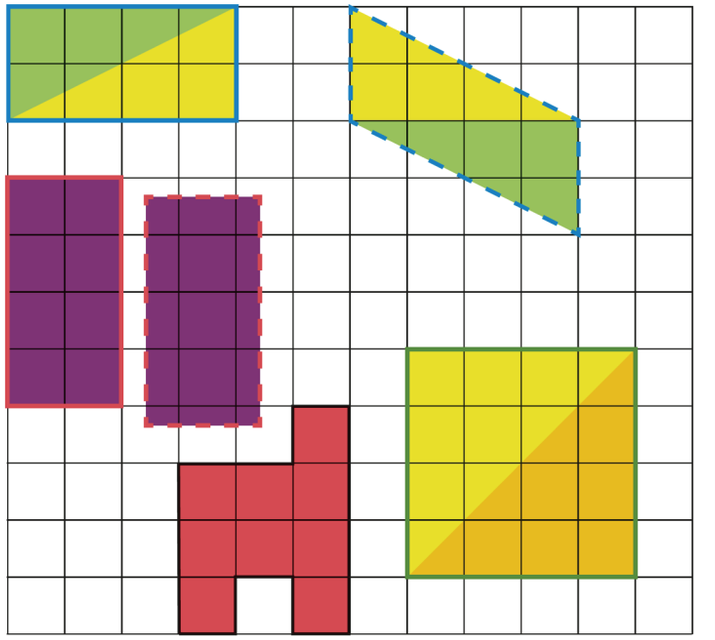
On a square grid each square is the same size. The side lengths of each square are the same size/length. The lines that connect opposite corners of a square are called diagonals. Many learners assume that the diagonals have the same length as the sides. This is not true. The diagonal of a square is always longer than the side of a square. To check, learners can measure the diagonal and the side lengths of the yellow square. If you draw a square on two sides of a square and on one of the diagonals, it shows this length difference quite clearly: see the diagram alongside.



Answers

3. (a) A: 8 grid squares P: 12 units/cm
   (b) A: 8 grid squares P: 12 units/cm
   (c) A: 16 grid squares P: 16 units/cm
4. (a) Yes
   (b) Yes
5. (a) 8 grid squares
   (b) Between 12,9 and 13 cm

3. Find the area and the perimeter of each of the following figures:
   (a) the rectangle with the blue edge
   (b) the rectangle with the solid red edge
   (c) the rectangle with the green edge



4. (a) Do you think the areas of the two purple rectangles are the same?
   (b) Do you think the areas of the red figure on this page and the red figure on the next page are the same?
5. (a) What is the area of the figure with the dotted blue edge?
   (b) Use your ruler to find the perimeter of this figure.

Mathematical notes

When learners are in Grade 7 they will use the fact that the diagonal divides the area of a rectangle in half to work out that the area of a triangle is half the area of a rectangle around it ($\frac{1}{2}$ base × height).

Teaching guidelines

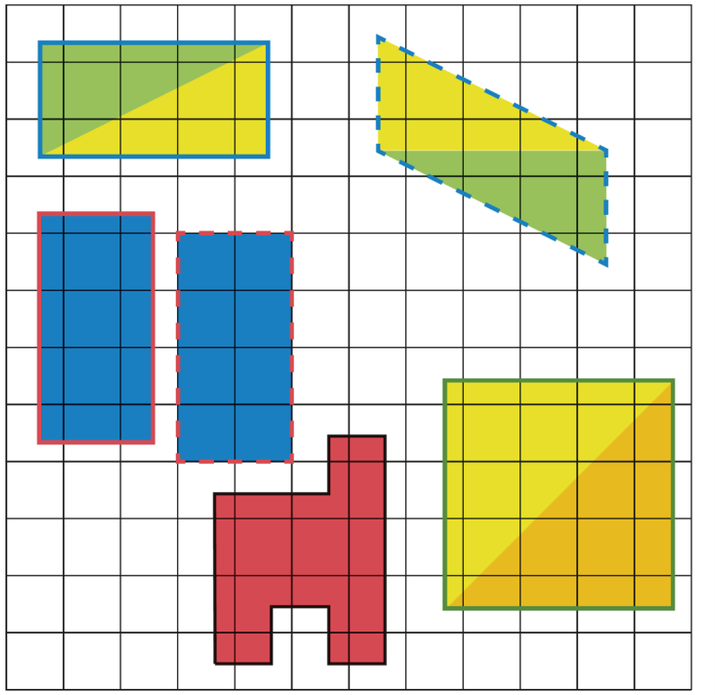
Learners might find it easier to answer question 6 by looking at the green and yellow rectangle in question 3 on page 321. Since the green and yellow triangles are visibly identical, each have an area half of the area of the rectangle.

Answers

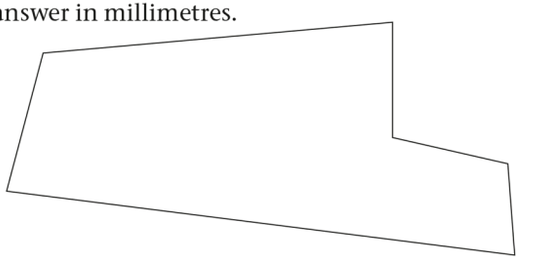
6. (a) Same area
   (b) 4 grid squares

7. 90 mm + 16 mm + 21 mm + 20 mm + 61 mm + 25 mm = 233 mm
   Some rulers may not have the millimetres accurately calibrated. Accept answers that range from about 230 mm to 236 mm.

6. The rectangle with the blue edge is divided into two parts.
   (a) Do the two parts have the same area or different areas?
   (b) What is the area of the yellow part?



7. Use your ruler to find the perimeter of the figure below. Give your answer in millimetres.



7.2 Calculate perimeter

Mathematical notes

Learners should by now understand and know that the perimeter is the distance around a shape. If a shape has many sides, you calculate the perimeter by adding the lengths of the sides.

Teaching guidelines

This is a short section. You could aim to complete it in about 1 hour. One possibility is to use

- question 1(a) for concept development,
- question 1(b) and (c) for classwork, and
- question 2 for additional practice.

You can start by asking learners how they could calculate the perimeter of the plot in question 1(a). Ask learners whether they think the perimeter will be more or less than 1 000 m.

Once learners have calculated the perimeter of the plot in 1(a), ask them whether they think the perimeter of the plot in 1(b) will be more or less than the perimeter of the plot in 1(a). Some learners may not notice that the measurements of plot (b) are given in kilometres.

Ask learners whether they think the perimeter of the plot in 1(c) will be more or less than the perimeter of the plot in 1(a). Ask them which estimated length below they think will be closest to the answer:

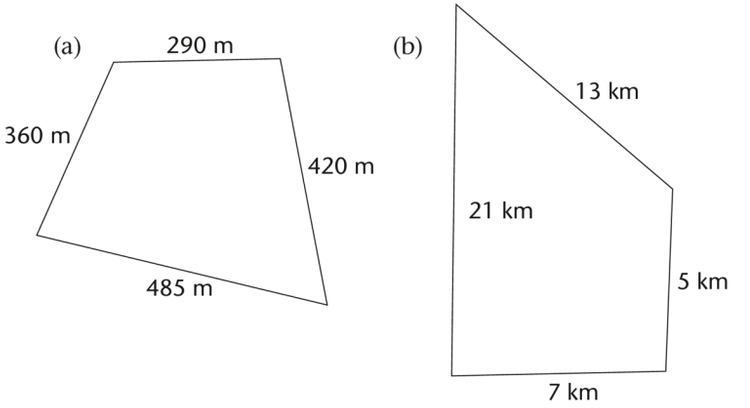
A. 9 000 m B. 10 000 m C. 8 000 m

Answers

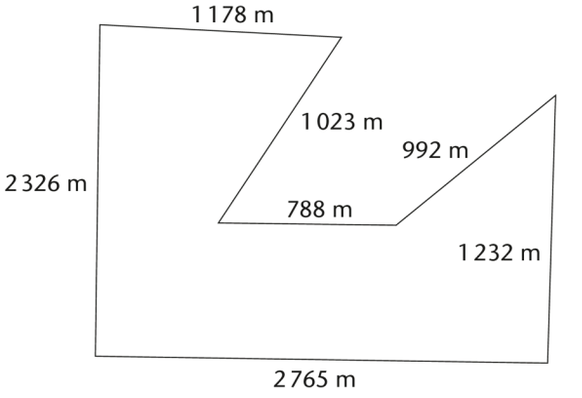
1. (a) 1 555 m (b) 46 km (c) 10 304 m
2. R9 330

7.2 Calculate perimeter

1. The figures below are maps of different plots of land. Calculate the perimeter of each plot and state your answer in the unit used on the map.



(c)



2. If fencing costs R6 per metre, how much will it cost to put a fence around the plot in question 1(a)?

7.3 Perimeter and area of curved figures

Mathematical notes

Not all shapes follow the lines on square grid paper. Square grid paper does not allow us to state exactly how big curved shapes are. We can, however, use the squares to describe the area by saying that it is more than a certain number of grid squares but less than a certain other number of grid squares. For example, in the shape on page 324 the area is more than 60 grid squares but less than 94 grid squares. The smaller the range we give, the more closely it will approximate the area of the shape.

Sometimes in mathematics we give the answer as lying in a range: between an upper limit and a lower limit.

Teaching guidelines

You could aim to complete this section in about 1 hour. You can use questions 1 and 2, and the yellow circle in question 3 for classwork. Learners who finish this quickly could estimate the perimeter of the green circle too.

In question 1 learners will find that the purple area (60 grid squares) is bigger than the blue area (34 grid squares).

Learners might find it easier to first answer question 2(b) before answering question 2(a). If the area of the curved figure is more than the number of purple squares it must also be more than the number of blue squares.

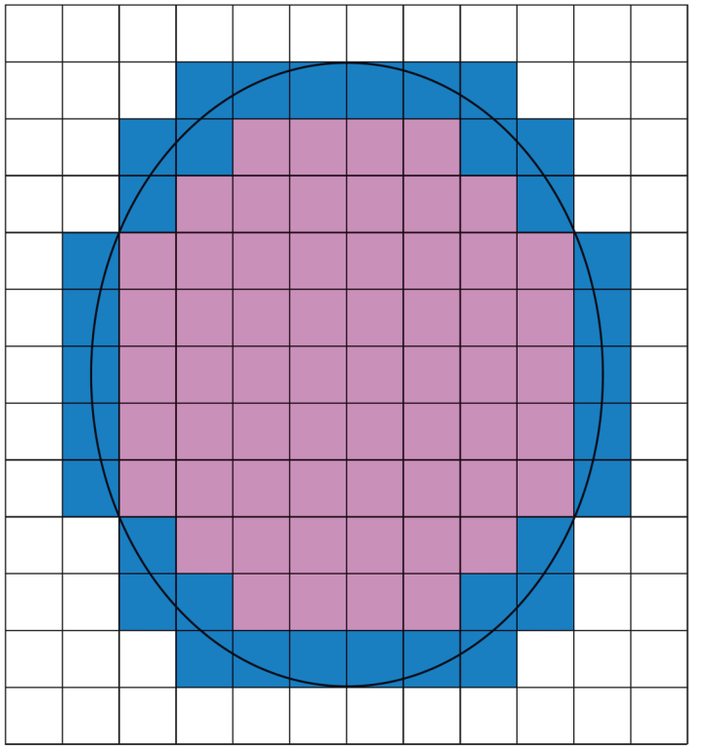
Questions 2(a) and (b) only allow learners to say that the area of the curved figure is bigger than 60 grid squares, but this could be any size: 100 grid squares, 1 000 grid squares, etc. Ask learners questions that will allow them to see that they can state that the curved figure is also smaller than a certain size. For example, you could ask them: "*How many squares are there altogether in the grid?*", "*Is the curved figure bigger or smaller than the whole grid?*", *"Imagine the total area of the coloured squares was one colour. How big would this figure be?"* (The size of the purple squares + blue squares = 94 grid squares.) *"Is the area of the curved figure more or less than the area of all the coloured squares put together?"*

Answers

1. Area of purple figure: 60 grid squares
   Area of blue figure: 34 grid squares
2. (a) Bigger
   (b) Bigger
   (c) Between 60 and 94 grid squares big

7.3 Perimeter and area of curved figures

1. Find the areas of the purple and blue figures.



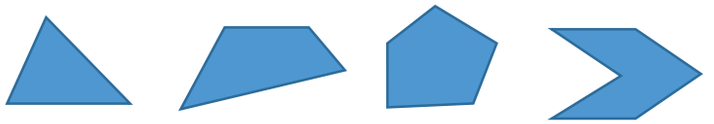
2. (a) Do you think the area of the curved figure is smaller or bigger than the area of the blue figure?
   (b) Do you think the area of the curved figure is smaller or bigger than the area of the purple figure?
   (c) If someone asks you what the area of the curved figure is, what will your answer be?

Mathematical notes

Learners only work formally with perimeters (and areas) of circles in Grade 8.

In question 3 learners estimate the perimeters of the circles by calculating the perimeters of polygons that fit closely around the edges of the circles. The circles will have slightly smaller perimeters (and areas) than the polygons.

A polygon is a two-dimensional closed figure with straight sides. Examples of polygons are triangles, quadrilaterals, pentagons, hexagons, etc.



A circle is not a polygon because it has a curved side, not straight sides.

The more sides a polygon has, the more closely it looks like a circle, and the closer the length of its perimeter to the perimeter of a circle, the closer its area will be to the area of a circle.

The perimeters of circles are usually called the circumference. Learners do *not* need to know this word in Grade 4.

Teaching guidelines

Remind learners that the 2 cm long straight lines make up polygons that fit around the outside of the circles. Once they have worked out the perimeters of the polygons, remind them that they are asked to estimate the perimeters of the *circles*. Ask learners questions like: *"Does the circle have exactly the same perimeter as the polygon?", "Is the perimeter of the circle more than or less than the perimeter of the polygon?"*

Learners could estimate the perimeter of the green circle for additional practice.

Possible misconceptions

Learners might forget that they are trying to find the perimeters of the circles. They might just add or multiply the lengths of the straight lines and forget what the question was.

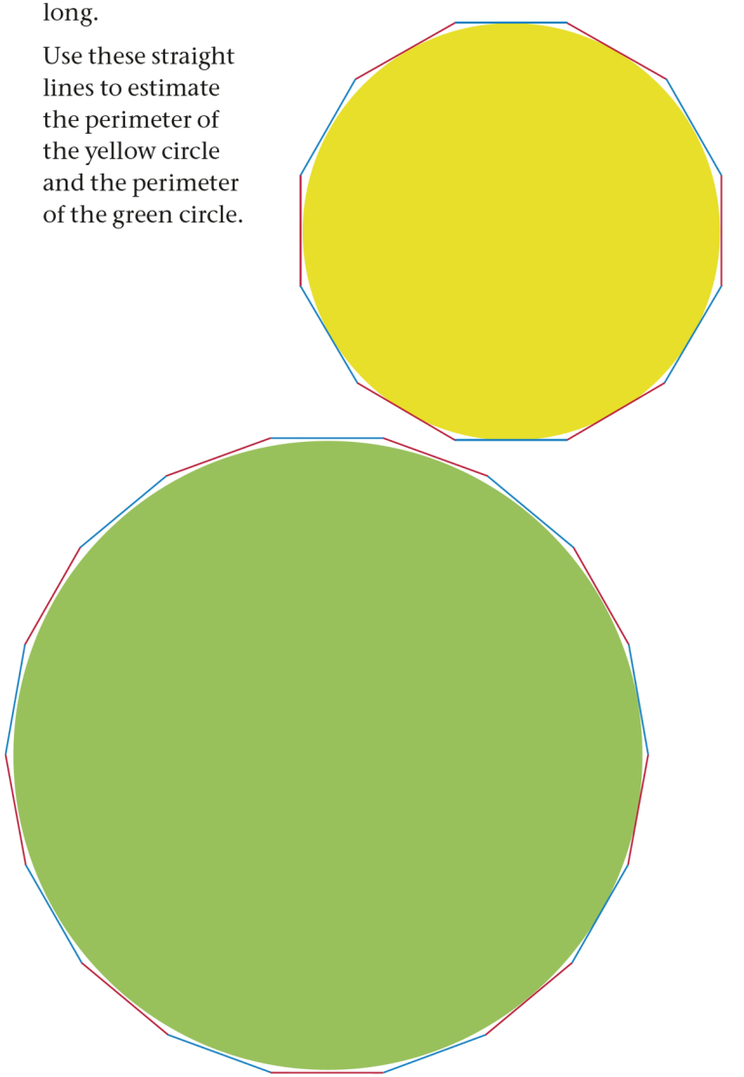
Answers

3. The perimeter of the polygon around the yellow circle is 24 cm.
   This means that the perimeter of the yellow circle is a bit less than 24 cm.

   The perimeter of the polygon around the green circle is 36 cm.
   This means that the perimeter of the yellow circle is a bit less than 36 cm.

3. The red and blue straight lines around the circles are all 2 cm long.

   Use these straight lines to estimate the perimeter of the yellow circle and the perimeter of the green circle.



GRADE 4: MATHEMATICS [TERM 4] 325

7.4 Capacity and volume

Mathematical notes

In this section learners work informally with the volume of a stack of blocks or small boxes, and fill larger boxes with small boxes or blocks to calculate informally (count) the capacity of the larger boxes. This lays the basis for an understanding of cubic units.

Learners are asked to estimate the sizes of various objects and spaces. Estimation is not random guessing. Estimation of size works best if you can compare the size (of what you are estimating) to the size of something that is known. For example, you need to have a good sense of the size of a metre before you can estimate lengths in metres.

Resources

Metre lengths of string; old newspapers or other large sheets of paper; sticky tape.

Cardboard boxes approximately the size of shoeboxes.

Small blocks, all the same size, or many empty matchboxes.

Teaching guidelines

Aim to spend about $2\frac{1}{2}$ hours on this section.

You can ask learners to estimate the number of grey cubes that will fill the green box. Rather than guessing wildly, encourage learners to use the three lengths of 5 cubes given in the drawing: they are drawn in the directions of the length, breadth and height of the green box to make it easier to estimate. Learners can return to question 1 after they have completed this section.

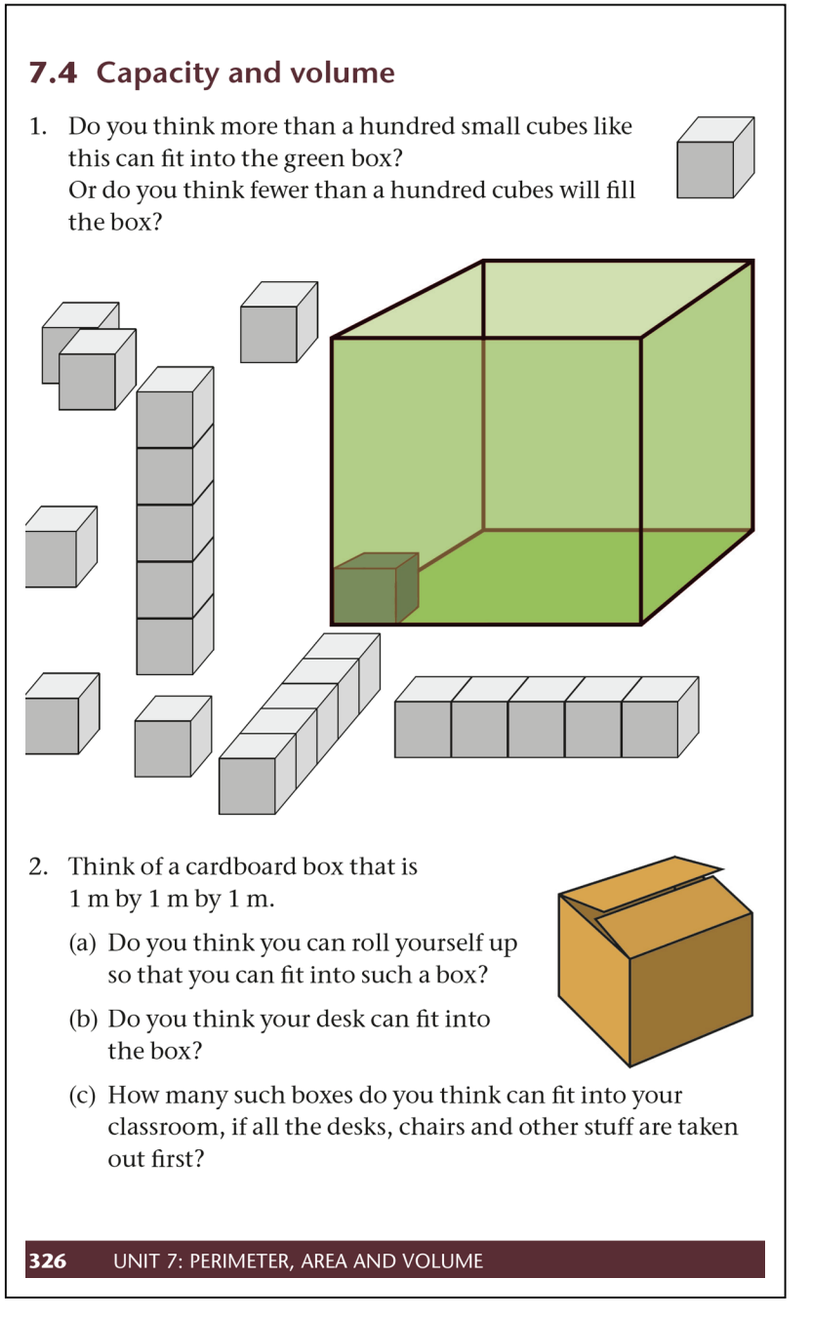
Learners have worked with metres since Grade 3. Ask them to measure out 1 m and cut a length of string this long. Then ask them to roll newspaper into 1 m long tubes. Groups of learners can then join 4 tubes to make a square shape that shows one square metre (1 m × 1 m). They can join six of these squares into a box shape that shows one cubic metre (1 m × 1 m × 1 m). They can use this to estimate their answers to questions 2(a), (b) and (c).

Notes on questions

In questions 1, 2 and 3 learners are asked for their opinions; their opinions will differ. Keep asking learners to motivate why they have given these answers. Focus on the reasons that learners give for their answers.

Answers

1. Some learners will say "yes", other learners will say "no".
2. See the next page.



7.4 Capacity and volume

1. Do you think more than a hundred small cubes like this can fit into the green box?
   Or do you think fewer than a hundred cubes will fill the box?

2. Think of a cardboard box that is 1 m by 1 m by 1 m.

   (a) Do you think you can roll yourself up so that you can fit into such a box?

   (b) Do you think your desk can fit into the box?

   (c) How many such boxes do you think can fit into your classroom, if all the desks, chairs and other stuff are taken out first?

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Answers

2. (a) Some learners will say "yes", other learners will say "no". Most Grade 4 learners should be able to roll themselves up into a space smaller than 1 cubic metre.
   (b) Some learners will say "yes", other learners will say "no". Most desks cannot fit into this space.
   (c) The sizes of classrooms differ. Learners' estimates will also differ.

Teaching guidelines

If you do not have enough cubes or matchboxes to let learners do question 3 individually or even in groups, do it as a classroom demonstration, but let learners do questions 3(a) and (d) individually.

Introduce learners to the term "volume". Volume is the amount of space something takes up, and it can be used to describe the size of a three-dimensional object such as a stack of cubes.

Answers

3. Task done by learners. Answers will differ. Consider all answers.

3. To do this task, you need many small blocks of equal size and a box. You could, for example, use many empty matchboxes and a shoebox.



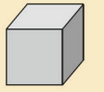
(a) Estimate, to the nearest hundred or ten, how many of your small blocks can be neatly packed into your box.

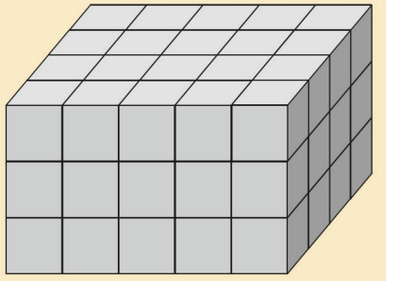
(b) Pack the small blocks neatly into your box, until it is full.

(c) Take out the small blocks and count them.

(d) How close was your estimate?

There are 60 small cubes in this stack.

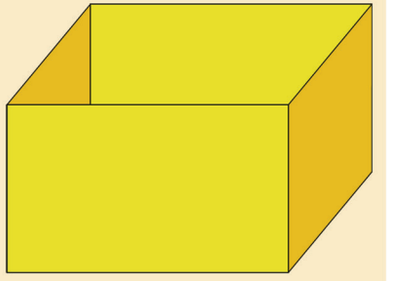




The **volume of the stack** is 60 cubes.

This stack of cubes fits exactly into the yellow box, and fills the yellow box.

The **capacity of the box** is 60 cubes.



Teaching guidelines

Learners can count the number of cubes in each stack and give the volume of each stack in number of cubes. Learners could work out the size of the stacks in questions 4(a), (b) and (e), and also the number of cubes in the stack in question 5(b) as classwork. They could do questions 4(c) and (d), and 5(a) as additional practice.

Notes on questions

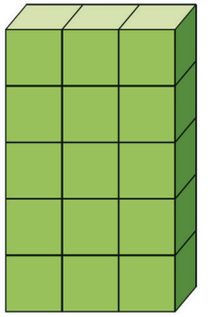
Question 5(b) is the more difficult question in this section. Learners are not given the illustration, so they cannot count the cubes. They have to imagine that there are 4 cubes in a row and 6 of these rows, which gives 24 cubes (learners could multiply or count in 4s or 6s). They have to imagine that there are 7 layers of 24 cubes. Here they need to multiply $7 \times 24 = 168$ cubes.

Answers

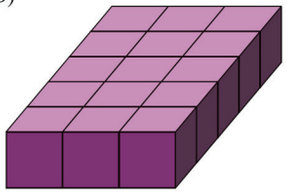
4. (a) 15 cubes (b) 15 cubes (c) 10 cubes
   (d) 30 cubes (e) 45 cubes
5. (a) 75 cubes (b) 168 cubes

4. What is the volume of each stack?

(a)



(b)



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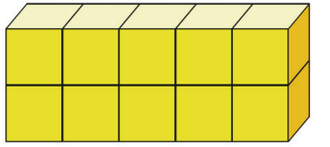
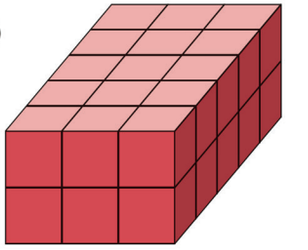
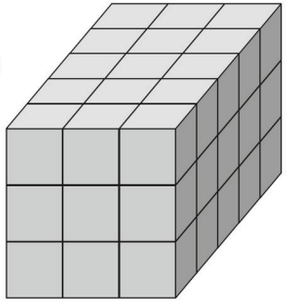


(d)



(e)



5. (a) How many cubes do you need to build a stack that is 3 cubes wide and 5 cubes long like in question 4(e), but 5 cubes high?

   (b) How many cubes do you need to build a stack that is 4 cubes wide, 6 cubes long and 7 cubes high?

Grade 4 Term 4 Unit 8 Position and movement

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
8.1 Positions on a grid	Placing a grid over a collection of objects in order to make it easy to say where they are	329 to 331
8.2 Make your own maps	A simple way to map a space using a grid of rows and columns	332

CAPS time allocation	2 hours
CAPS page references	24 and 115

Mathematical background

It is not easy to describe where each of a large number of objects is. A way to make this much easier is to put a grid over the surface on which the objects are resting. One can then look at the cell in which a particular object lies and give the cell position (row this, column that). This is a very, very important basic idea in mathematics, one without which many uses of mathematics would not exist.

8.1 Positions on a grid

Mathematical notes

This section shows that putting a grid over a surface allows us to say where a particular object is. When the grid is placed over the surface each object will be mostly in particular cells of the grid. For any particular object we only have to say which row and which column of the grid it lies in. Once we have these two pieces of information we can say with confidence where the object is. The grid is therefore a position-locating tool. Note that we usually use a square grid.

Teaching guidelines

First let learners use everyday terms and referents to describe the position of people and things in the classroom. Then introduce learners to the idea that we can impose a grid over the surface (or situation), and explain the terms *row, column* and *cells* as used in grid referencing: see the tinted passage on page 330 and the summary bar on page 331.

This is a very important first experience of finding positions using a grid. Be sure to let your learners appreciate the idea that *we* place the grid on the surface to *make* it easier to say where each thing is.

You can use the position of learners in your classroom to draw a diagram on the board similar to the one shown on page 329. You can then impose a grid onto your diagram so that it will look similar to the diagram on page 330.

Possible misconceptions

Learners may confuse the term column (up–down) with row (across). This is simply a matter of carefully remembering which name goes with which direction.

In many classrooms learners' desks are arranged in rows running from the front to the back of the classroom. If this is the case in your classroom, then learners may use the term row to describe where learners sit. For example, learners may say: "Sophia sits in front in the row on the left of the classroom; Nkhosi and Busi sit at the back of this row." They may then get confused when they turn over the page to page 330 and see that on the grid this is labelled Column A, and the Sophie, Miriam and Jack sit in Row 7.

Answers

1. to 4. Learners' own work – ensure that learners use a reference point to describe positions.

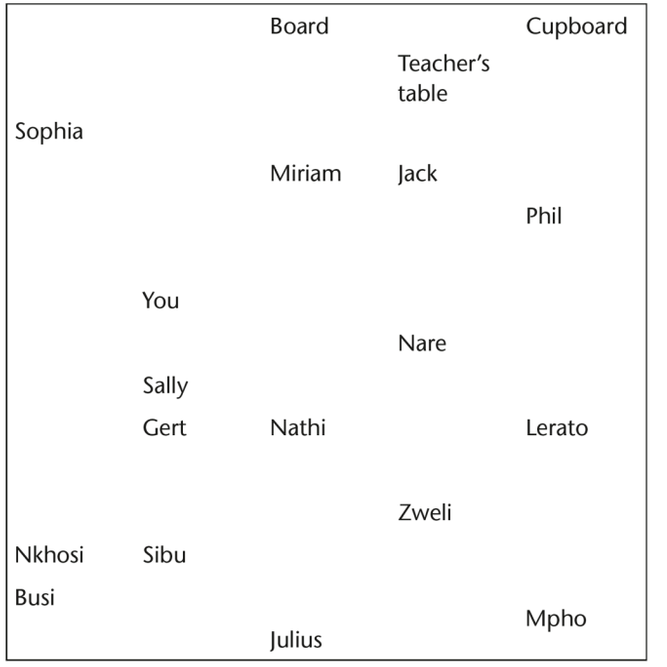
UNIT 8

POSITION AND MOVEMENT

8.1 Positions on a grid

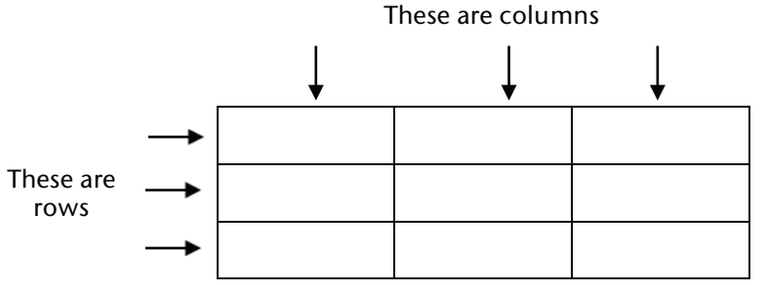
The map below shows where different learners sit in a classroom.

1. Describe in words where you sit in the classroom.
2. Describe where Nathi sits.
3. Describe where Phil sits.
4. Describe where the teacher's table is.



You will now learn how to make a better map, so that it is easier to say where people or objects are.

Mathematical notes



Answers

5. Column D
6. Row 7

There is a grid on the map below.
The grid has 8 rows, marked 1 to 8 from bottom to top.
The grid has 5 columns, marked A to E from left to right.

Nathi sits in Column C, in Row 4.

5. In which column does Jack sit?
6. In which row does Jack sit?

	A	B	C	D	E
8			Board	Teacher's table	Cupboard
7	Sophia		Miriam	Jack	
6					Phil
5		You		Nare	
4		Sally Gert	Nathi		Lerato
3				Zweli	
2	Nkhosi Busi	Sibu			
1			Julius		Mpho

Critical knowledge

Note that in Sections 8.1 and 8.2 two different kinds of questions are asked. In questions 7, 8, 10, 11 and 12 learners are given the name of the person (or the object) and asked in which cell they are sitting (or appear). In question 9 learners are given the cell number and asked who sits in that cell. Section 8.2 asks more questions in the latter form (learners are given the cell reference number to go to).

Learners need to be able to answer both kinds of questions: they have to find a stated object and give the cell name, and they have to find the named cell and say what is in it.

Answers

7. Column E Row 4

8. (a) Column A Row 7 (b) Column A Row 2
   (c) Column C Row 7 (d) Column D Row 5
   (e) Column B Row 4 (f) Column B Row 4

9. Nkhosi and Busi

10. C8

11. E8

12. (a) A2 (b) B2 (c) C1 (d) E1

13. 8 rows

14. 5 columns

15. 8 × 5 = 40 cells

16. Learners can list any five of the following cells:
    A1; A3; A4; A5; A6; A8
    B1; B3; B6; B7; B8
    C2; C3; C5; C6
    D1; D2; D4; D6
    E2; E3; E5; E7

All the questions on this page are about the map on the previous page.

7. In which column, and in which row does Lerato sit?
8. In which column and in which row do these learners sit?
   (a) Sophia (b) Busi
   (c) Miriam (d) Nare
   (e) Sally (f) Gert

The squares in a grid are called **cells**.

Zweli sits in Cell D3.
This means Zweli sits in Column D, in Row 3.

9. Who sits in Cell A2?
10. In which cell is the board?
11. In which cell is the cupboard?
12. In which cells do the following learners sit?
    (a) Nkhosi
    (b) Sibu
    (c) Julius
    (d) Mpho
13. How many rows are there on this map?
14. How many columns are there on this map?
15. How many cells are there on this map?

Cell C3 is empty.

16. Name five other cells that are also empty.

8.2 Make your own maps

Teaching guidelines

Ask learners to complete questions 1(a) and (b) on their own. In question 1(a) learners draw and label a grid or photocopy the grid in the Addendum on page 467. In question 1(b) learners fill in the stated numbers in particular cells. In question 1(c) learners need to use logic and counting in multiples to work out the answers. Let learners first work out what numbers go into B6 and C4. Then discuss with them how they got the answers. This will help them to find the other answers.

Answers

Learners are not asked to write in the numbers shown in black in the grid below, but they will need to say these (aloud or in their heads) to get the answers to question 1(c).

1. (a) Learners draw the grid and label it.
   (b) See the red numbers.
   (c) See the blue numbers.

	A	B	C	D	E	F	G	H
8	26	28	30	32	34	36	38	40
7	23	25	27	29	31	33	35	37
6	20	22	24	26	28	30	32	34
5	17	19	21	23	25	27	29	31
4	14	16	18	20	22	24	26	28
3	11	13	15	17	19	21	23	25
2	8	10	12	14	16	18	20	22
1	5	7	9	11	13	15	17	19

2. Learners' own work. Answers will differ from class to class.

8.2 Make your own maps

1. (a) Draw a grid with 8 columns and 8 rows on a clean sheet of paper. Use most of the sheet for the grid.
   Label the columns A, B, C, D, E, F, G and H from left to right.
   Label the rows 1 to 8 from bottom to top.

   (b) Now work as follows:

Write 5 in Cell A1.	Write 10 in Cell B2.
Write 13 in Cell B3.	Write 26 in Cell A8.
Write 20 in Cell A6.	Write 17 in Cell A5.
Write 30 in Cell F6.	Write 13 in Cell E1.
Write 12 in Cell C2.	Write 7 in Cell B1.
Write 11 in Cell A3.	Write 11 in Cell D1.

   (c) What numbers do you think should be in each of the following cells?

   B6 C4 E3 H1 H8

2. (a) How many rows will you need to make a map of your own classroom?

   (b) How many cells will you need to make a map of your own classroom?

   (c) Draw a map of your own classroom on a clean sheet of paper.
   On your map, indicate where the board is, and where the teacher's table is.

   (d) In which cell do you sit in your classroom?

   (e) On your map, write the names of ten of your classmates in the correct cells.

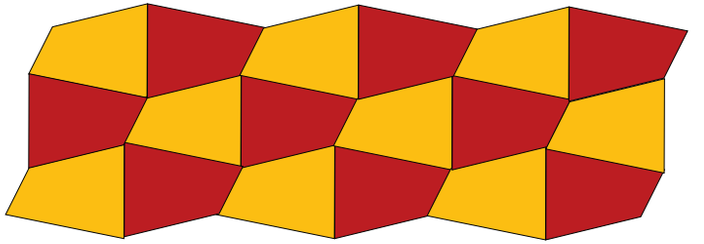
Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
9.1 Tessellations	Tessellations around us: in nature, in human structures and in art	333 to 336
9.2 Tessellate	Making tessellations from basic shapes	337 to 338

CAPS time allocation	3 hours
CAPS page references	23 and 116

Mathematical background

A flat surface (like a floor, wall or table top) and evenly rounded surfaces (like the skin of a fish or snake) can be divided into or covered by one or more identical flat shapes that do not overlap or leave any gaps. A pattern formed in this way is called a tessellation (the Latin word for *tile* is *tessel*).

Any quadrilateral can be used to make a tessellation pattern:



Resources

Each learner or pair of learners needs 12 of each of the shapes on page 338 (if you have access to a photocopier, you can photocopy the templates in the Addendum on pages 469 to 471).

A demonstration set of large copies of the shapes that you can stick to the board when discussing solutions.

Scissors.

9.1 Tessellations

Mathematical notes

Tessellations are patterns formed by arranging shapes to fit neatly next to each other. There are no gaps in a tessellation and none of the shapes overlap (see the summary bar on page 337).

Each of the tessellations in this section is made up of a single shape. Any triangle or quadrilateral will tessellate. This is an important fact that can be very useful to you as a teacher.

Teaching guidelines

Aim to spend about 1 hour on this section.

You can start by explaining to learners what a tiling or tessellation pattern is. Ask learners to look around to see whether they can see any tessellations. Brickwork, floor tiles, fences and sometimes window panes make tessellating patterns. Ask learners to name or describe the shape (tile) that is repeated to make the pattern. You can use the tinted passage on page 333 to consolidate the concept of a tessellation and the tiles that are used to make up the patterns.

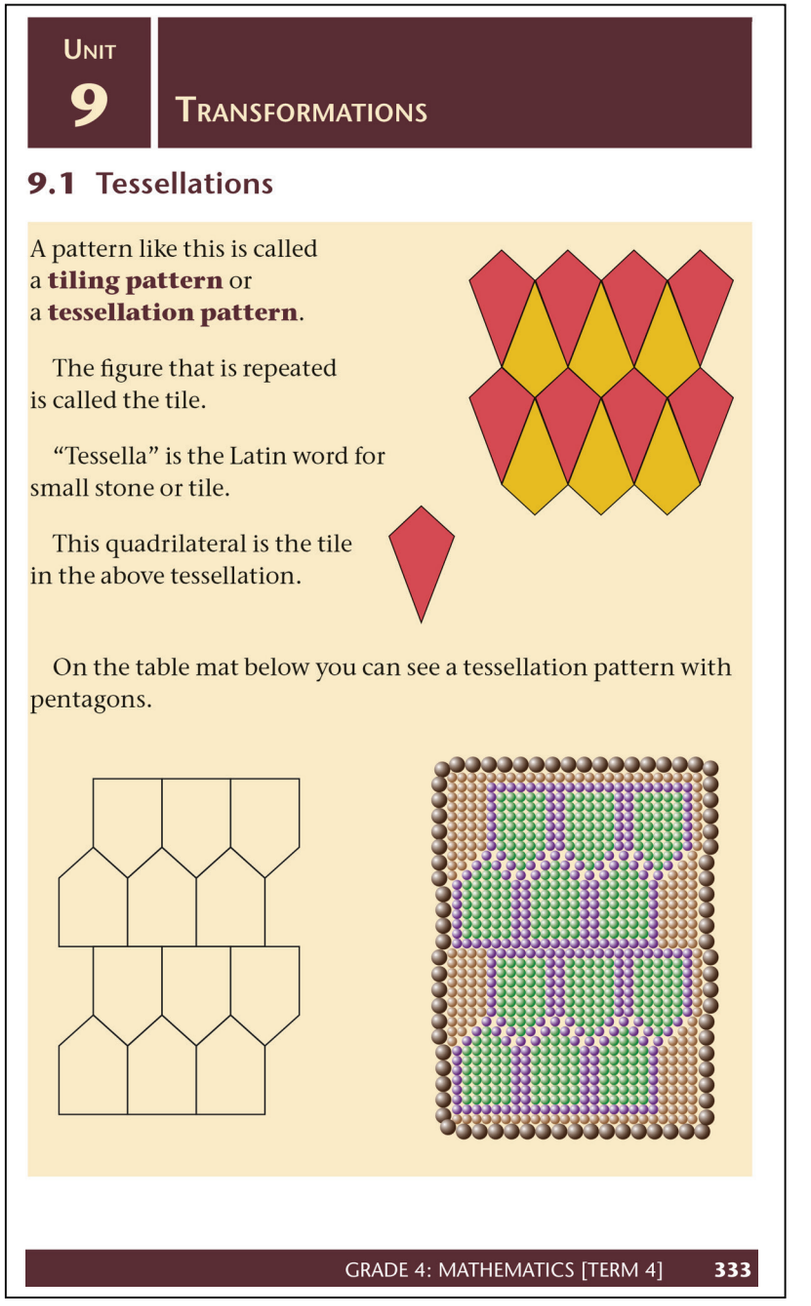
Drawing the patterns helps learners to focus more clearly on how the tessellation is made. This is a focus in questions 1, 2 and 3. Describing the patterns not only helps learners to develop the language to talk about the patterns, but also helps them to think more clearly about the patterns.

Learners can trace and cut out the shape in question 4, or you can photocopy the one in the Addendum (page 468).

If time permits, you can include additional tile shapes to tessellate. Printing and cutting out many copies of the same triangle or quadrilateral will provide you with additional tessellation activities.

Possible misconceptions

It is possible to arrange tiles so that they do not tessellate, i.e. that there are gaps between the tiles. If learners do this, remind them that there are no gaps in a tessellation. Otherwise they may believe that the activities are about arranging tiles in any pattern, including non-tessellating patterns.



UNIT 9 TRANSFORMATIONS

9.1 Tessellations

A pattern like this is called a **tiling pattern** or a **tessellation pattern**.

The figure that is repeated is called the tile.

"Tessella" is the Latin word for small stone or tile.

This quadrilateral is the tile in the above tessellation.

On the table mat below you can see a tessellation pattern with pentagons.

GRADE 4: MATHEMATICS [TERM 4] 333

Mathematical notes

The scales on real fish overlap. You can draw a pattern of fish scales that do not overlap: see page 391. This is a simplified model of the scales.

Answers

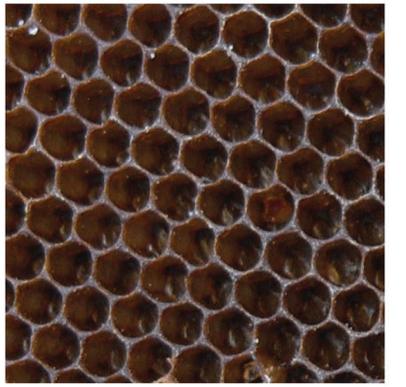
1. (a) Learners' own drawings of the tessellating pattern in a honeycomb.
   (b) Learners' descriptions will differ. Learners cannot be expected to produce good descriptions. The purpose of the question is to challenge them to think about the pattern and to try to find words and phrases that can be used to describe it.



The small drawing alongside shows the tessellation pattern with a curved shape that you can see on the fish.



1. (a) Make a drawing of part of the pattern that you can see in this piece of honeycomb.
   (b) Describe the pattern in words.



Notes on questions

Drawing the patterns and talking about it help learners to focus more clearly on how the tessellation is made. Talking about two or more different patterns helps learners to see and talk more clearly about how each pattern is made.

You can ask learners what is *the same* about the brick patterns (they are both made up of bricks; each pattern uses one kind of brick). Then you can ask learners what is *different* about the patterns (in the top photograph the bricks have straight edges and are all laid in the same direction; in the bottom photograph the bricks have wavy/curvy edges and some bricks lie "north–south" and others "east–west").

Answers

2. (a) Learners' own drawings of the tessellation of bricks.
   (b) Learners' descriptions will differ, but they could say something like: "The bricks make a pattern of rows. All bricks have straight edges and are identical. Each brick touches the two bricks next to it along a short side, and the bricks above and below it along a long side."
3. (a) Learners' own drawings of the tessellation of bricks.
   (b) Learners' descriptions will differ, but they could say something like: "The bricks make a tessellating pattern because there are no gaps or overlaps. These bricks do not make a pattern of rows, because some bricks have the long side going up and down, and other bricks have the long side going across. All bricks have curvy sides and are identical to each other."

2. (a) Make a drawing of the pattern that you can see in the photograph of the brick wall below.
   (b) Describe the pattern in words.



3. (a) Make a drawing of the pattern that you can see in the photograph of the pavement below.
   (b) Describe the pattern in words.

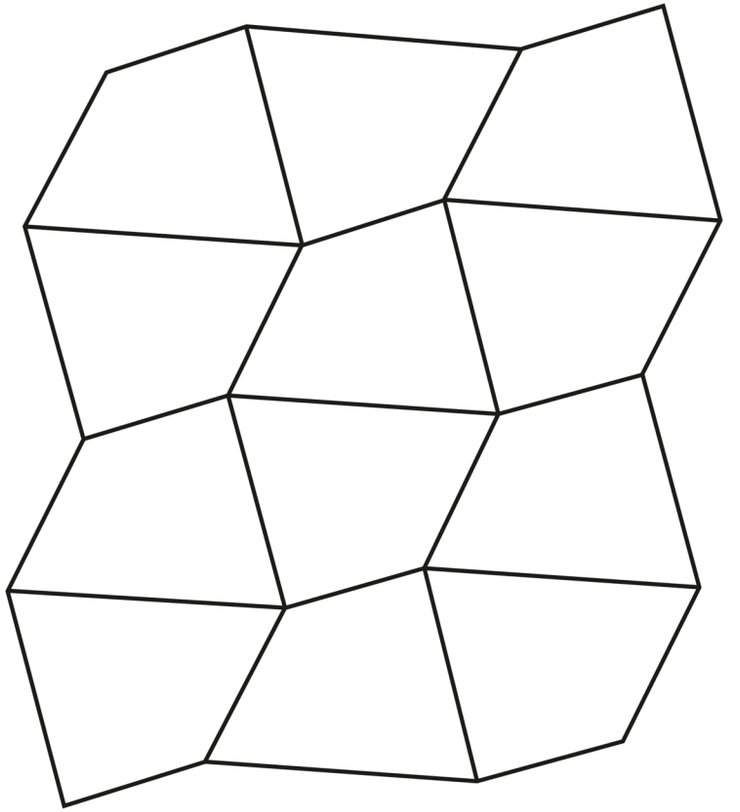


GRADE 4: MATHEMATICS [TERM 4] 335

Answers

4. (a) to (c) Learners' own work

4. (a) Put a clean sheet on top of this page and trace the figure below.



(b) Cut out all 12 tiles.

(c) Arrange the 12 tiles on your desk, so that they fit together neatly to form a copy of the above tessellation.

9.2 Tessellate

Mathematical notes

Some of the tessellations on page 337 involve two or more different tiles. Some tiles (shapes) cannot tessellate (see the example in question 1 on page 338).

Resources

Each learner or pair of learners needs 12 of each of the shapes on page 338. If you have access to a photocopier, you can photocopy the templates in the Addendum (pages 468 to 470).

A large demonstration set of the shapes that you can stick to the board when discussing solutions.

Scissors.

Teaching guidelines

Aim to spend about 2 hours on this section.

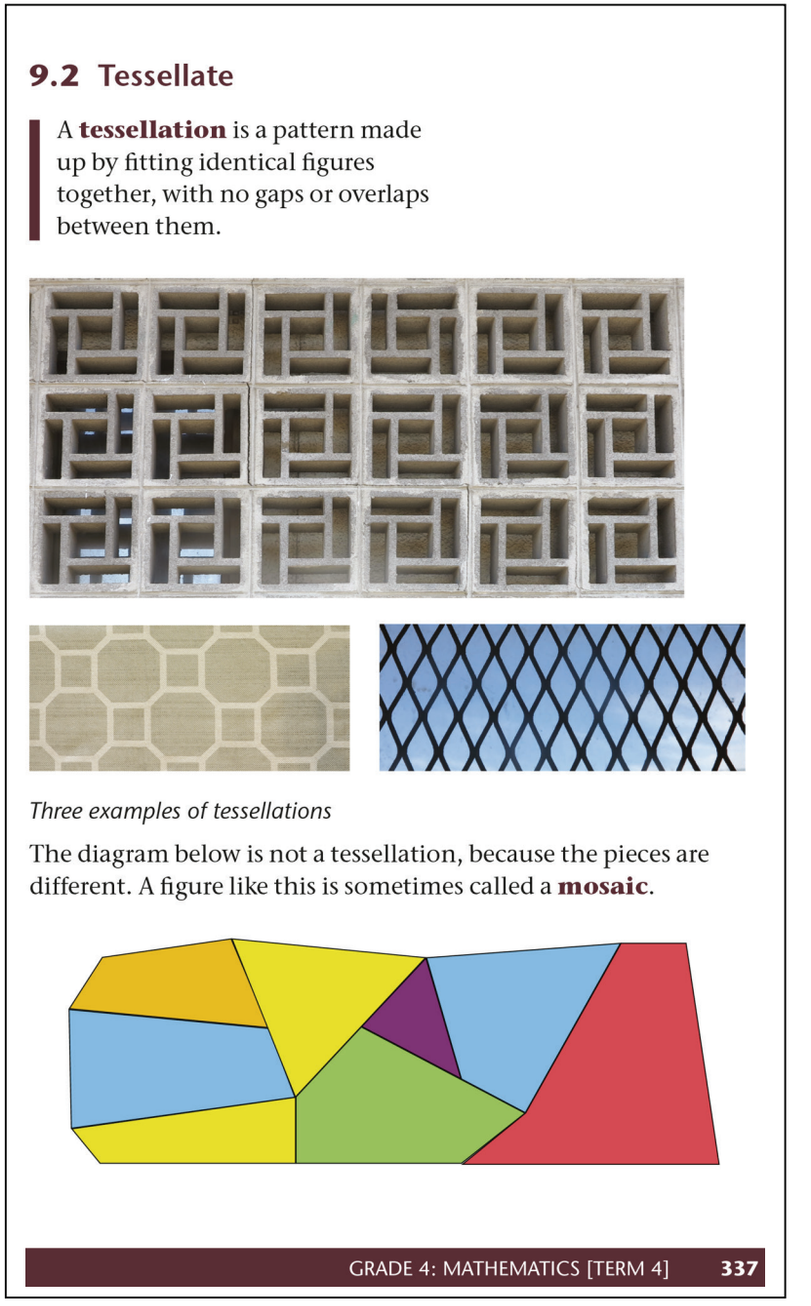
You might like to start by reminding learners what a tessellation is: see the description at the top of page 337. You can ask learners what they see in the photographs. Then focus their attention on the shapes that make up the patterns. One way to do this is to ask them to copy the pattern. The fence is a tessellating pattern of quadrilaterals. The other two tessellations are both made up of two shapes. Ask learners to talk about these shapes and their arrangements. They can use drawings to explain the patterns. The diagram at the bottom of page 337 is a mosaic and not a tessellation, because none of the shapes are repeated.

In question 1 learners use the shapes to try to make tessellating patterns. If you have access to a photocopier, photocopying the templates in the Addendum (pages 468 to 471) will save time. If possible, allow learners to engage in structured discussions about the activities. Since most of the activities in this section are open-ended, it is valuable for learners to compare and evaluate their approaches.

Notes on questions

Question 4 of Section 9.1 about the tessellation of quadrilaterals (page 336) should help to prepare learners to tessellate with the shapes in question 1 on page 338.

Question 2 is challenging because learners have to decide whether each tile will tessellate, and explain their decision, without cutting out the shapes. Take advantage of any learner responses that use drawing to help explain their thinking. It is important that learners realise that drawings can be tools to help them understand things, and that they may choose to make one or more drawings to help their thinking whenever they wish.



9.2 Tessellate

A **tessellation** is a pattern made up by fitting identical figures together, with no gaps or overlaps between them.

Three examples of tessellations

The diagram below is not a tessellation, because the pieces are different. A figure like this is sometimes called a **mosaic**.

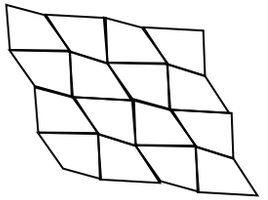
GRADE 4: MATHEMATICS [TERM 4] 337

Once learners have tried to imagine whether the shapes can tessellate, ask them to trace 12 of the shapes, cut them out and try to tessellate with them, or photocopy the templates provided in the Addendum (pages 472 to 474) to save time.

Answers

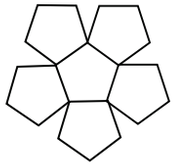
1. (a) Learners make 12 copies of each tile/shape.

   (b) Learners' tessellations should look something like the diagram below.

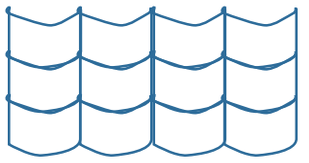


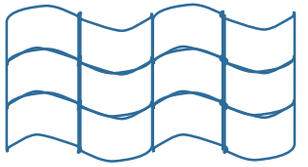
   (c) The pentagon does not tessellate. When you place pentagons next to each other there are either gaps between them or they overlap.

   The rounded tile will tessellate – see (d) below.

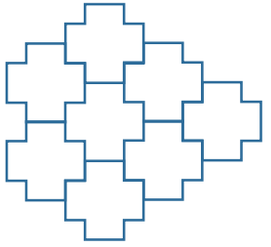


   (d) Sample sketches are shown. Note that learners might place tiles vertically rather than horizontally as shown below.

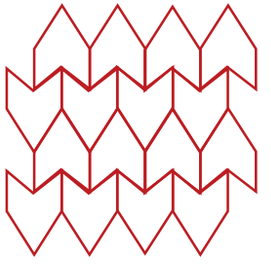


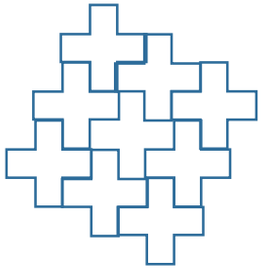


2. (a) All the shapes tessellate.

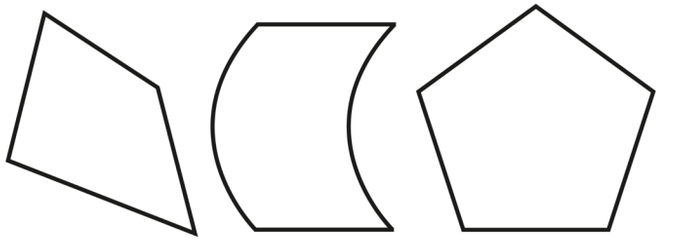


   (b) All the shapes tessellate.





1. (a) Trace and cut out 12 copies of each of the figures below.

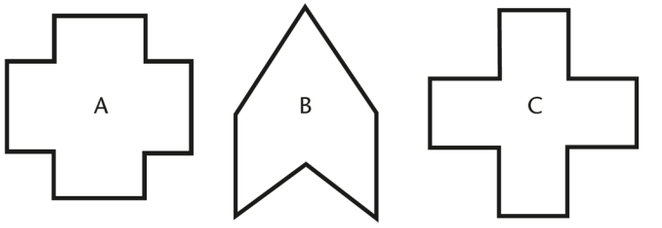


   (b) Try to tessellate with the quadrilateral tile.
   This means you have to arrange the 12 tiles in such a way that they fit together to form a tessellation.

   (c) Try to tessellate with the other two tiles.

   (d) You can tessellate in two different ways with the rounded tile. Find out what the two ways are, and make rough drawings to show how they differ.

2. (a) Which of these figures do you think will work to make a tessellation pattern?



   (b) Explain why you think the other figures will not work. Also make drawings to show why you think they will not work.

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
10.1 Investigate and extend patterns	Visual recognition and generalisation of structure in geometric figures	339 to 340
10.2 Investigate and extend more patterns	Visual recognition and generalisation of structure in geometric figures	341
10.3 From tables to flow diagrams	Tables and flow diagrams as different equivalent representations of patterns	342

CAPS time allocation	2 hours
CAPS page references	19 and 117

Mathematical background

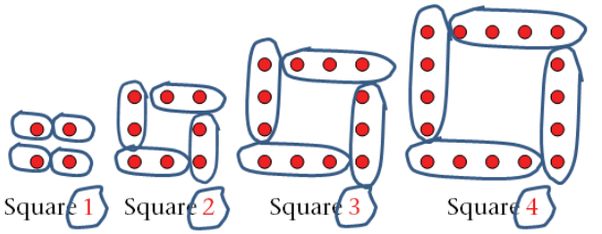
While providing opportunities to develop understanding of patterns, continuing the sequences or completing the tables according to a pattern also contributes to the development of the Mental Mathematics section of the CAPS.

All the sequences in Section 10.1 are sequences of multiples (e.g. 3, 6, 9, 12, ...) and therefore reinforces the times tables. All the sequences in Section 10.2 are based on increasing differences (e.g. +3, +4, +5, ...) and therefore reinforces mental addition strategies.

The approach in this unit is not to reduce the work on geometric patterns to numerical patterns in tables – that too – but to capitalise on the *visual* aspects of geometric representations as a method to find **rules** based on the *structure* of the geometric figures.

This implies that you should help learners not only to *count* the number of dots in a figure (counting in ones), but to use "clever counting" by identifying appropriate larger units. Then they should not actually *count* the larger units, but rather write down a **numerical expression** (calculation plan or **rule**) expressing the structure of the figure. It is very important that learners should learn to study the *structure* of such an expression, and not to jump into calculation.

To find a general rule for the pattern requires a second level of pattern recognition, namely recognising the structure in a series of numerical expressions – what is unchanged (is **constant**) and what changes (is **variable**). This process is illustrated below, and on the next page.

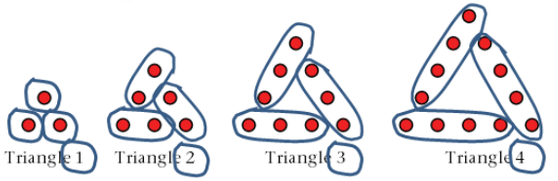


10.1 Investigate and extend patterns

Mathematical notes

All the geometric patterns in this section can be transformed to numeric patterns by completing a table. The resulting numeric patterns are all sequences of multiples. We studied patterns in sequences of multiples in Term 1, so it is important that you help learners to see that these are different representations of the same pattern, and that they bring the knowledge that they developed in Term 1 to this "new" work.

However, the focus in this unit should be on solving the problems directly in the geometric context by "seeing" the *structure* in the geometric representations. Learners should continue the mindset of "clever counting" developed in Term 2: the way to "see" structure is to understand that in Triangle 4 we try to identify a unit of 4, in Triangle 3 we try to identify a unit of 3, in Triangle 2 a unit of 2, etc. For example:

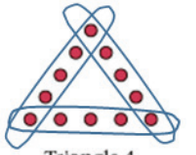


← ← T1 = 3×1 ← ← T2 = 3×2 ← ← T3 = 3×3 four, four, four T4 = 3×4

Pattern recognition now proceeds at a different level: we need to see the *structure* – not in a number sequence or in a figure, but in a series of related *specific* calculation plans. This requires us to be very clear about what is changing (the *variable*) and what remains unchanged (*constant*). If you present it vertically it may help learners to see the pattern. It is this structure that must be *generalised* to a *general calculation plan* (rule):

T1 = 3×1
T2 = 3×2
T3 = 3×3
T4 = 3×4
So T30 = 3×30

Note that some learners may use clever counting by using the triangle sides as a unit. But they may then say that Triangle 4 has 5 dots on each of the 3 sides, and reason that the total number of dots in Triangle 4 is 3×5 = 15. However, this would be wrong, because we have in fact now counted each of the corner dots twice!

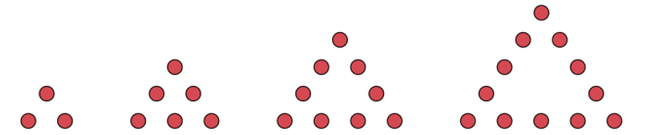

Triangle 4

Continued on the next page

UNIT 10 GEOMETRIC PATTERNS

10.1 Investigate and extend patterns

1. (a) Describe in words how this growing pattern of triangles is made.


Triangle 1 Triangle 2 Triangle 3 Triangle 4

(b) Describe Triangle 6 and Triangle 7 in words. How many dots are there in Triangle 6 and how many in Triangle 7?

(c) Describe Triangle 60 and Triangle 70 in words. Calculate the number of dots in Triangle 60 and in Triangle 70.

(d) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Triangle number	1	2	3	4	5	6	30
Number of dots	3	6					

2. On the next page you can see growing patterns of squares, pentagons and hexagons. Investigate each pattern by answering these questions.

(a) Describe in words how the growing pattern is made.

(b) Describe Figure 6 (that is Square 6, Pentagon 6 and Hexagon 6) and Figure 7 in words. How many dots are there in Figure 6 and how many in Figure 7?

Instead of abandoning the method, we can simply build in the correction and construct an alternative *equivalent* rule:

T1 = 3×2 – 3
T2 = 3×3 – 3
T3 = 3×4 – 3
T4 = 3×5 – 3
So T30 = 3×31 – 3

The squares, pentagons and hexagons on page 340 can be approached in the same way.

Answers

1. (a) Each triangle has three more dots than the previous triangle.
   (b) Triangle 6: 3×6 dots = 18 dots; Triangle 7: 3×7 dots = 21 dots
   (c) Triangle 60: 3×60 dots = 180 dots; Triangle 70: 3×70 dots = 210 dots
   (d) 3, 6, 9, 12, 15, 18; Triangle 30: 3×30 = 90 dots

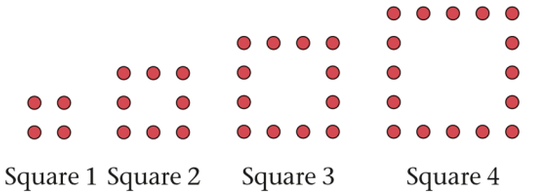
2. (a) To make the next figure, add one dot to each side of the previous figure.
   Squares: (b) Square 6: 4×6 dots = 24 dots Square 7: 4×7 dots = 28 dots
   (c) Square 60: 4×60 dots = 240 dots Square 70: 4×70 dots = 280 dots
   Pentagons: (b) Pentagon 6: 5×6 dots = 30 dots Pentagon 7: 5×7 dots = 35 dots
   (c) Pentagon 60: 5×60 dots = 300 dots Pentagon70: 5×70 dots = 350 dots
   Hexagons: (b) Hexagon 6: 6×6 dots = 36 dots Hexagon 7: 6×7 dots = 42 dots
   (c) Hexagon 60: 6×60 dots = 360 Hexagon70: 6×70 dots = 420 dots

   (d)

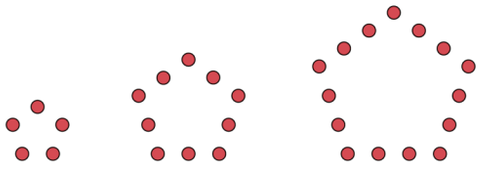
Figure no.	1	2	3	4	5	6	30
No. of dots in square	4	8	12	16	20	24	120
No. of dots in pentagon	5	10	15	20	25	30	150
No. of dots in hexagon	6	12	18	24	30	36	180

   The sequences are multiples of the number of sides of the figure (4×, 5× and 6× tables).

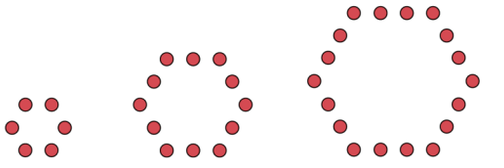
3. The patterns are all the same in the sense that they are all multiples.
   The patterns are all different in the sense that they are different multiples.



Square 1 Square 2 Square 3 Square 4



Pentagon 1 Pentagon 2 Pentagon 3



Hexagon 1 Hexagon 2 Hexagon 3

(c) Describe Figure 60 and Figure 70 in words. Calculate the number of dots in Figure 60 and in Figure 70.

(d) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Figure number	1	2	3	4	5	6	30
No. of dots in square	4	8					
No. of dots in pentagon	5	10					
No. of dots in hexagon	6						

3. How are the patterns of triangles, squares, pentagons and hexagons the same, and how are they different?

10.2 Investigate and extend more patterns

Notes on questions

In question 1 the next few numbers are easily found mentally by using the beautiful and useful horizontal increasing differences pattern, like this:

Triangle no.	1	2	3	4	5	6	7	8	9	10
No. of dots	3	6	10	15	21	28	36			

+3 +4 +5 +6 +7 +8

However, it may be useful not to calculate a running total, but to rather manipulate the numerical *structure* in the picture, e.g. Triangle 10 = 1+2+3+4+5+6+7+8+9+10+11.

Once the numerical expression is written down, we can manipulate it to an equivalent expression to make calculation easier, for example:

Triangle 10: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11

= (1+10) + (2+9) + (3+8) + (4+7) + (5+6) + 11 (the order does not matter)

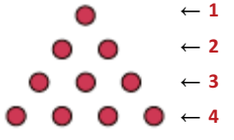
= 11 + 11 + 11 + 11 + 11 + 11= 6 × 11 = 66

In question 2 we can also use the horizontal increasing differences pattern +5, +7, +9, ... for the next few numbers, but it will be unproductive to continue like this all the way to 30. A more efficient strategy is to look at the *structure* of the pictures and to imagine in your head what Square 30 will look like. The pattern can be developed like this:

Square 1	Square 2	Square 3	Square 4	Square 5	...	Square 30
2×2	3×3	4×4	5×5	6×6	...	31×31

Answers

1. (a) In each triangle every row has one more dot than the previous row: from the top the number of dots is 1, 2, 3, 4, ...



(b) Triangle 6 has 7 dots in the bottom row, and Triangle 7 has 8.

Triangle 6: 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28; Triangle 7: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36

(c)

Triangle no.	1	2	3	4	5	6	7	8	9	10
No. of dots	3	6	10	15	21	28	36	45	55	66

2. (a) Each square has one more dot in a side that the Square number.

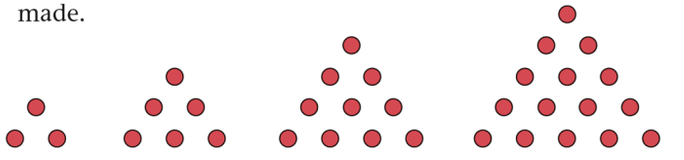
(b)

Square no.	1	2	3	4	5	6	7	8	30
No. of dots	4	9	16	25	36	49	64	81	961

3. In Section 10.1 all the sequences had a constant difference. Here the difference is not constant.

10.2 Investigate and extend more patterns

1. (a) Describe in words how this growing pattern of triangles is made.



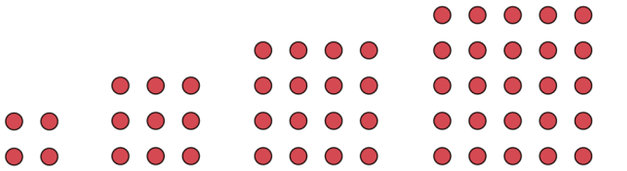
Triangle 1 Triangle 2 Triangle 3 Triangle 4

(b) Describe Triangle 6 and Triangle 7 in words. How many dots are there in Triangle 6 and how many in Triangle 7?

(c) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Triangle no.	1	2	3	4	5	6	7	8	9	10
No. of dots	3	6								

2. (a) Describe in words how this growing pattern of squares is made.



Square 1 Square 2 Square 3 Square 4

(b) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Square no.	1	2	3	4	5	6	7	8	30
No. of dots	4	9							

3. How are these patterns different from those in Section 10.1?

10.3 From tables to flow diagrams

Teaching guidelines

You should continue to emphasise different approaches to "see" the patterns that are useful to generalise towards a rule, so that the rule can then be used to calculate missing output values. Using a rule is usually easier than other methods.

Here are some alternatives approaches for making rules:

- *Visually, using the structure in the pictures.* For example:

Pink 1	Pink 2	Pink 3	Pink 4	Pink 5	Pink 6	Pink 30
1	2+1	3+2	4+3	5+4	6+5	30+29

Green 1	Green 2	Green 3	Green 4	Green 5	Green 6	Green 30
2+1	3+2	4+3	5+4	6+5	7+6	31+30

Although the above structures can be used directly to calculate output values, they do not fit the flow diagram representation. Therefore, instead of quick calculation, it is useful to first transform the numerical structure into a more useful equivalent structure, for example

Green 3 = 4+3 = 1+3+3 = 2×3 + 1; ... that can be generalised to Green n = 2×n + 1; ...

- *Numerically, using horizontal patterns.* For example, for Pink and Green: +2; +2; +2; ...
- *Numerically, using vertical patterns.* For example, for Total: *Total no. = Figure no.* × 4
- *Using relationships between the sequences*, often first using an easy known sequence like the even numbers, and then using it to find the others. See the example below.

Answers

1. Learners to describe and discuss methods and patterns in the table.

Figure number	1	2	3	4	5	6	30
Number of pink triangles	1	3	5	7	9	11	59
First use an easy sequence: Even numbers	***2***	***4***	***6***	***8***	***10***	***12***	***60***
Number of green triangles	3	5	7	9	11	13	61
Total number of triangles	4	8	12	16	20	24	120

−1
+1

Due to space limitations we use the one-line flow diagram notation, for teachers only:

2. 1, 2, 3, 4, 5, 6, 30 – [× 2] – [− 1] → 1, 3, 5, 7, 9, 11, 59
3. 1, 2, 3, 4, 5, 6, 30 – [× 2] – [+ 1] → 3, 5, 7, 9, 11, 13, 61
4. 1, 2, 3, 4, 5, 6, 30 – [× 4] – [+ 0] → 4, 8, 12, 16, 20, 24, 120

10.3 From tables to flow diagrams

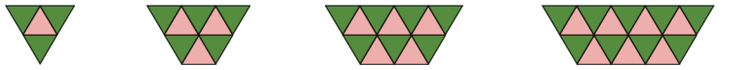
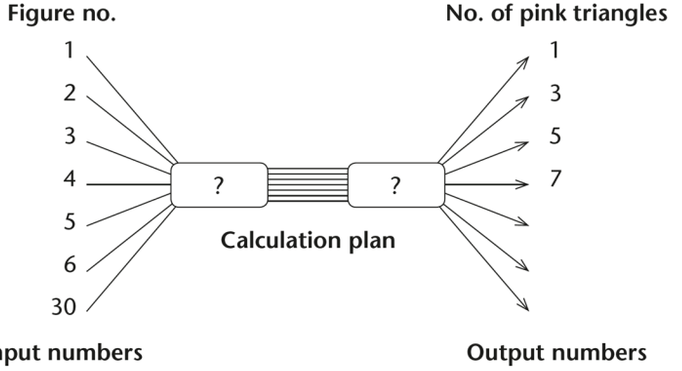


Figure 1 Figure 2 Figure 3 Figure 4

1. Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Figure number	1	2	3	4	5	6	30
Number of pink triangles	1	3	5	7			
Number of green triangles	3	5					
Total number of triangles	4	8					

2. Make a flow diagram to show how the number of pink triangles can be calculated. Use the *Figure numbers* as the *input numbers* and the number of pink triangles as the *output numbers*. Complete all missing parts.



3. Repeat question 2, but this time make a flow diagram to show how the number of *green triangles* can be calculated.
4. Repeat question 2 again but show how the *total number* of triangles can be calculated.

Grade 4 Term 4 Unit 11 Whole numbers: Addition and subtraction

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
11.1 Practise addition and subtraction	Addition and subtraction calculations	343 to 344
11.2 Find information	Problem solving, calculating, rounding off and estimating	345
11.3 One number for many	Estimation and addition, developing a feel for averages	346

CAPS time allocation	3 hours
CAPS page references	14 to 15, 69 to 71 and 118

Mathematical background

Learners revise addition and subtraction. For most of the calculations and problems that they solve, they may use any method for adding and subtracting. They also practise rounding off and estimating.

11.1 Practise addition and subtraction

Mathematical notes

Questions 6 and 7 demonstrate the associative (grouping) property of addition. It does not matter how we group numbers *within a calculation* when we add, the answer remains the same. Question 4 is another demonstration of the associative property of addition, but here the parts of the numbers are grouped differently *within the numbers* in a calculation.

Learners do *not* have to know the names of this property, just how to use it.

Teaching guidelines

Aim to cover this section in $1\frac{1}{2}$ hours. One possibility is to use

- questions 1(a), (c), (e) and (g), 2, 4, 5, 10, 11 and 12 for concept development and classwork, and
- questions 1(b), (d), (f) and (h), 3, 6, 7, 8 and 9 for additional practice.

Notes on questions

In question 4 learners are asked which questions they think will have the same answer. They are not asked to give answers. It is useful to discuss why these answers are the same.

In questions 4(a) and (d) the thousands, tens and units parts of both numbers are the same. The hundreds parts of the first and second number are swapped around. If learners separate out the hundreds parts of the numbers they will get 5 074 + 3 098 + 600 + 200 and 5 074 + 3 098 + 200 + 600. These will clearly give the same answer. If learners struggle to see this, you can demonstrate that the totals will be the same by using place value cards.

In questions 4(a) and (c) learners can separate out the thousands parts of the numbers and get 5 000 + 674 + 3 000 + 298 = 3 000 + 674 + 5 000 + 298 (the thousands parts are swapped).

In question 4(f) the same thousands, hundreds, tens and units are added, only the order is different (5 000 + 3 000) + (200 + 600) + (70 + 90) + (8 + 4).

Answers

1. (a) 7 453 (b) 3 678 (c) 7 453 (d) 3 678
   (e) 7 453 (f) 3 678 (g) 7 453 (h) 3 678
2. Learners check their answers and correct any mistakes.
3. Learners check their answers and correct any mistakes.
4. (a), (c), (d) and (f)
5. (a) 8 972 (b) 9 674 (c) 8 972 (d) 8 972 (e) 9 170 (f) 8 972
6. All four will have the same answer. (Because you can add numbers in any order without changing the answer.)
7. (a) to (d) all add up to 7 887.

UNIT 11

WHOLE NUMBERS: ADDITION AND SUBTRACTION

11.1 Practise addition and subtraction

1. Calculate.
   (a) 4 677 + 2 776 (b) 8 677 – 4 999
   (c) 3 867 + 3 586 (d) 7 867 – 4 189
   (e) 5 586 + 1 867 (f) 5 506 – 1 828
   (g) 6 195 + 1 258 (h) 6 135 – 2 457
2. If your answers for questions 1(a), (c), (e) and (g) are not the same, you have made mistakes. If this is the case, check your calculations and correct the mistakes.
3. If your answers for questions 1(b), (d), (f) and (h) are not the same, you have made mistakes. If this is the case, check your calculations and correct the mistakes.
4. Don't do the actual calculations. Just say which of these you *think* will have the same answer.
   (a) 5 674 + 3 298 (b) 5 746 + 3 928
   (c) 3 674 + 5 298 (d) 5 274 + 3 698
   (e) 5 896 + 3 274 (f) 5 278 + 3 694
5. Now do the calculations to check your answer for question 4.
6. Which of these calculations *do you think* will have the same answer, if the calculations inside brackets are done first?
   (a) 3 476 + (2 328 + 2 083)
   (b) (3 476 + 2 328) + 2 083
   (c) 2 328 + (2 083 + 3 476)
   (d) 2 083 + (3 476 + 2 328)
7. Now do the calculations to check your answer for question 6.

Notes on questions

In questions 8 and 9 it is only 8(b) that has a different answer. The answer to 9(b) is a negative number. Grade 4 learners have not yet learnt about negative numbers. They will probably say that "you cannot subtract a bigger number from a smaller number". Some learners might say that the answer is zero. You can either let learners know that they will learn about numbers smaller than zero in later years, or you could use this opportunity to explain and explore negative numbers. You can start with counting backwards from zero and then do some calculations with smaller numbers, for example 2 – 3; 4 – 6; 1 – 9, etc.

Answers

8. Learners make a judgement.
9. (a) 3 721 (b) –935
   (c) 3 721 (d) 3 721
   (e) 3 721 (f) 3 721
10. (a) Rounded to the nearest 1 000:

1 000	2 000	1 000	2 000	1 000
1 000	0	1 000	1 000	1 000

    Estimated total: 11 000

    (b) Rounded to the nearest 100:

1 000	1 700	900	1 600	500
1 300	400	800	1 000	600

    Estimated total: 9 800
11. Learners may make different transfers. Check all their answers.
12. If learners worked accurately, their total should be 9 894.

8. Which of these calculations *do you think* will have the same answer, if the calculations inside brackets are done first?
   (a) 3 476 + (2 328 – 2 083)
   (b) 3 476 – (2 328 + 2 083)
   (c) (3 476 – 2 083) + 2 328
   (d) (3 476 + 2 328) – 2 083
   (e) 2 328 + (3 476 – 2 083)
   (f) (2 328 – 2 083) + 3 476
9. Now do the calculations to check your answer for question 8.
10. Janice has to add all these numbers together:

976	1 721	881	1 648	546
1 254	447	764	1 034	623

    (a) Round each number off to the nearest 1 000 and use the rounded figures to make an estimate of the total.
    (b) Round each number off to the nearest 100 and use the rounded figures to make a better estimate of the total.
11. Janice decides not to calculate the accurate total of the numbers in question 10 by working in her normal way. She decides to transfer parts between numbers so that she can end up with easy work. For example:
    - Janice transfers the 4 from the 1 034 to the 976 to make 1 030 and 980.
    - Janice also transfers the 20 from the 623 to the 881 to make 603 and 901.

    Rewrite the numbers in question 10 in your book. Transfer parts like Janice did until it is very easy to add up the new numbers.
12. Now find the total of all the new numbers that you made in question 11.

11.2 Find information

Teaching guidelines

Aim to cover this section in 45 minutes. One possibility is to use

- questions 1, 2 and 5 for classwork, and
- questions 3 and 4 for additional practice.

Let learners work on the problems on their own. As they work, go around and check that they understand the questions by asking them to retell the questions in their own words.

Possible misconceptions

Check that learners understand that in question 1 there were 7 352 books *after* the book sale. This means that there were *more books before* the book sale. They need to add 7 352 + 2 659.

Notes on questions

Note that question 2 involves multiplication. The inclusion was deliberate. It is important that learners develop the habit and skill to think critically when they read and analyse problems so that they choose an appropriate operation to solve the problem.

Answers

1. 10 011 books
2. 384 beds
3. R6 940
4. 889 bricks
5. 94 mm; 87 mm; 68 mm; 79 mm; 105 mm; 112 mm; 94 mm
   Total: 639 mm

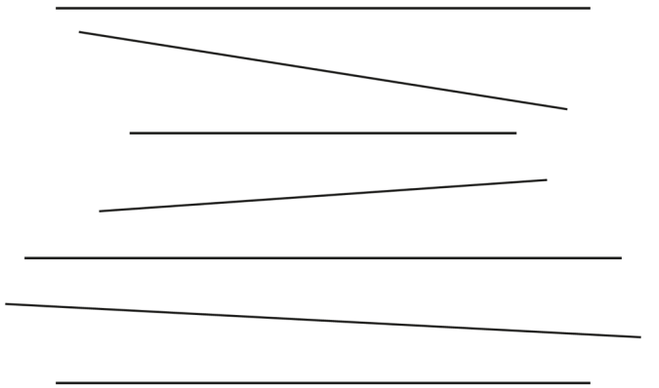
11.2 Find information

1. In December, a hospice shop sold 2 659 of its books at a bargain sale. After the book sale, there were 7 352 books left on its shelves. How many books were there before the sale?
2. Twenty-four new clinics are built in a certain province, each with sixteen beds for patients. How many beds is this in total?
3. Gabieba has R9 800 in her savings account. She withdraws R840, R475, R910 and R635 on different days. How much money is left in her account?
4. Kaliep laid 8 773 bricks and Solly laid 7 884 bricks. How many more bricks did Kaliep lay than Solly?
5. How long are all these line pieces together?

   First make an estimate.

   Then measure and calculate.

   Give your answer in millimetres.



11.3 One number for many

Mathematical notes

While this section involves practice in addition and subtraction, it also serves to develop an intuitive sense of the concept "average" (mean), which is part of the content of Data Handling.

Learners will also be rounding and estimating.

Teaching guidelines

Aim to cover this section in 45 minutes. One possibility is to use

- questions 1 to 7 for classwork, and
- question 8 for additional practice.

Answers

1. No, because most numbers are bigger than 500.
2. Learners choose a number and estimate the total.
3. 7 400
4. Bennie's estimate of the sum was 5 000.

   Learners' answers will differ, but they need to see whether the difference between their estimate and the exact answer is more than 2 400 or less than 2 400.
5. $10 \times 800 = 8\,000$

   Learners make a judgement, but they need to see whether the difference between their estimate and the exact answer is more than 600 or less than 600.
6. 700 or 750 would be a good choice.
7. 740 would have been a "perfect" choice.
8. (a) A number between 1 700 and 1 800 would be a good choice.
   (b) 8 654

11.3 One number for many

Bennie has to find the sum of the ten numbers below.

To "find the sum" means you have to add all the numbers together.

He decides to make an estimate by choosing an easy number that is close to these numbers and multiplying it by 10.

887	734	639	729	901
663	781	809	585	672

Bennie chooses 500 and multiplies it by 10, so his estimate of the sum is 5 000.

1. Do you think Bennie made a good choice for a single number? If not, explain why you think 500 is not a good choice.
2. Choose a number that you think is better than 500, and use it to make your estimate of the sum of the above numbers.
3. Now add up the ten numbers accurately.
4. Was your estimate of the sum better than Bennie's estimate?
5. Investigate whether the single number 800 would have produced a better estimate than the number you chose.
6. Which of the following numbers would have been the best choice for a single number?

   600 650 700 750 800
7. Which number would have been a "perfect" choice for a single number?
8. (a) Choose a single number and use it to estimate the sum of 1 354, 2 007, 1 785, 1 576 and 1 932.

   (b) Add the numbers up accurately to check how close your estimate was.

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
12.1 How does it work to flip a coin?	Chance, likelihood of a certain result	347
12.2 An experiment with flipping a coin	Two possible outcomes with equal chances of occurring, but experimental results vary	348 to 350
12.3 An experiment with rolling a die	Six possible outcomes with equal chances of occurring, but experimental results vary	350 to 352

CAPS time allocation	2 hours
CAPS page references	31 and 119

Mathematical background

When a coin is tossed, one of two things can happen: the coin can come to rest on one side or on the other side. Stated differently, there are two possible outcomes. The terms HEADS and TAILS are often used to distinguish the two sides of a coin. When a normal coin is tossed many times, there is no reason to expect that the one outcome (HEADS) will occur more often than the other outcome (TAILS). We say the two possible outcomes are "equally likely" – this is a way of saying that one would expect more or less the same number of HEADS and TAILS if a coin is tossed many times. The same applies to the rolling of a die, though in this case there are six different equally likely outcomes.

When a fair coin is tossed (or an unbiased die is rolled) once, it is impossible to predict with any confidence what the outcome of the event will be. Although the range of possible outcomes is known, no grounds exist to predict that one outcome rather than another will occur. Any of the outcomes is exactly as likely to occur as any other. Hence the outcome of the event is **unpredictable**. Such events are called **random events**.

Although the outcome of a random event is completely unpredictable, predictions can be made about approximately how often a particular outcome will occur if the event is repeated many times. For example, if a coin is tossed many, many times, it will end up on one side for about half of the time and on the other side for about half of the time. If an ordinary die is rolled many, many times, the number 4 (or any other number in the range 1, 2, 3, 4, 5, 6) can be expected to occur roughly one sixth of the time. Suppose another die is not marked 1, 2, 3, 4, 5, 6 on its six faces, but coloured red on one face, blue on two faces and yellow on three faces. If such a coloured die is rolled many, many times, red can be safely predicted to come on top about 1 sixth of the time, blue to come on top roughly one third of the time and yellow to come on top roughly half of the time.

The activities in this unit provide learners with experiences of repeated **random events**, with a view to provide them with experiences where the different possible outcomes occur **approximately** the same number of times.

Resources

A coin for each pair of learners.

A die for each pair or group of learners.

Square grid paper to record results.

12.1 How does it work to flip a coin?

Mathematical notes

When we flip a coin there are two possible outcomes: HEADS and TAILS. Each outcome has the same chance of occurring every time you flip the coin. The focus of this section is on fairness. If there is an equal chance of getting any of the results, it does not matter which possibility you choose because there is no way of predicting what the outcome will be.

Teaching guidelines

When learners do the experiments their results will differ. That is the nature of probability experiments. Lots of discussion is needed about what learners expect. The discussion in this section focuses on whether there is an equal chance of getting the result of your choice. It also focuses on the understanding that only if there is an equal chance of getting any of the results, is the mechanism/method a fair way to make a decision.

Aim to spend about 30 minutes on this section.

Possible misconceptions

Learners may think that they can accurately predict the result when flipping a coin. They may predict correctly now and then, but that is "luck".

Notes on questions

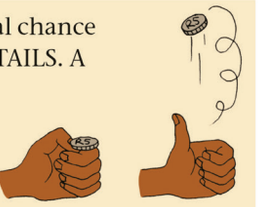
Let learners draw up tally tables to record the results of their experiments.

Answers

1. (a) Discussion about fairness. A coin flip is a fair way to decide because both teams have the same chance to win the toss.
   (b) Answers will differ. There are different ways to make such decisions in different cultures. Learners may talk about methods such as choosing the fist with the stone, or counting rhymes they use on the playground. Counting rhymes have a pattern and one can work out what the result will be. Therefore counting rhymes are not fair in the way a coin flip is.

UNIT 12 PROBABILITY

When you flip a coin, then there is an equal chance that it lands on HEADS or that it lands on TAILS. A coin can only land on one of these.



If you say "heads" when your teacher flips a coin, and it lands on HEADS, then you win. If it lands on TAILS, you lose.

Flipping a coin helps us make some decisions.

12.1 How does it work to flip a coin?

1. (a) When two teams are playing each other, they agree to flip a coin to make a decision. The person who wins the toss starts the game. Do you think this is fair? Why do you say so?

   (b) What else do people use when they may not agree with each other and they have to make a fair decision? Discuss other ways of helping to make a decision, like drawing straws (the person who draws the short straw is usually the loser). Write down at least one other way.

A coin has two sides called HEADS and TAILS. HEADS is the side with the coat of arms of South Africa, and TAILS is the side with the number that tells the value of the coin. We say a **coin flip** has **two outcomes**: either HEADS or TAILS.



HEADS



TAILS

Sometimes people think they can predict which side the coin will land on. People think one side is luckier than the other side, or that one HEAD must be followed by one TAIL. We will do an experiment to better understand how it works to flip a coin.

12.2 An experiment with flipping a coin

Mathematical notes

There is no way to tell whether you will get exactly the same results as before when you repeat an experiment with coins. We all know that the chances are small to get exactly the same number of HEADS and TAILS when you compare 20 flips of a coin. However, it is not impossible to get the same result again.

Teaching guidelines

Let learners work in pairs. One learner collects the data of the flips and the other shades squares in a tally table.

For question 3 you may prepare a table on the board in which to write each learner's results so that you can compare the results as a class.

Aim to spend about 45 minutes on this section.

Possible misconceptions

Learners may think that exactly half of the flips *must* be HEADS and half *must* be TAILS.

Critical knowledge

When we flip a coin there are two possible outcomes. Each outcome has the same chance to occur every time we flip the coin.

Answers

1. Answers will differ. If there is time, allow more flips.
2. (a) The total number of flips each learner recorded may differ.
   (b) The total number of HEADS will differ from one learner to the next. Some may have the same total.
   (c) The fraction of HEADS and the fraction of TAILS must add up to 20 out of 20 ($\frac{20}{20}$) if they made a total of 20 flips.
3. (a) Results will differ. It is not possible to get a bigger fraction than $\frac{20}{20}$ HEADS. It is possible to get $\frac{20}{20}$ and $\frac{0}{20}$ HEADS. Both results are unlikely – we will be surprised to see such a result.
   (b) The results differ because we cannot predict how the coin will land.

12.2 An experiment with flipping a coin

Work on your own to flip a coin many times and record what happens each time. You need a sheet of squared paper and a coin.

Make a key on your sheet of paper:

HEADS ☐ TAILS ☐

Each time the coin lands HEADS up, shade in a square with your pencil. Each time the coin lands TAILS up, leave the block unshaded.

Below is an example of 20 flips of a coin that were recorded by shading blocks:

T	H								

In the example above, the HEADS are 11 out of 20, and the TAILS are 9 out of 20.

1. Now do your own experiment. Flip a coin at least 20 times and record the results. If you have time, flip the coin 100 times.
2. Work with your own data.
   (a) How many flips did you do?
   (b) How many times did you get HEADS in your experiment?
   (c) What fraction of the flips was HEADS and what fraction of the flips was TAILS?
3. Compare your data with that of other classmates.
   (a) What fraction of HEADS did they get in their experiments?
   (b) Why do you think your data are not all the same?

When you record the way the flipped coin landed, this is your **data**. We can analyse the data to understand coin flipping better.

Answers

4. (a) Each learner calls out the number of HEADS in his or her experiment so that the tally table can be completed.

   Example tally table (the data from your class will differ):

Number of HEADS in 20 flips per learner

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		/	/		//		/	///	//	////	~~////~~	////	///	//	///		/			

(b) Example of pictograph (the data from your class will differ):

											x									
										x	x	x								
								x		x	x	x	x		x					
					x			x	x	x	x	x	x	x	x					
		x	x		x		x	x	x	x	x	x	x	x	x		x			
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Number of HEADS in 20 flips per learner

(c) The graph shows that most of the fractions are close to the middle of the range.

5. (a) Example of data table (the data from your class will differ):

Number of flips	Fraction HEADS	Fraction TAILS
After the first 20 flips	$\frac{6}{20}$	$\frac{14}{20}$
After 40 flips	$\frac{24}{40}$	$\frac{16}{40}$
After 60 flips	$\frac{26}{60}$	$\frac{34}{60}$
After 80 flips	$\frac{35}{80}$	$\frac{45}{80}$
After 100 flips	$\frac{46}{100}$	$\frac{54}{100}$

(b) Answers will differ. In general, the more flips that are combined the closer to half of the total the fractions can be expected to be.

(c) See question 4(b). It is unlikely to get very small numbers of HEADS out of 100 flips. The number line must start at the smallest number obtained in the class and go up to the biggest number obtained in the class.

Sometimes people think the number of HEADS and the number of TAILS must be the same in any experiment. This is not true. Only when we flip a coin many, many, many times can we expect that the total number of HEADS and TAILS may be the same. We cannot expect that in short experiments. Why is this?

4. Work with the rest of the class.

   (a) Make a tally table with rows from 0 to 20 to show how many HEADS each classmate got out of 20 flips of a coin. (Nobody can get more than 20 HEADS in 20 flips.)

   (b) Make a pictograph. Draw a number line that runs from 0 to 20. Make a cross above the correct number to show how many HEADS each one got.

   (c) Write a short paragraph about the story of the graph.

5. Combine the data of some classmates until you have data of 100 flips.

   (a) Complete the table with the data of 100 flips.

Number of flips	Fraction HEADS	Fraction TAILS
After the first 20 flips		
After 40 flips		
After 60 flips		
After 80 flips		
After 100 flips		

   (b) Compare your table with the results of another group of classmates with 100 combined flips. How much different are your data after 20 flips? And after 100 flips?

   (c) Make a pictograph of the number of HEADS out of 100 flips that different groups of classmates got. Think before you start: Does your number line have to run from 1 to 100?

Answers

5. (d) Answers may differ. For example: The number of HEADS out of 100 flips is mostly between___ and ___. A few groups got more than ____ HEADS out of 100, and some got fewer than ___ HEADS out of 100.
   (e) Answers will vary. The more flips you do, the closer the fraction of HEADS is expected to get to half of the total number of flips. Still, we cannot predict with certainty what the exact result will be.

6. (a) Possible, but unlikely.
   (b) Possible, but unlikely.
   (c) Answers will differ. It is quite likely to get 15 HEADS in 20 flips.
   (d) Answers will differ. Although it is not unlikely to get 20 HEADS in 100 flips, it will be unusual. We expect a fraction that is closer to half of 100.
   (e) Answers will differ. We cannot predict exactly, but we can expect a fraction close to half of 100.

7. (a) 20 – 12 = 8. 8 TAILS
   (b) 100 – 56 = 44. 44 TAILS
   (c) The number of HEADS and TAILS must add up to the total number of flips.

12.3 An experiment with rolling a die

Critical knowledge

When we roll a die there are six possible outcomes. Each outcome has the same chance to occur every time we roll the die.

Notes on questions

For question 7 learners may combine their data to get the results of 100 rolls instead of rolling 100 times.

Answers

1. Possible numbers: 1, 2, 3, 4, 5, 6.
2. Learners' lists of numbers will vary. We cannot predict accurately, but it is possible to get all the numbers.

(d) Write a short paragraph to tell the story of the pictograph.
(e) What do you think will happen after 100 flips? Write why you say so.

6. Think about your experiment.
   (a) Do you think it is possible to get 100 HEADS in 100 flips of a coin? Write why you say so.
   (b) Do you think it is possible to get 0 HEADS in 100 flips of a coin? Write why you say so.
   (c) Will you be surprised to see 15 HEADS in 20 flips?
   (d) Will you be surprised to see 20 HEADS in 100 flips?
   (e) If your class repeated the experiment of flipping a coin, about how many HEADS would you expect to get in 100 flips?
7. (a) If you flipped a coin 20 times and you got 12 HEADS, how many TAILS did you get?
   (b) If you flipped a coin 100 times and you got 56 HEADS, how many TAILS did you get?
   (c) How did you know the answers to (a) and (b) without actually flipping a coin?

12.3 An experiment with rolling a die

Games with dice are very popular, and are played by children and adults. This is because we cannot predict what number on the die we will get for any specific throw. We will gather data about rolling a die to learn how it works.

1. Write down the possible numbers that you can get if you roll a die.
2. Write down what you think. If you roll one die 10 times, what numbers do you think you will get? Why do you say so?



Teaching guidelines

Let learners work in pairs. One learner records the data of the rolls in a grid and the other records them in a tally table. The information in the tally table does not give the results of the rolls one after the other. Use the grid blocks to discuss that there is no fixed pattern or order in which you get numbers.

For question 3 you may prepare a table on the board in which to write each learner's results so that you can compare the results as a class. Similarly, with question 4 you may prepare the "frame" of the pictograph on the board while learners roll their dice.

Aim to spend about 45 minutes on this section.

You can remind learners that because a die has six identical faces shaped like squares, it looks like a cube.

Answers

3. Make a tally table that runs from 1 to 6.
4. Answers will differ.
   (c) Answers will differ. For example: I got the same number of ___ and ___. I was surprised to get so many ____, and so few ____.

Prepare to gather data about rolling a die.

Work on squared paper. Each time you roll the die write the number on the die in one block.

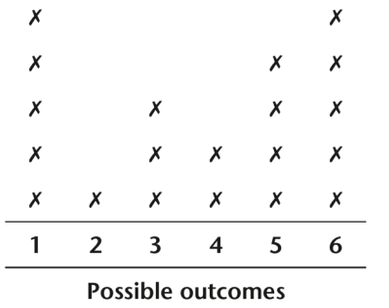
Below is an example of the results of rolling a die 20 times.

1	4	4	1	5	6	5	5	3	6
6	6	3	3	6	2	1	1	5	1

3. Do your own experiment. Roll one die at least 20 times. If you have time, roll the die many times. Record your data in a tally table.

When you roll one die there are six possible outcomes.

Below is a pictograph of the data in the table above.



4. Now make a pictograph to show how many of each number you got in *your* experiment.
   (a) Draw a number line in your book that runs from 1 to 6.
   (b) Make crosses above the correct numbers to show your results.
   (c) Write a short paragraph about the story of your graph.

Answers

5. (a) Answers will differ.
   (b) The graphs are different because you rolled only 20 times each. The more times you roll the more you can expect your graphs to start looking the same.
6. There still will be only 6 possible outcomes. You can expect the graph to start evening out, and the differences between the different outcomes to be smaller than with 20 rolls.
7. Results will differ.
8. Graphs will differ. In general you expect the results to even out.
9. (a) Did that happen to anyone in the class? If you roll 20 times, you should not be too surprised if you get no 2s. You should be surprised if you get no 2s in 100 rolls.
   (b) Did that happen to anyone in the class? Yes, you should be surprised, especially if you roll many times.
   (c) Did that happen to anyone in the class? No, you should not be surprised if you roll about equally many of each number. Each number has the same chance with every roll.
   (d) Did that happen to anyone in the class? Yes, you should be surprised, especially if you roll many times.
   (e) Did that happen to anyone in the class? You should not be surprised.

5. Compare your pictograph with that of a classmate.
   (a) Explain the differences between your graphs.
   (b) Why do you think your results are different?
6. How will your graph change if you record the results of 100 rolls of one die? Why do you say so?
7. If you have not yet done so, roll a die 100 times and record the results on a pictograph.
8. Write a short paragraph to tell the story of your experiment. Say how much your data about the number of each outcome differ.
9. Interpret your pictograph to answer these questions. Give reasons from your data.
   (a) If you do the experiment again, would you be surprised if you throw no 2s?
   (b) Would you be surprised if you roll the same number all the time?
   (c) Would you be surprised if you roll almost the same number of all the possible outcomes?
   (d) Would you be surprised if one number is never repeated?
   (e) Would you be surprised if you roll the same number three times in a row?

Addendum

General resources

Rulers 414
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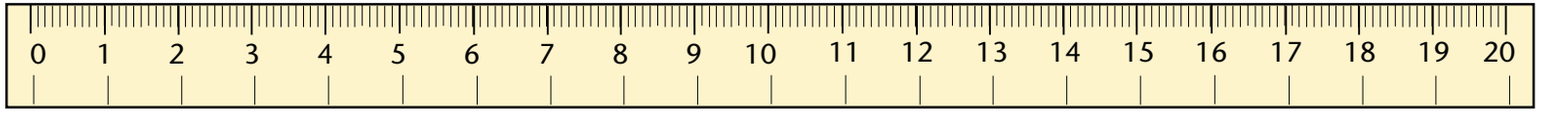
Place value cards for teachers 419
Grid paper 433
Dotted paper 434

Resources per term

Term 1 Resources 435
Term 2 Resources 447
Term 3 Resources 454
Term 4 Resources 462

If you do not have access to a photocopy machine, ask learners to prepare grids or tables beforehand, for example as homework, so that they do not waste class time.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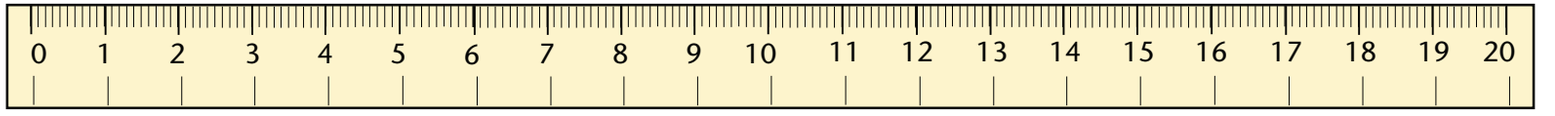
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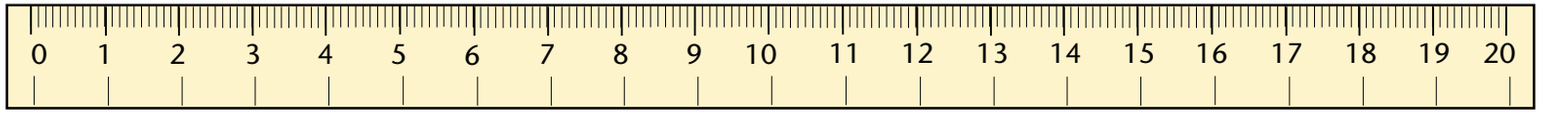
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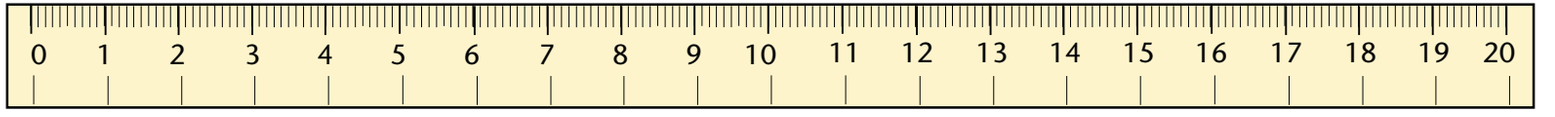
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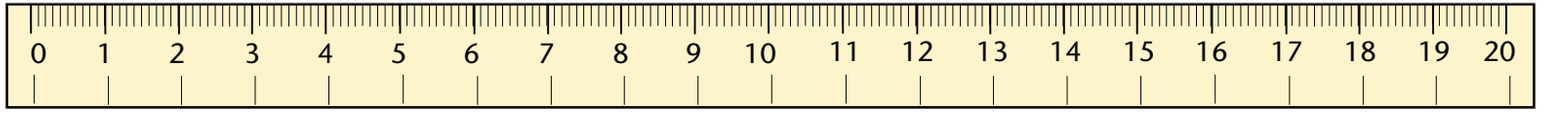
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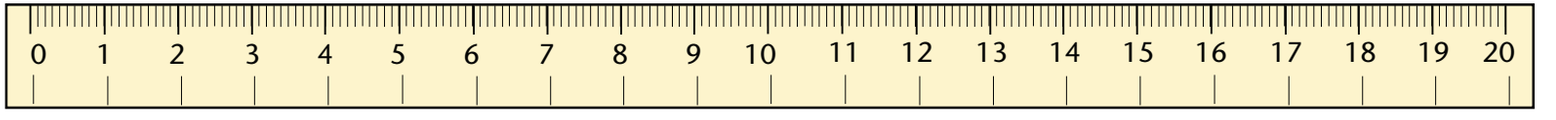
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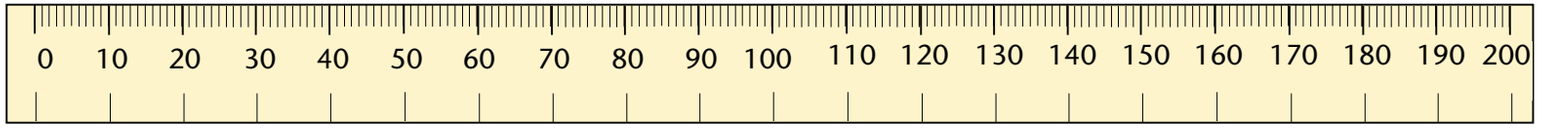
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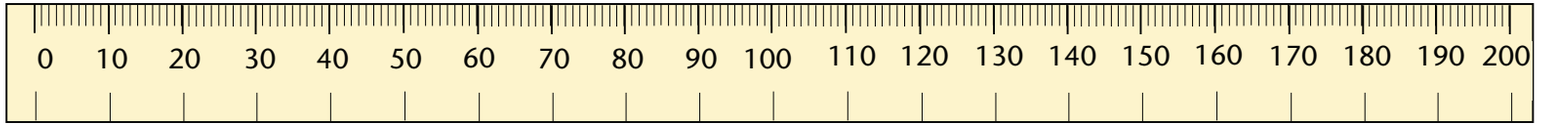
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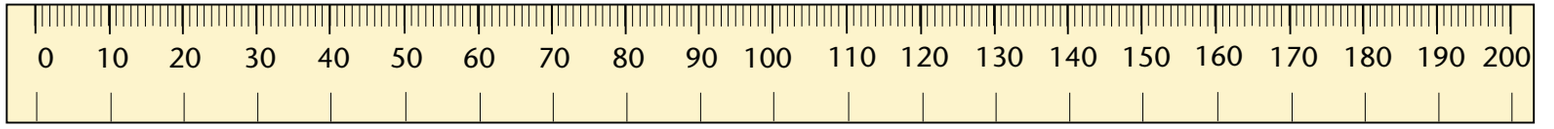
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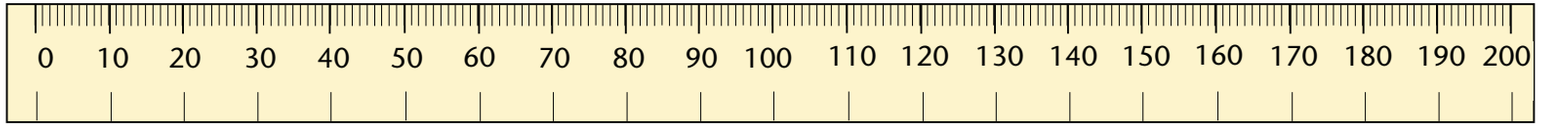
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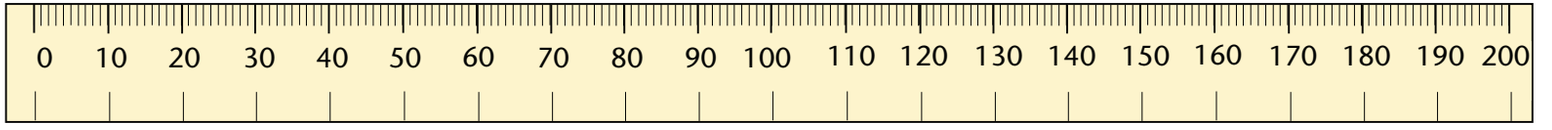
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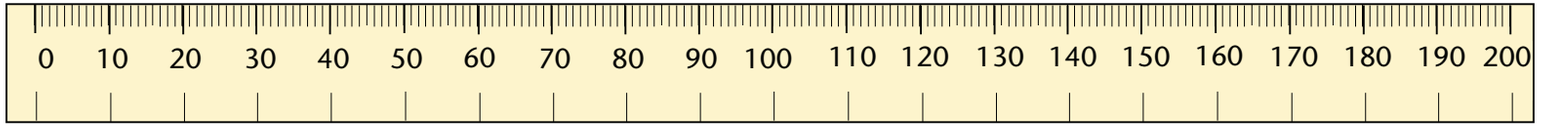
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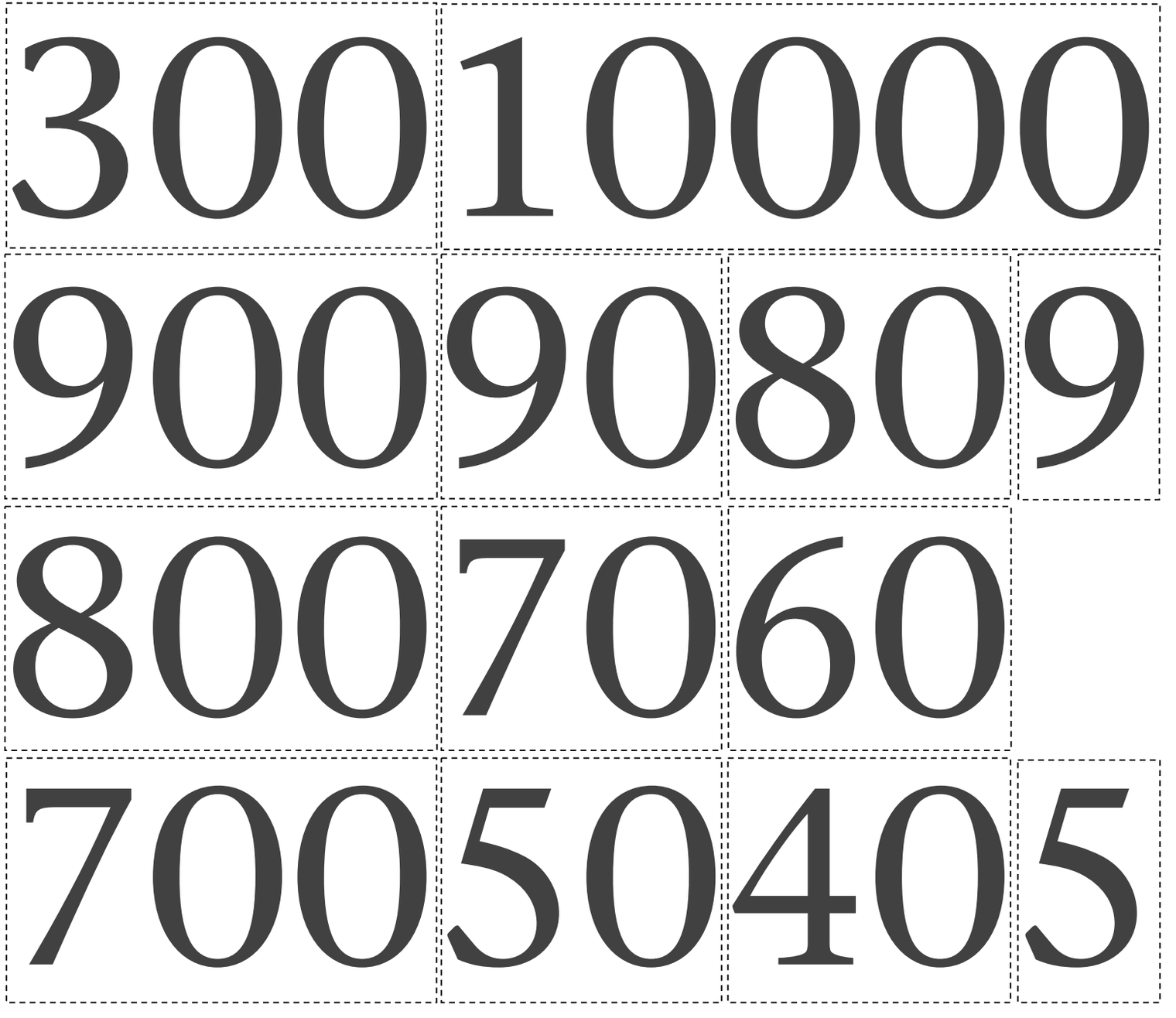
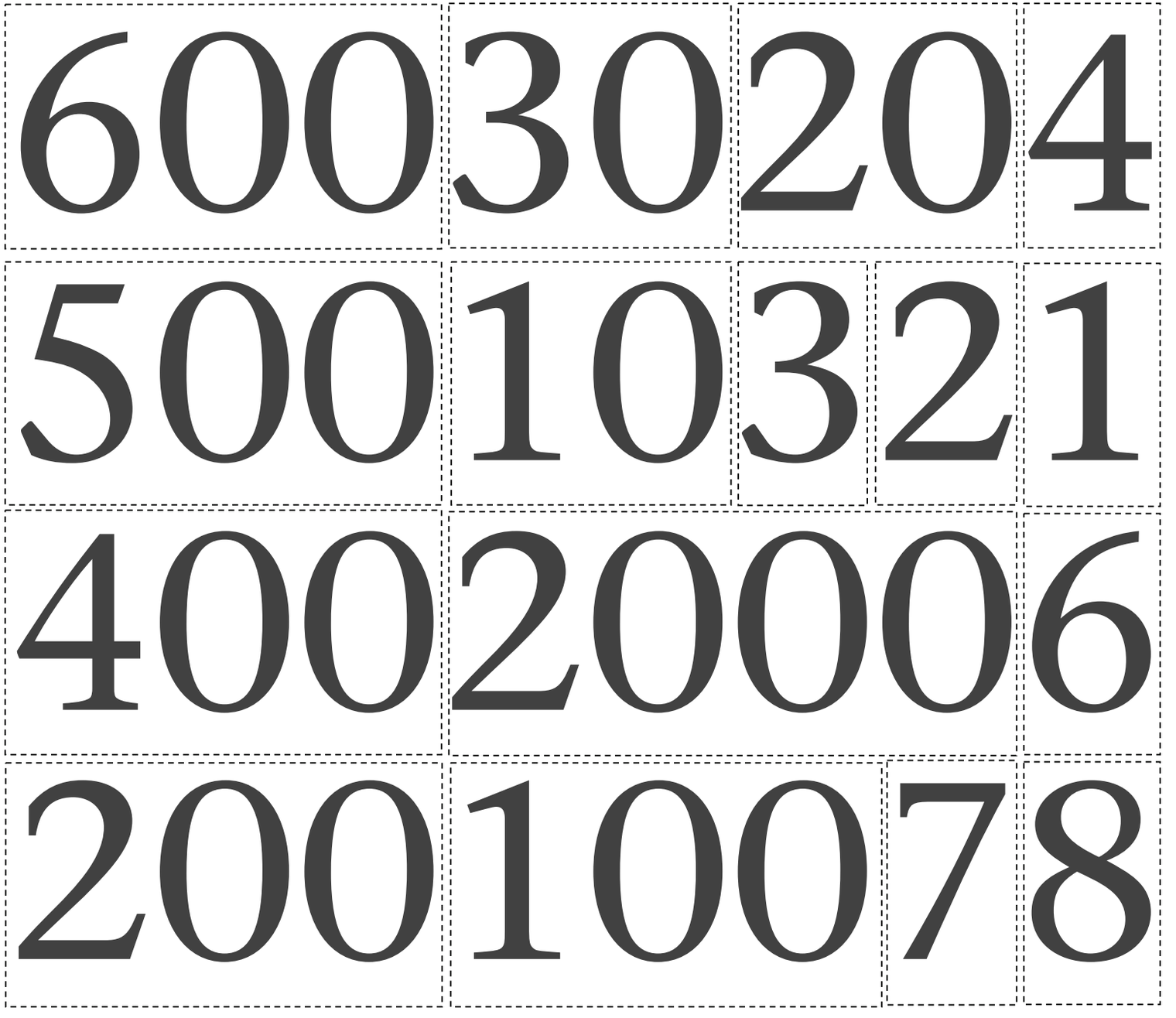


Place value cards for learners

(3 pages = 1 set)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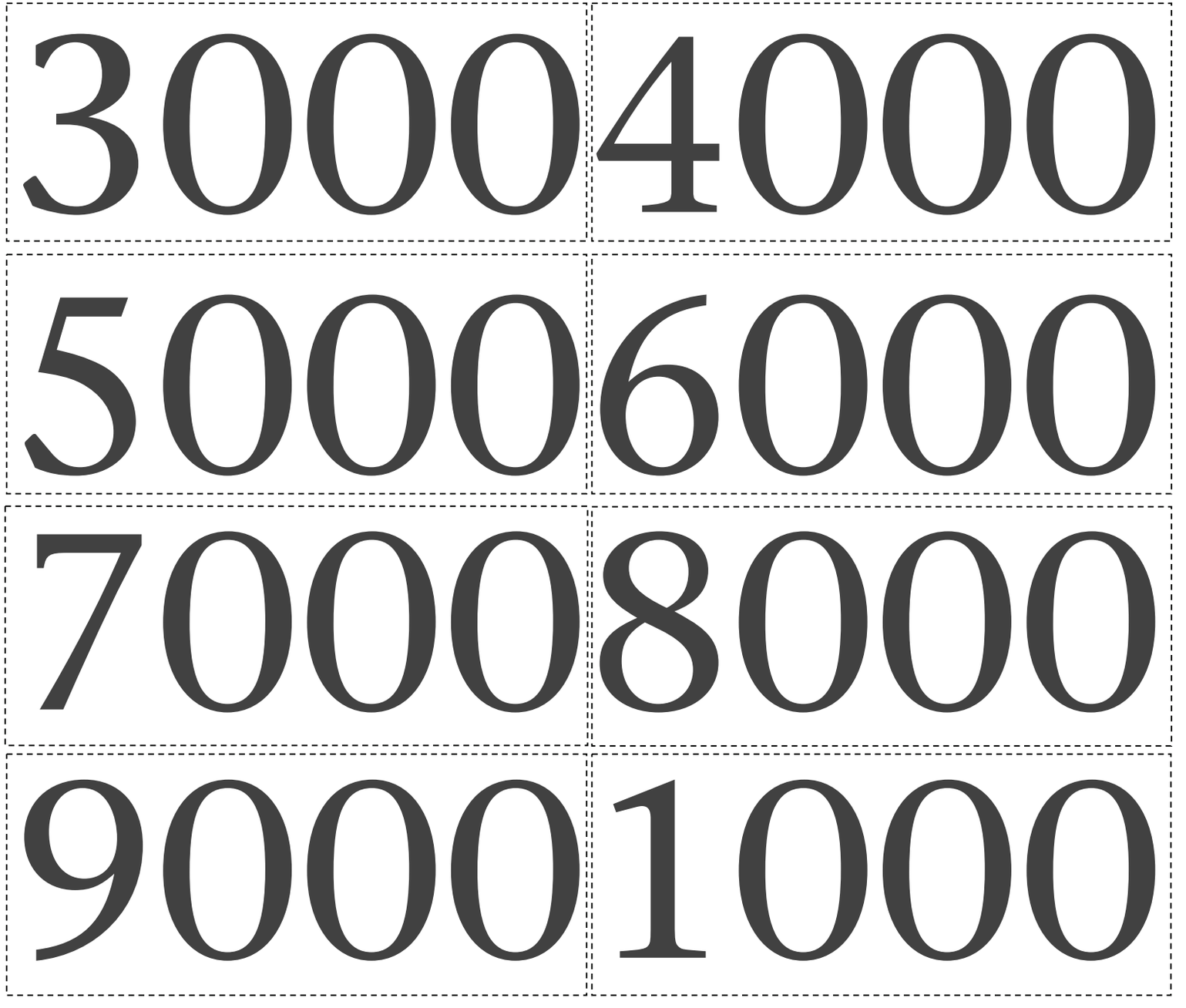
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Place value cards
for teachers

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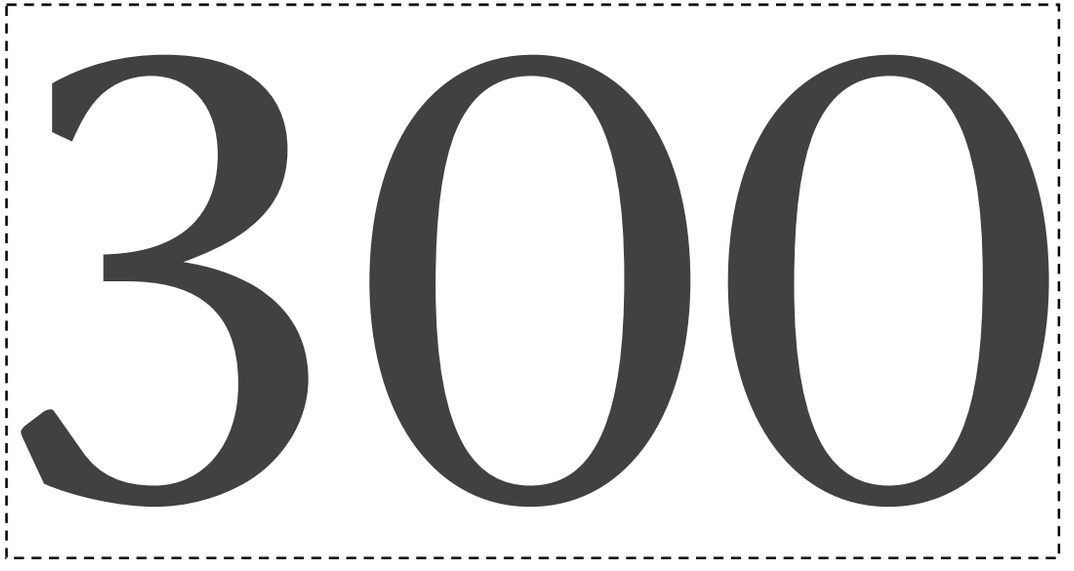
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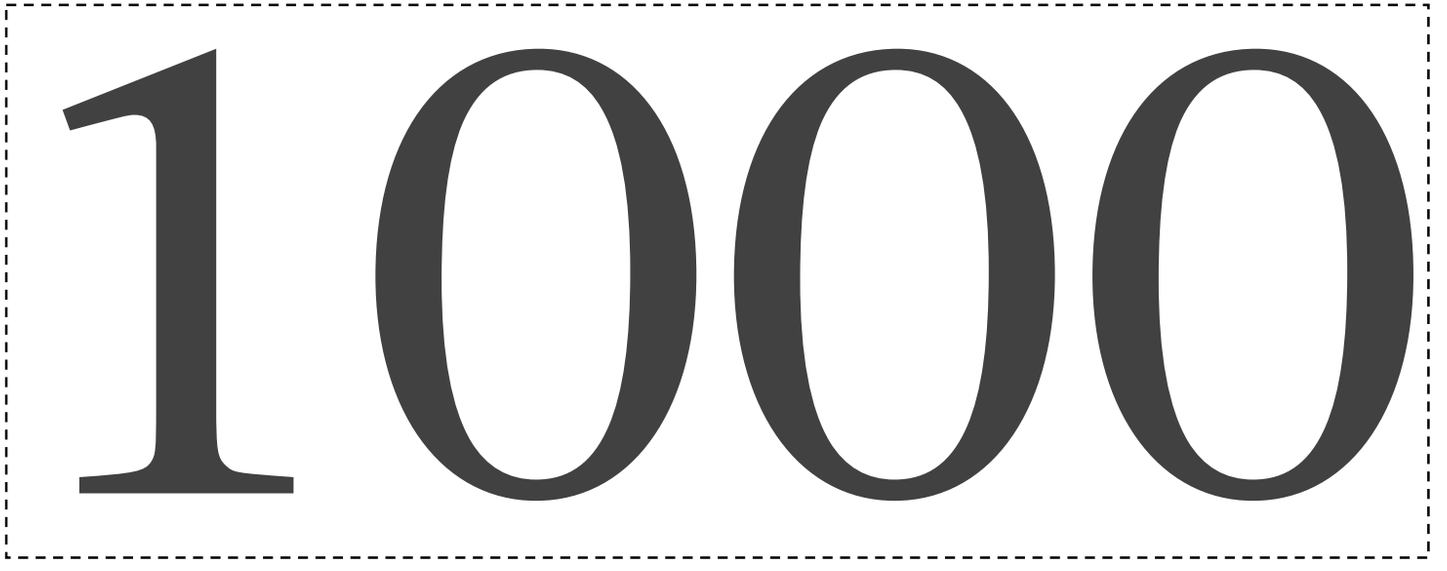
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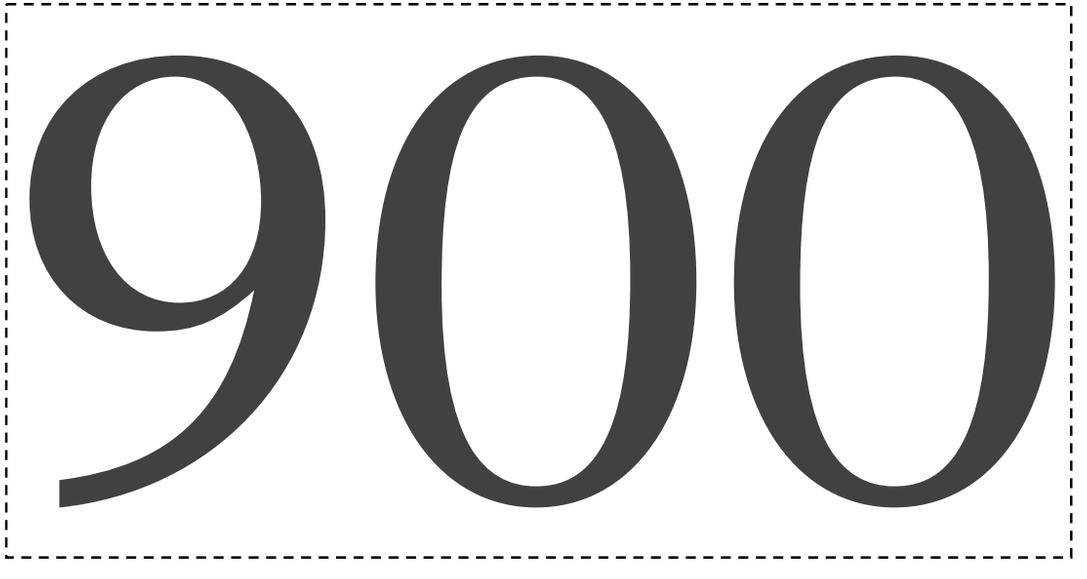
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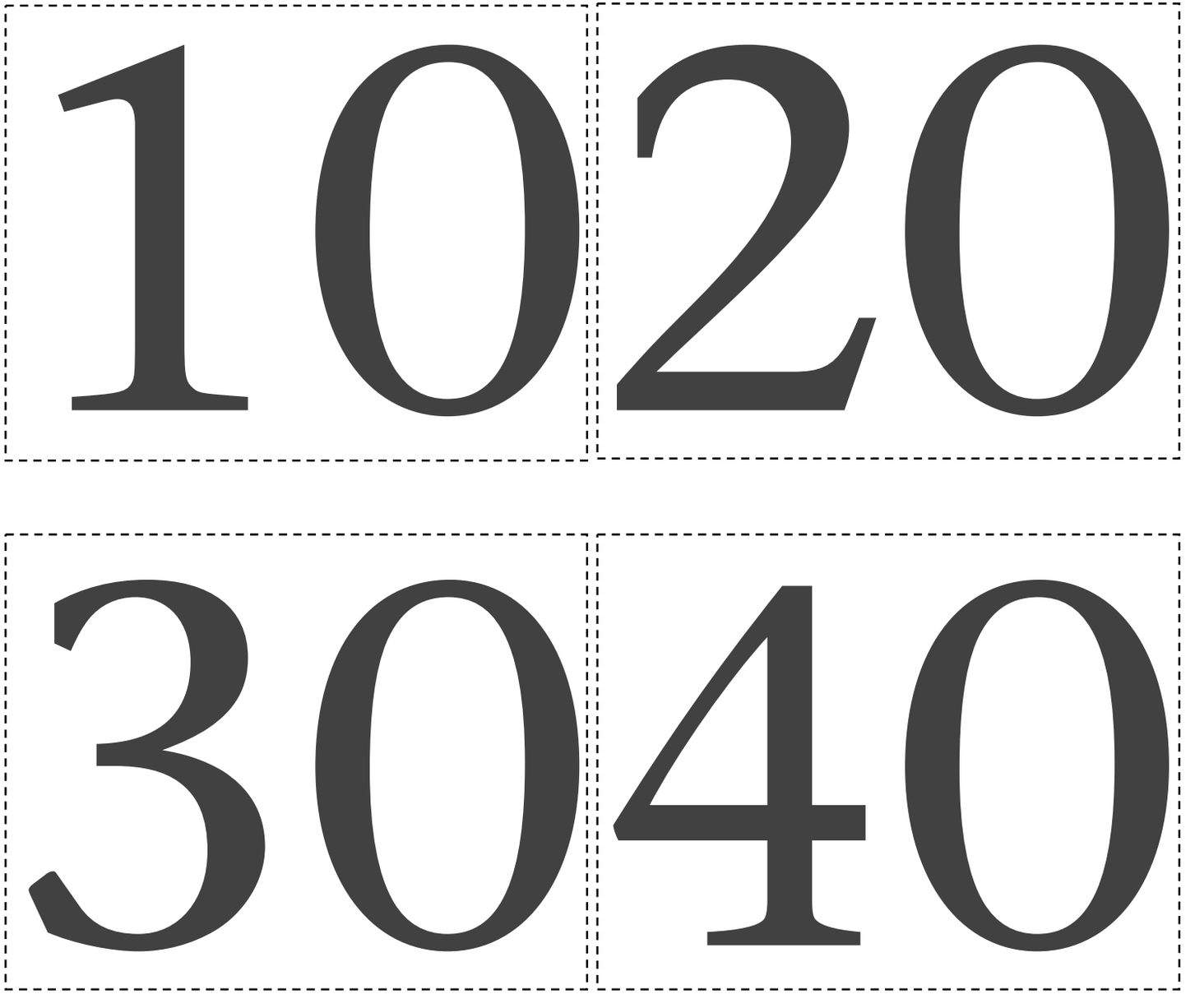
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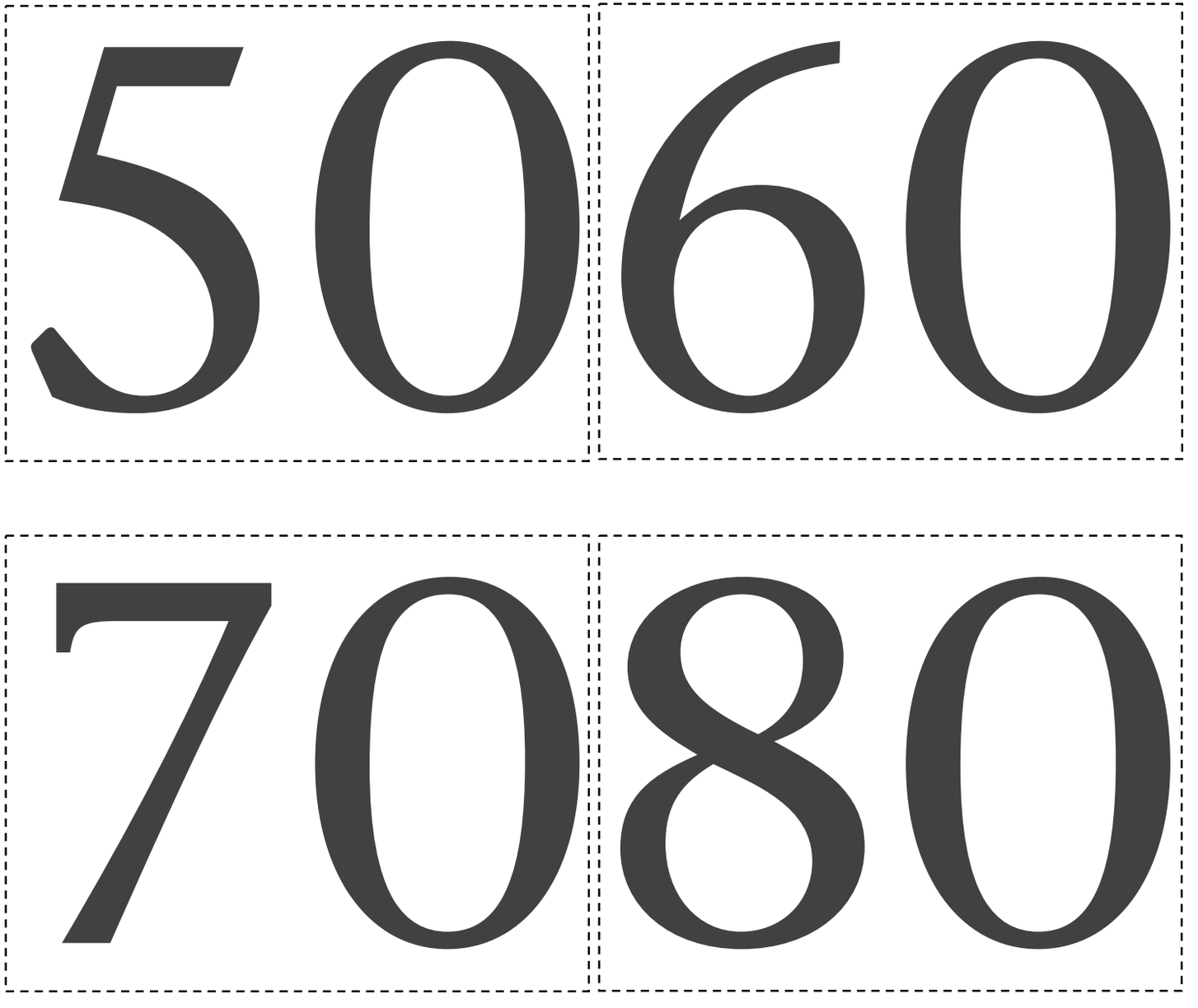
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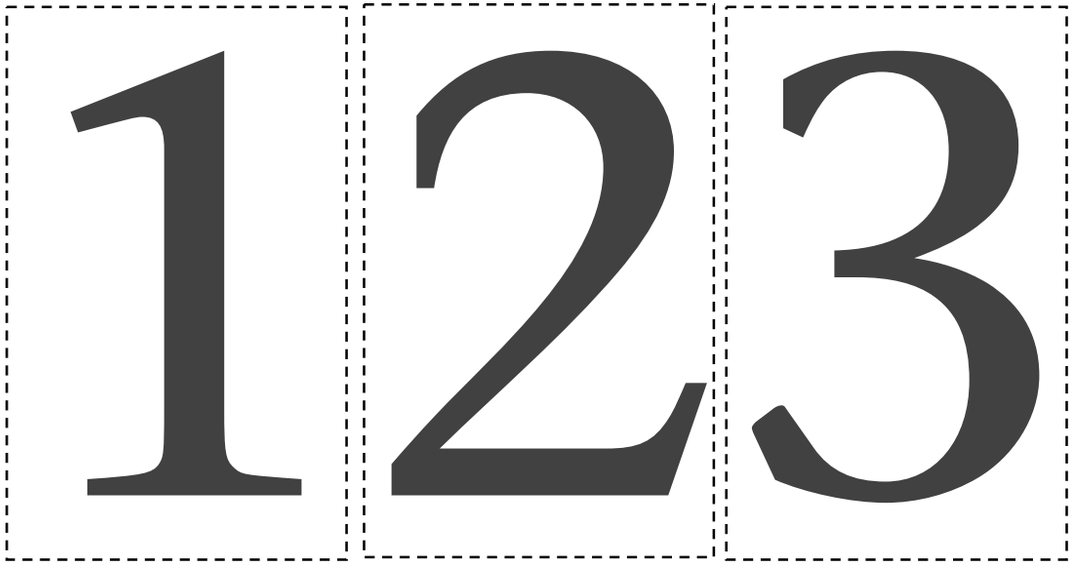
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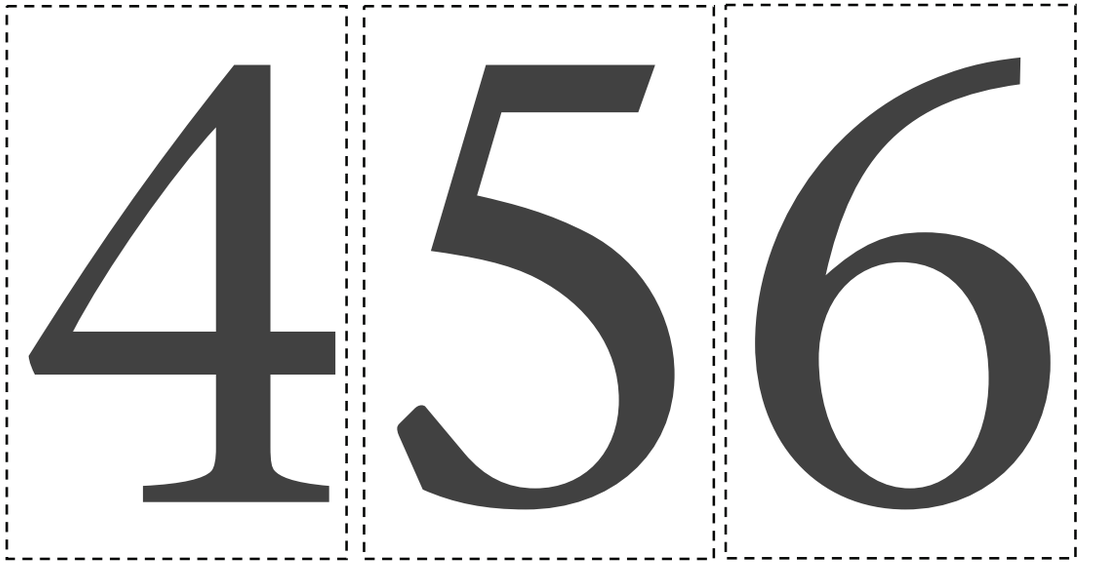
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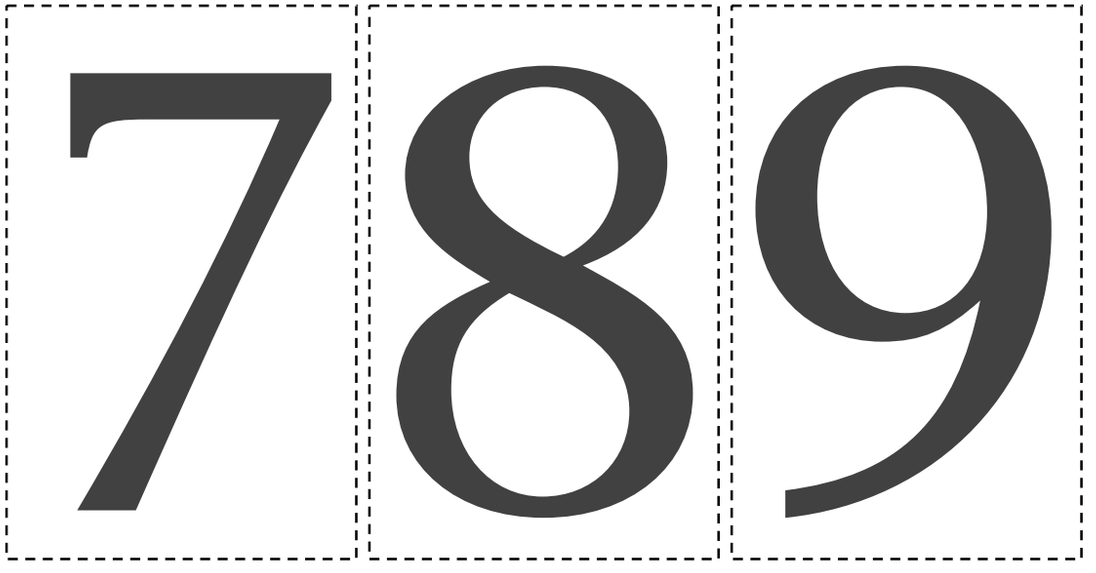
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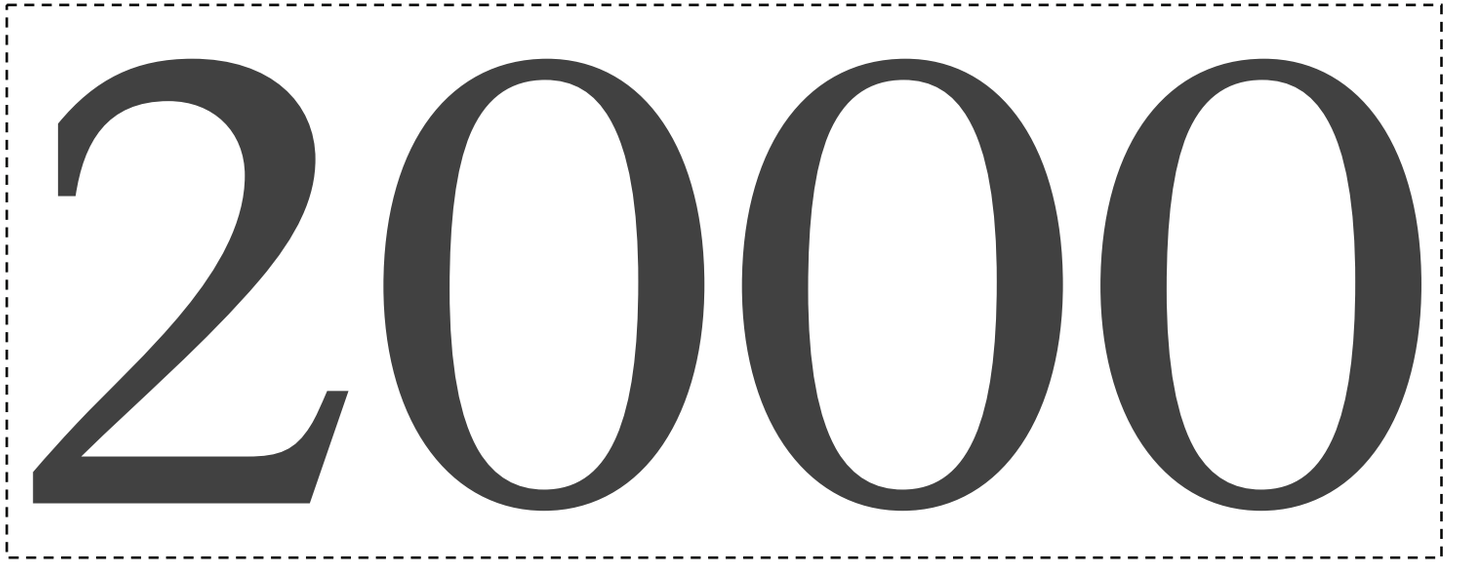
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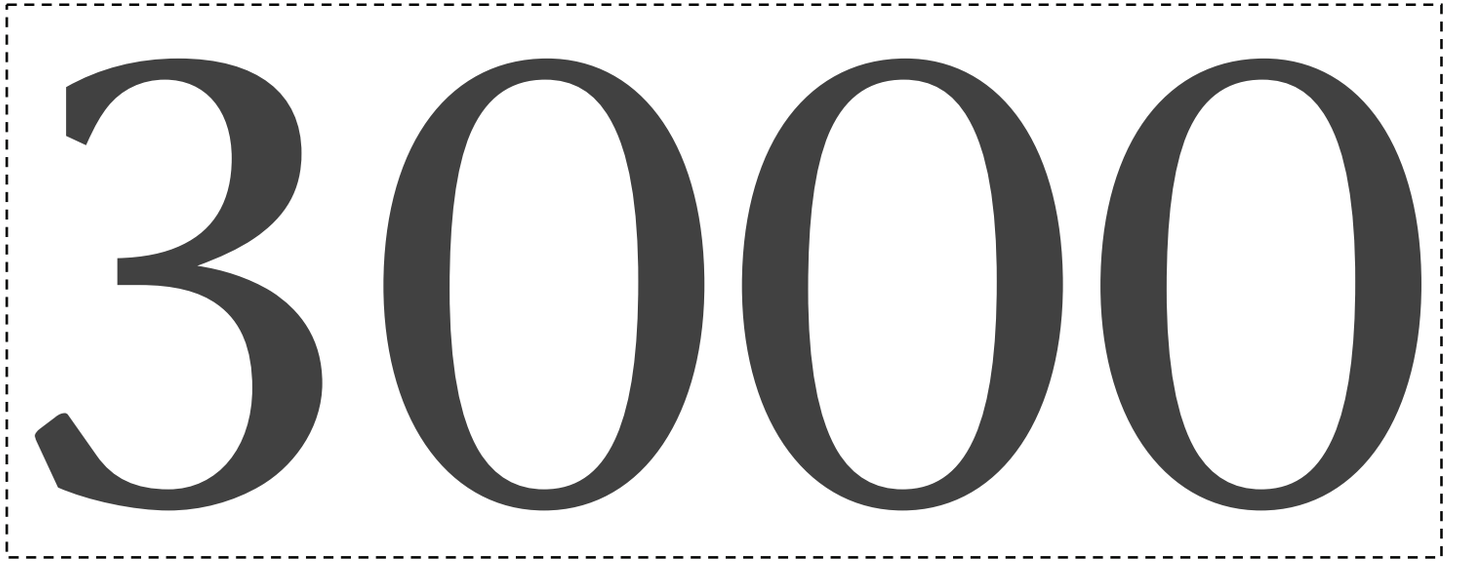
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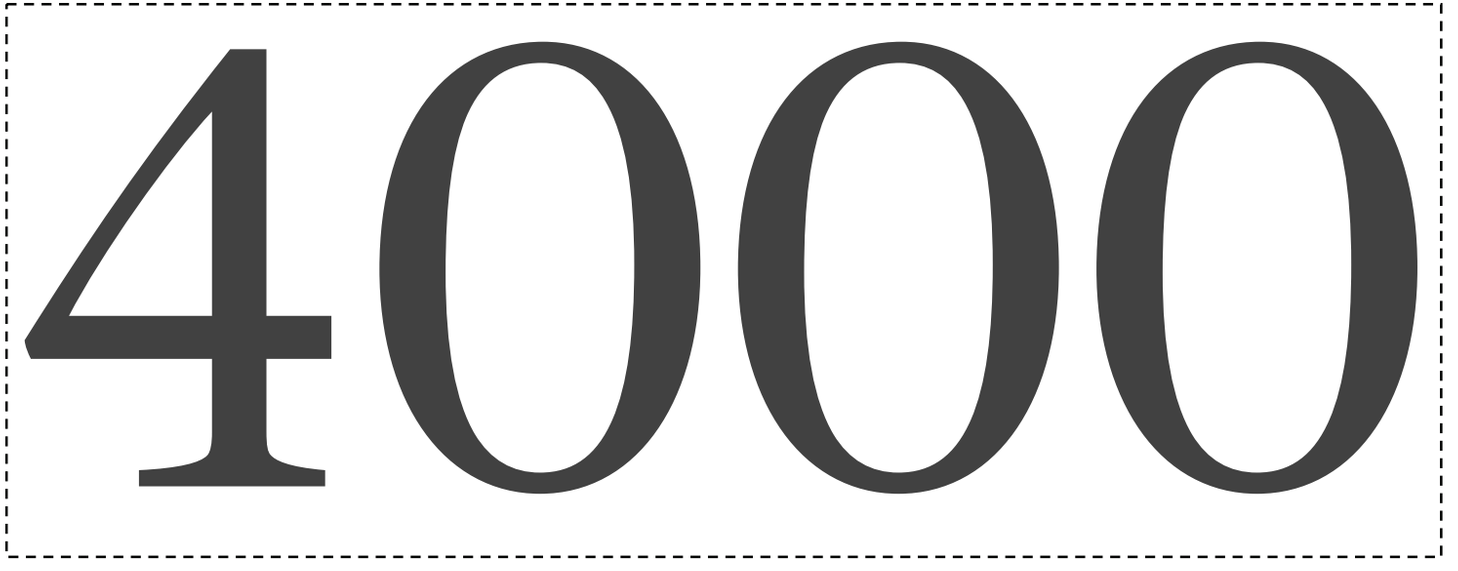
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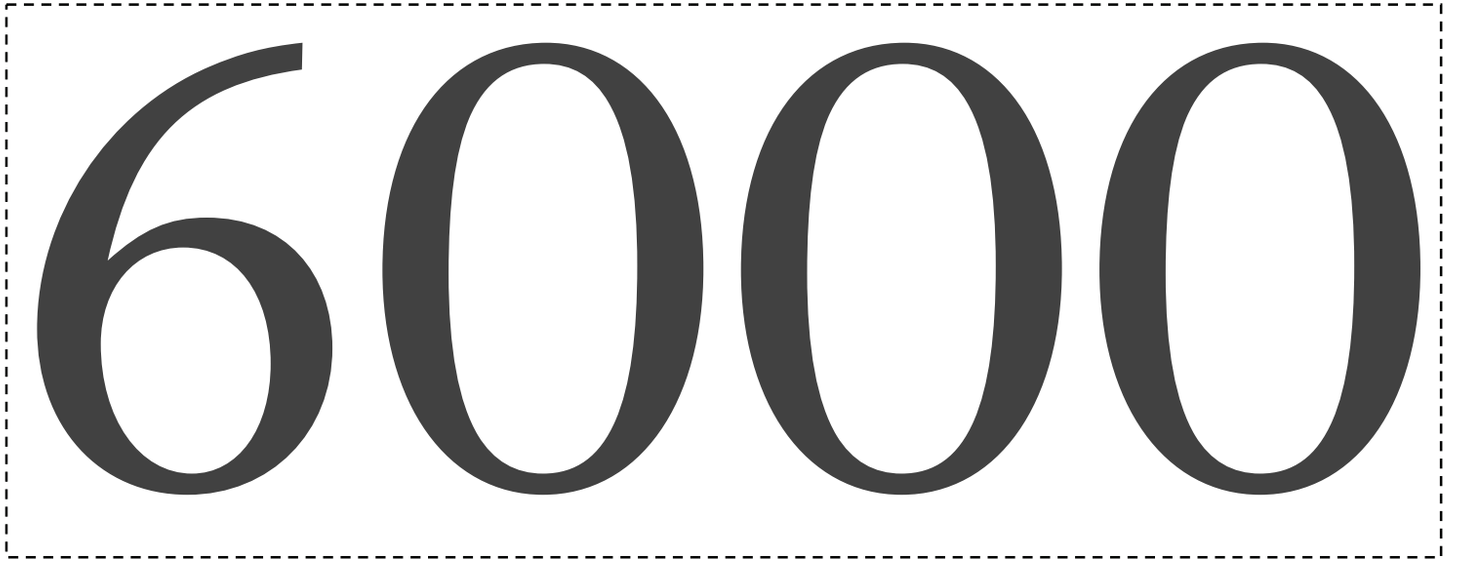
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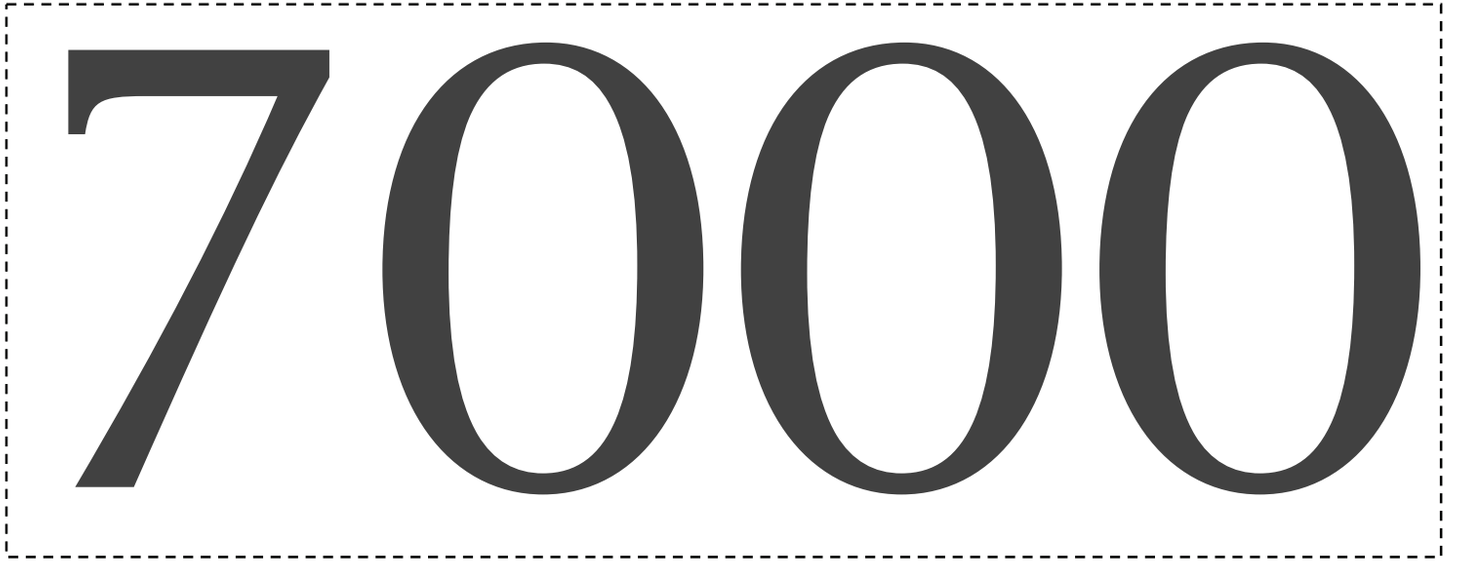
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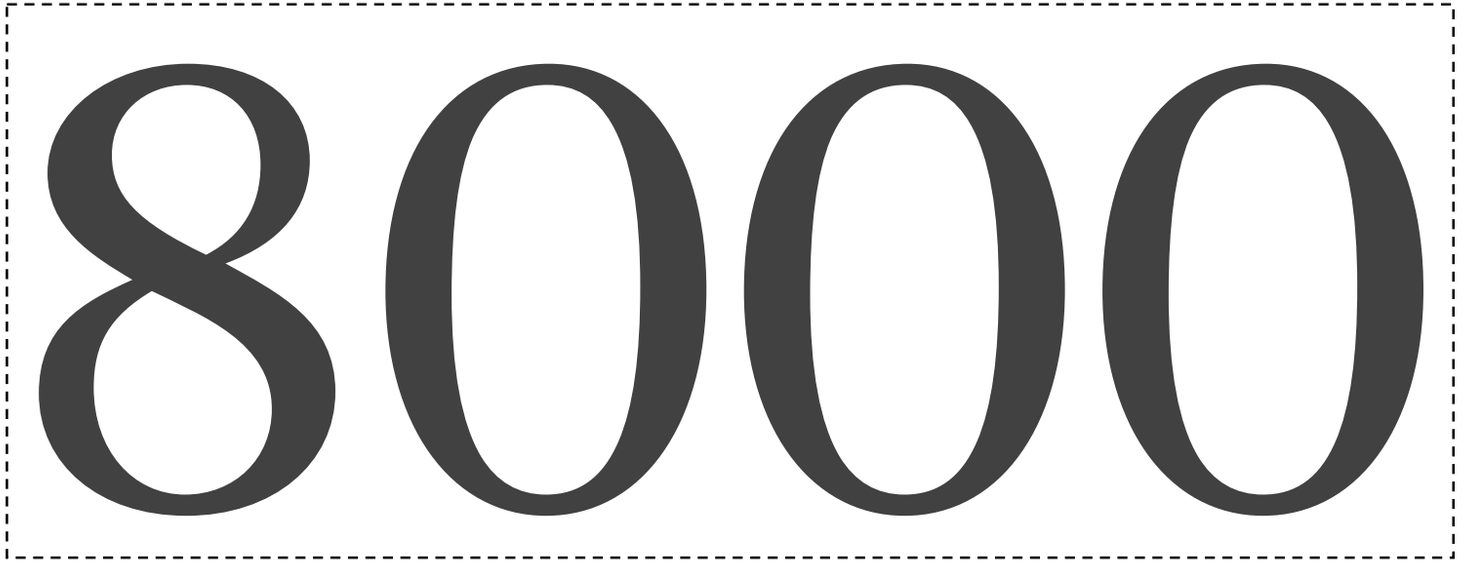
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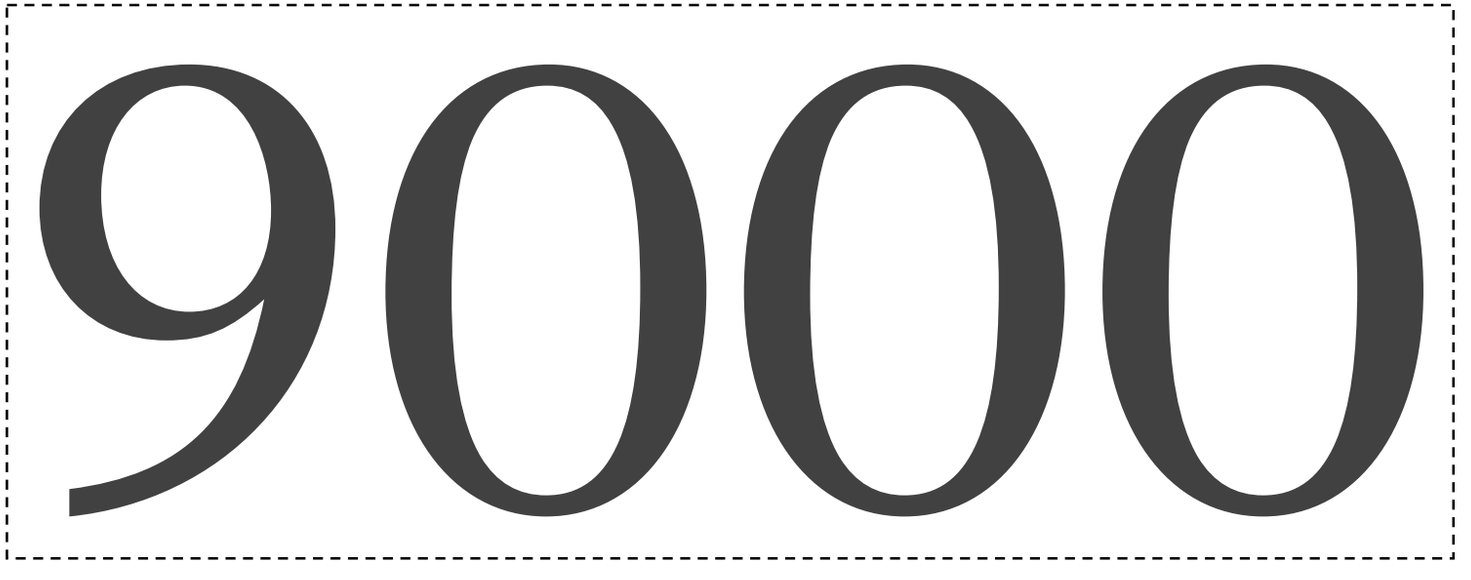
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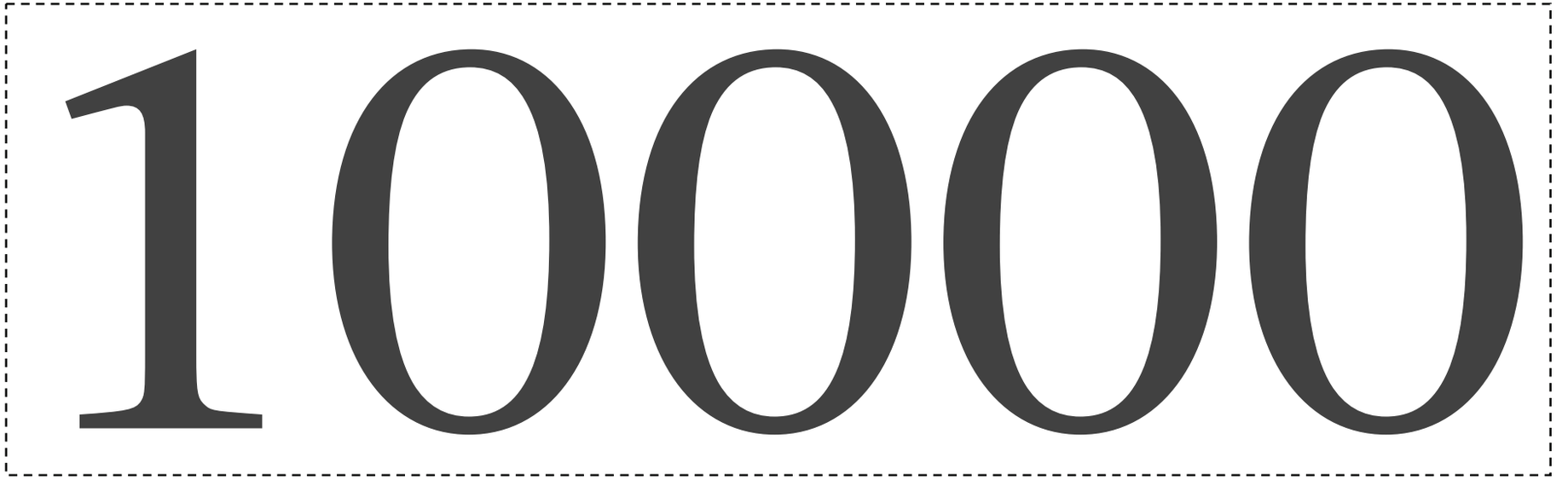
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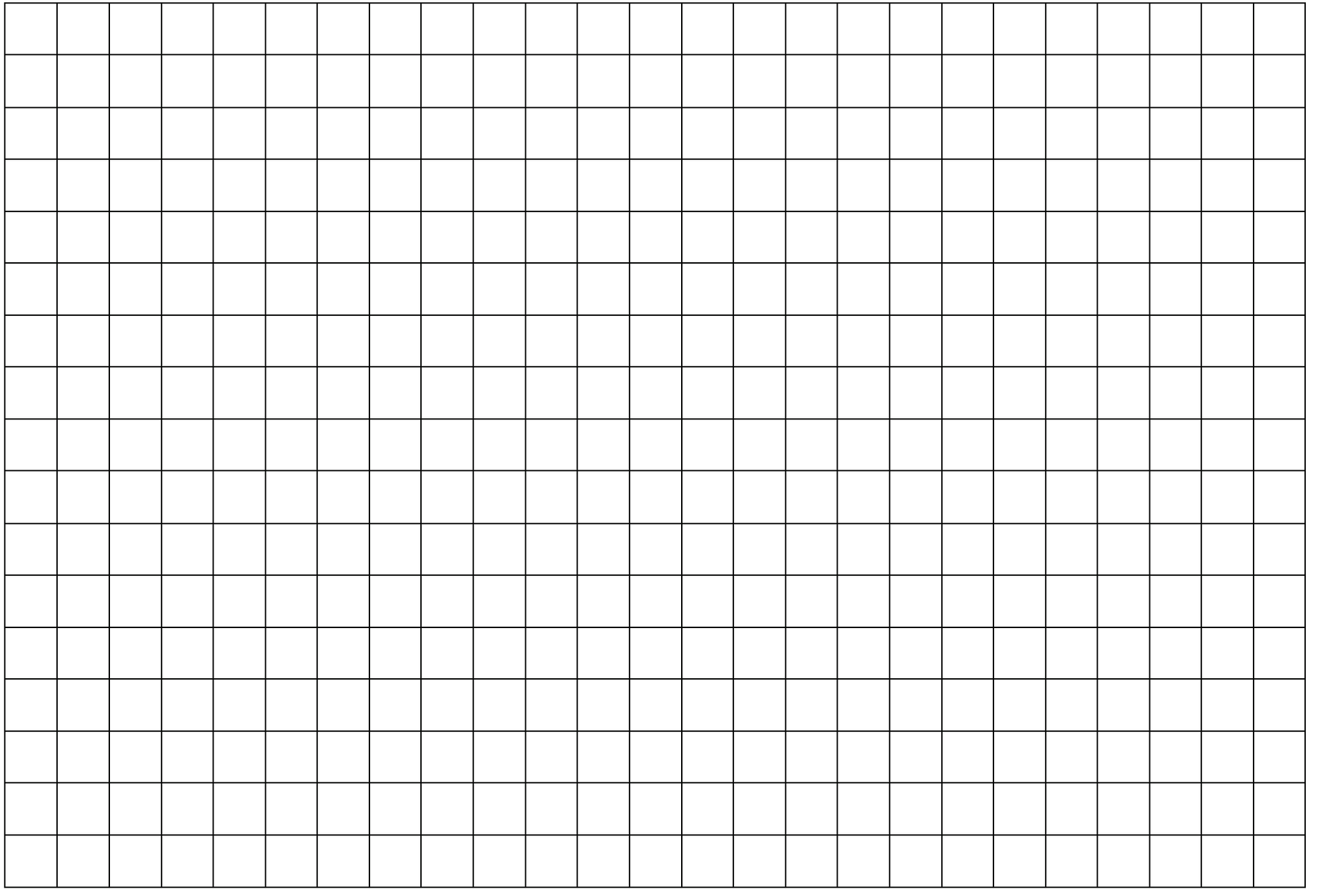
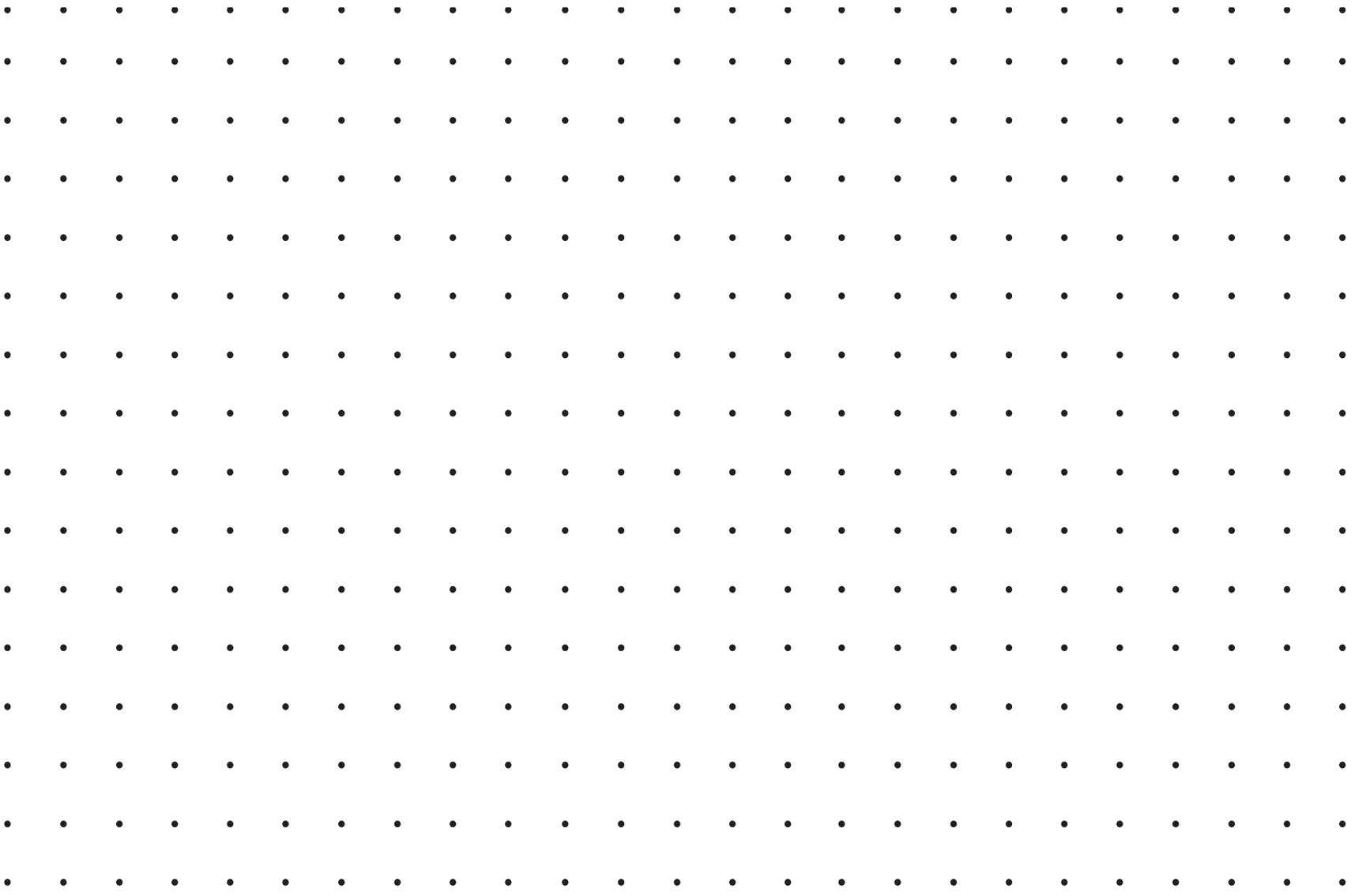
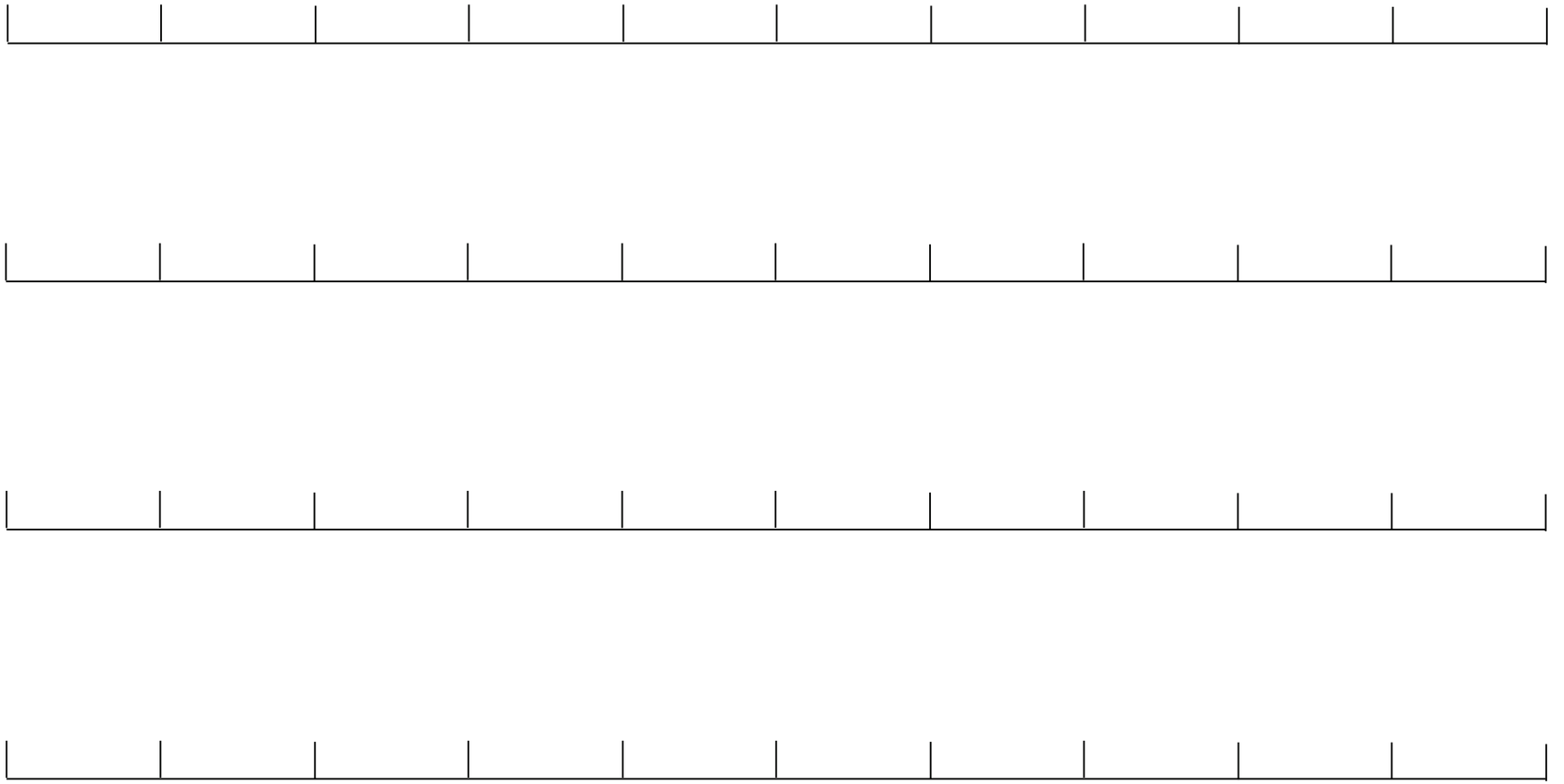




Term 1

Term 1 Unit 1: Section 1.1, question 7 (TG p. 7; LB p. 6)

Term 1 Unit 1: Section 1.1, questions 10 to 13 (TG p. 7; LB p. 6)



Term 1 Unit 4: Section 4.5, question 1 (TG p. 62; LB p. 58)

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15					
4	4	8	12							
5	5	10	15	20	25					
6	6	12	18							
7	7	14	21							

Term 1 Unit 5: Section 5.2, question 7 (TG p. 67; LB p. 62)

×	1	2	3	4	5	6	7	8	9	10
3	3	6	9	12						

Term 1 Unit 5: Section 5.3, question 4 (TG p. 69; LB p. 64)

×	1	2	3	4	5	6	7	8	9	10
4	4	8	12	16						

Term 1 Unit 5: Section 5.3, question 7 (TG p. 70; LB p. 65)

×	1	2	3	4	5	6	7	8	9	10
5	5	10	15	20						

Term 1 Unit 5: Section 5.3, question 11 (TG p. 71; LB p. 66)

×	1	2	3	4	5	6	7	8	9	10
10	10	20	30	40						

Term 1 Unit 5: Section 5.3, question 12 (TG p. 71; LB p. 66)

×	1	2	3	4	5	6	7	8	9	10
6										

Term 1 Unit 5: Section 5.3, question 14 (TG p. 71; LB p. 66)

×	2	4	8	3	6	6	7	5	10	9
10	20		80							
5										
3										
6		24								
4			32				28			

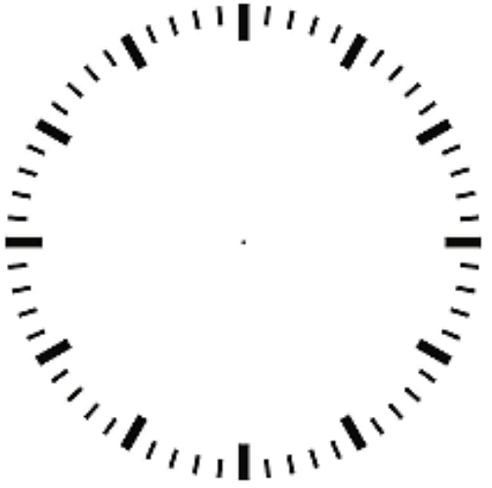
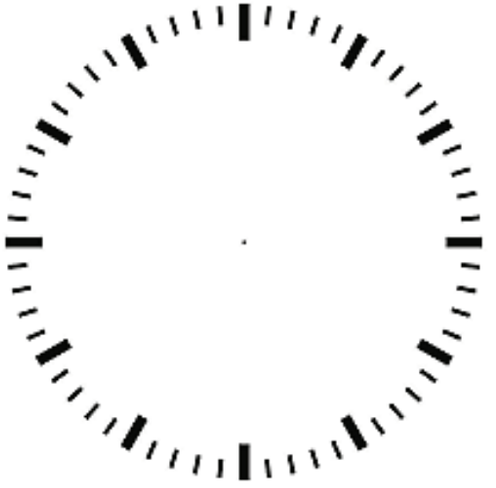
Term 1 Unit 5: Section 5.4, question 1 (TG p. 72; LB p. 67)

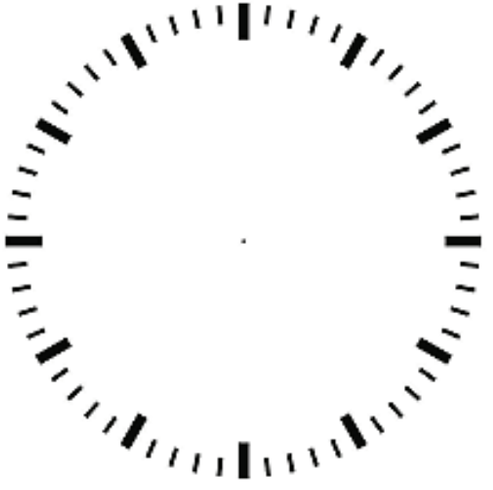
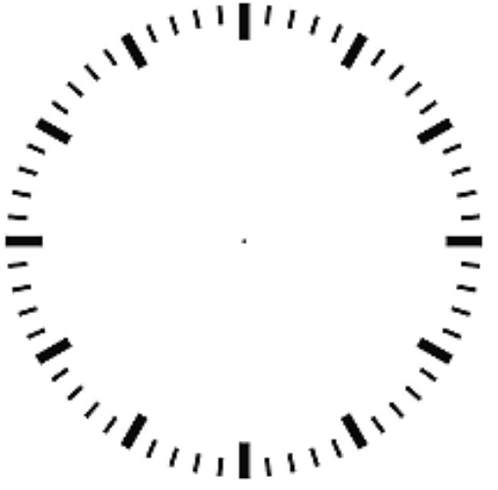
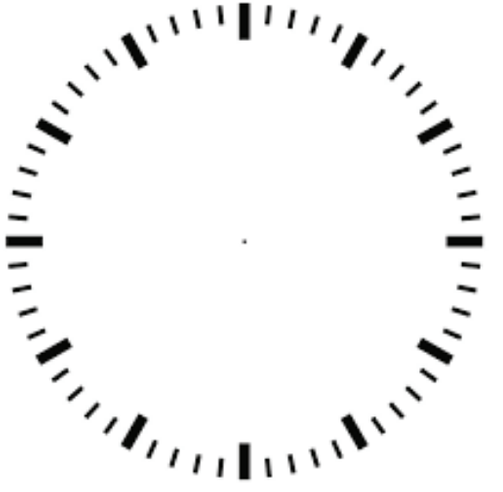
×	2	3	4	8	1	6	7	5	10	9
8										
7										
9										

Term 1 Unit 5: Section 5.4, question 6 (TG p. 73; LB p. 68)

×	1	2	4	8	3	6	7	5	10	9
8				64						
3										
6										
7										
9										

Term 1 Unit 6: Section 6.3, question 5 (TG p. 87; LB p. 81)

Term 1 Unit 9: Section 9.1, question 7 (TG p. 115; LB p. 105)

×	2	3	4	5	6	7	8	9	10
30	60	90	120						
50				250					

Term 1 Unit 9: Section 9.2, question 2 (TG p. 116; LB p. 106)

×	2	3	4	5	6	7	8	9	10
20	40	60	80						
30				150					
40									
50									

Term 1 Unit 9: Section 9.4, question 1 (TG p. 118; LB p. 108)

×	2	3	4	5	6	7	8	9	10
60									
70									
80									
90									
100									

Term 1 Unit 9: Section 9.4, question 4 (TG p. 119; LB p. 109)

×	10	50	40	80	30	60	70	100	90
10				800					
5									
2									
4			160						
8				640					
3									
6									
7									
9									

Term 1 Unit 9: Section 9.4, question 5 (TG p. 119; LB p. 109)

Number of pancakes	1	2	3	4	5	6	7	8	9
Cost (in rands)	7	14							

Number of pancakes	10	20	30	40	50	60	70	80	90
Cost (in rands)	70	140							

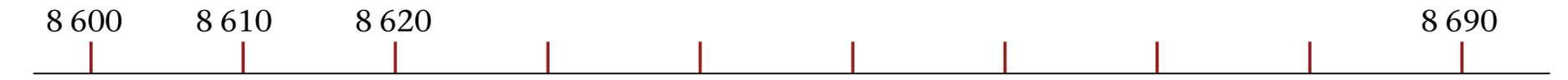


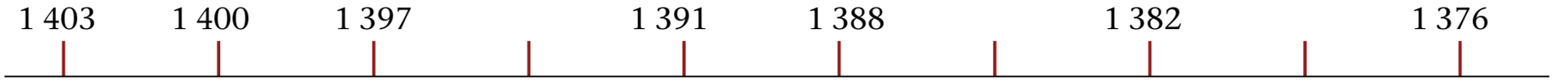
Number of pancakes	1	2	3	4	5	6	7	8	9
Cost (in rands)	7	14							

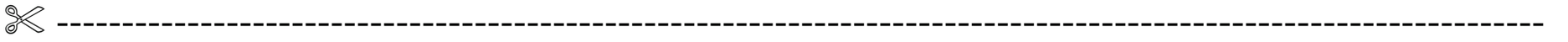
Number of pancakes	10	20	30	40	50	60	70	80	90
Cost (in rands)	70	140							

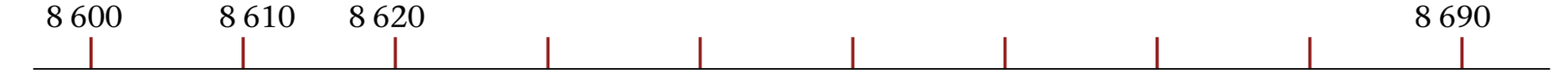
Term 2

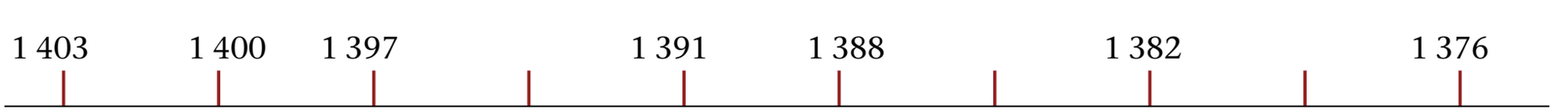
Term 2 Unit 1: Section 1.4, question 5 (TG p. 138; LB p. 127)





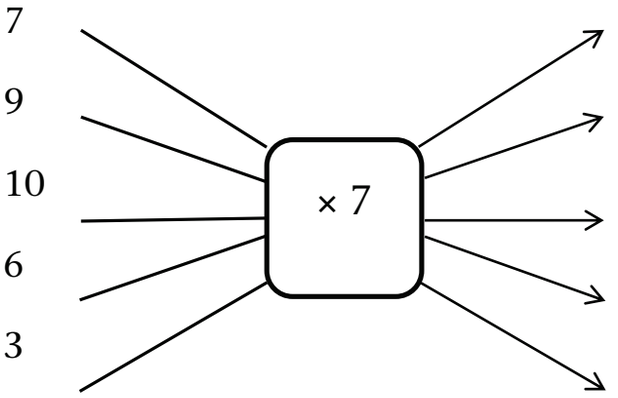




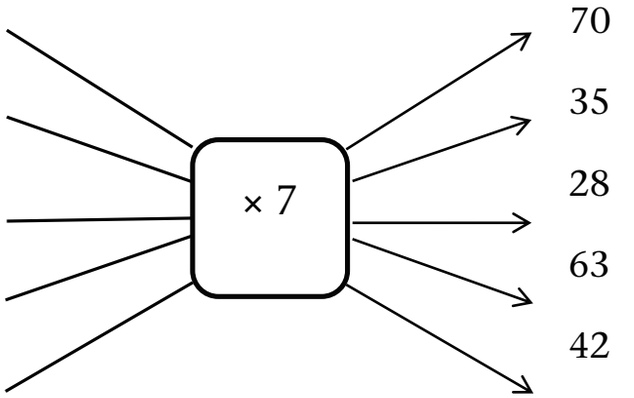


Term 2 Unit 5: Section 5.1, question 1 (TG p. 178; LB p. 161)

(a)



(b)



Term 2 Unit 5: Section 5.1, question 2 (TG p. 178; LB p. 161)

Number	7	9	10	6	3					
Number × 7						70	35	28	63	42

Term 2 Unit 5: Section 5.1, question 3 (TG p. 179; LB p. 162)

(a)

Number	20	40	80				
Number × 7				490	210	700	350

(b)

Number	30	60	90				
Number × 6				480	300	240	420

(c)

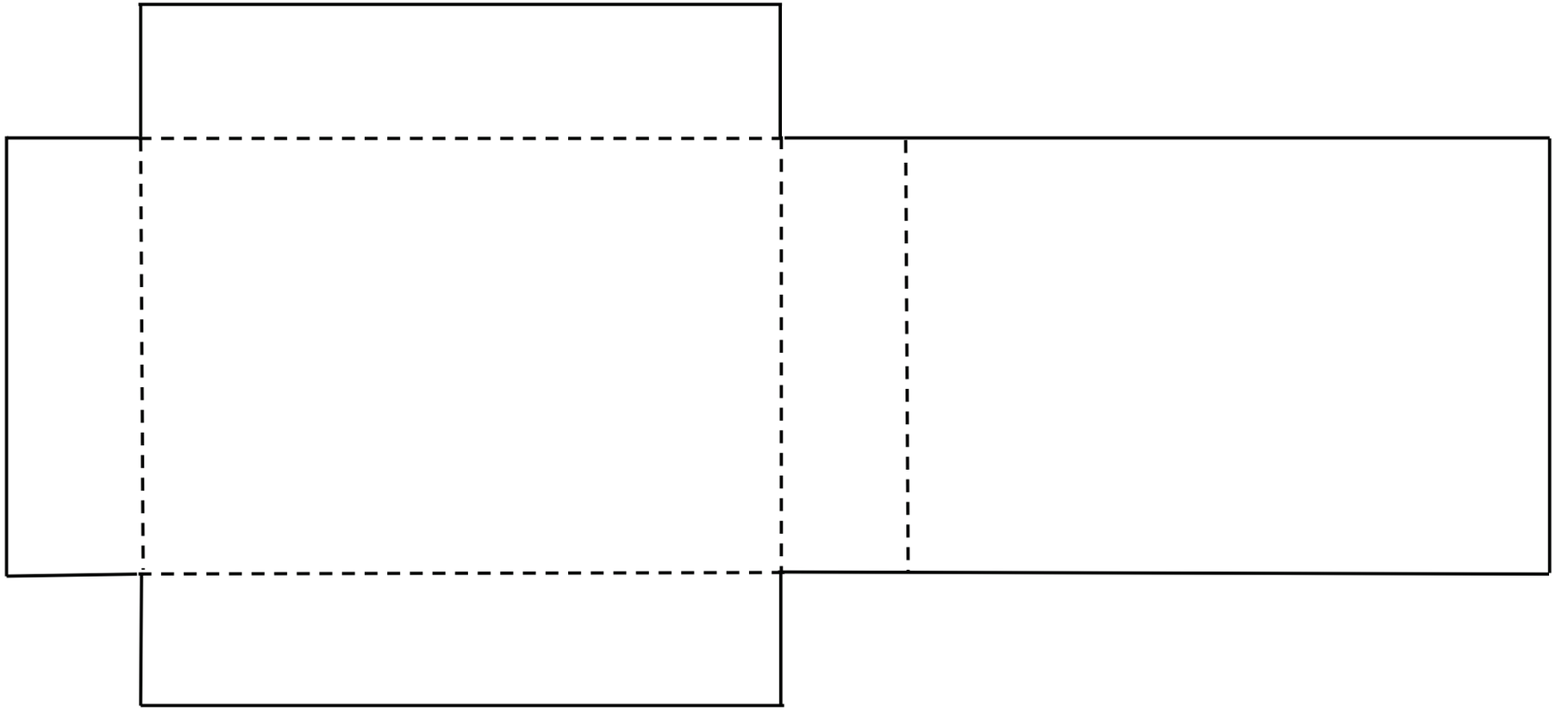
Number	100	60	50				
Number × 8				320	640	720	560

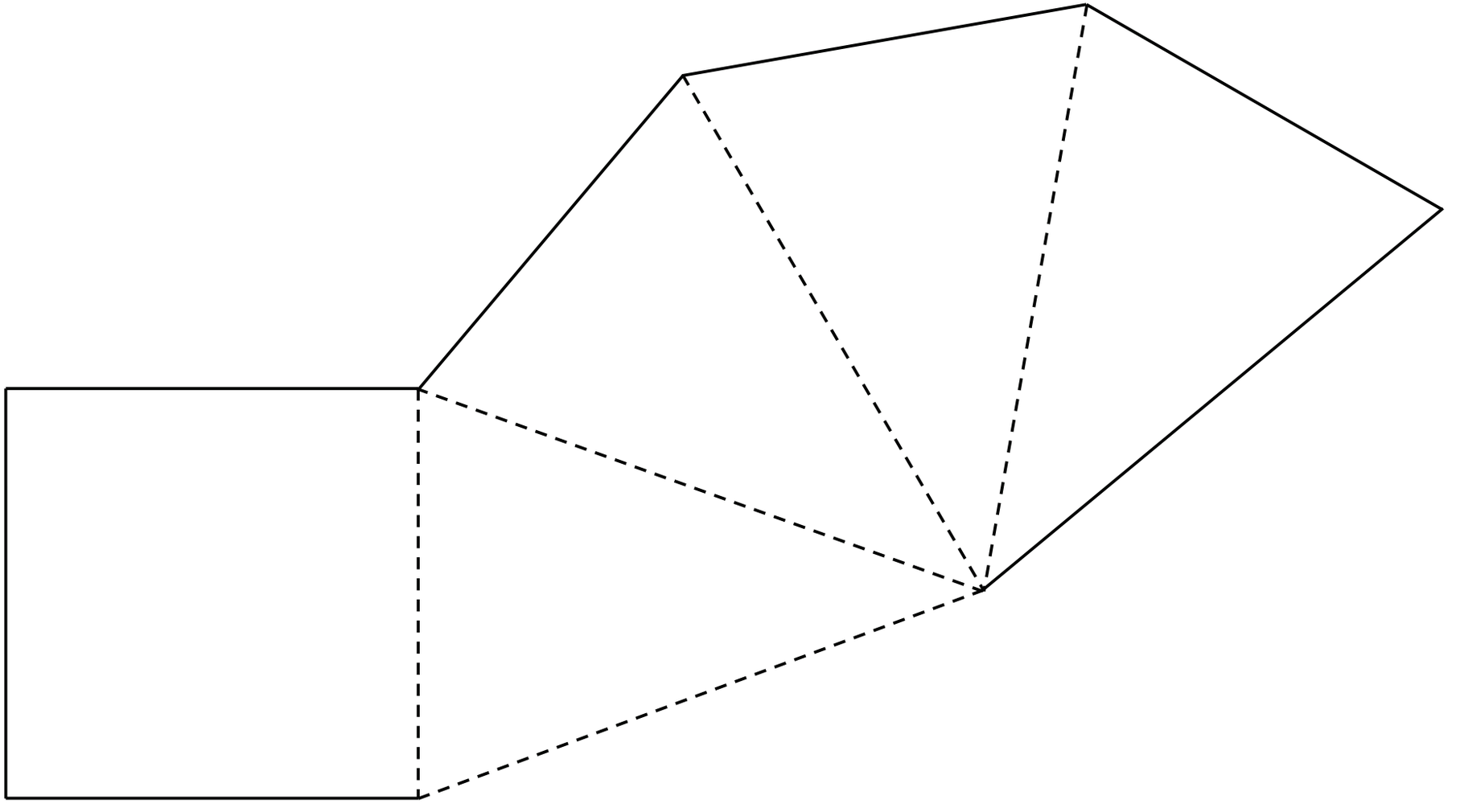
(d)

Number	40	80	60				
Number × 9				450	900	630	540

Term 2 Unit 5: Section 5.2, questions 1, 2, 4, 5, 6 and 7 (TG p. 182; LB p. 164)

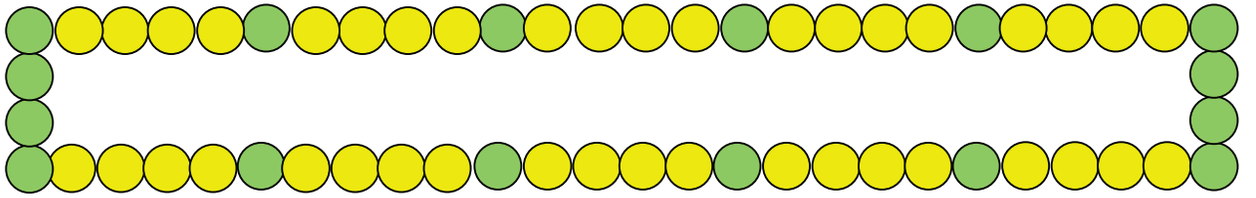
×	10	20	30	40	50	60	70	80	90	100
2										
20										
3										
30					1 500					
4										
40										
5										
50										
6										
60					3 000					
7										
70										
8										
80										
9										
90										
10										
100										



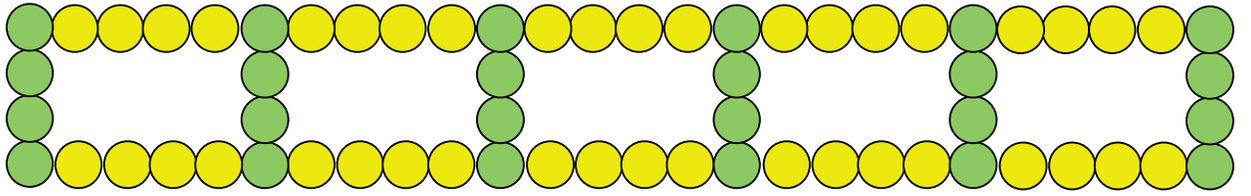


Term 2 Unit 7: Section 7.2, question 2 (TG p. 202; LB p. 182)

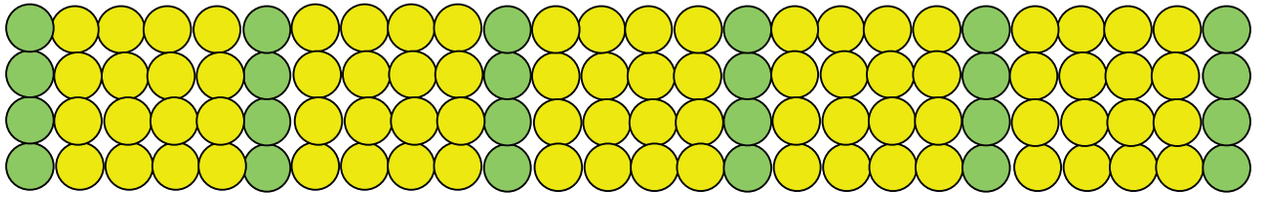
(a)



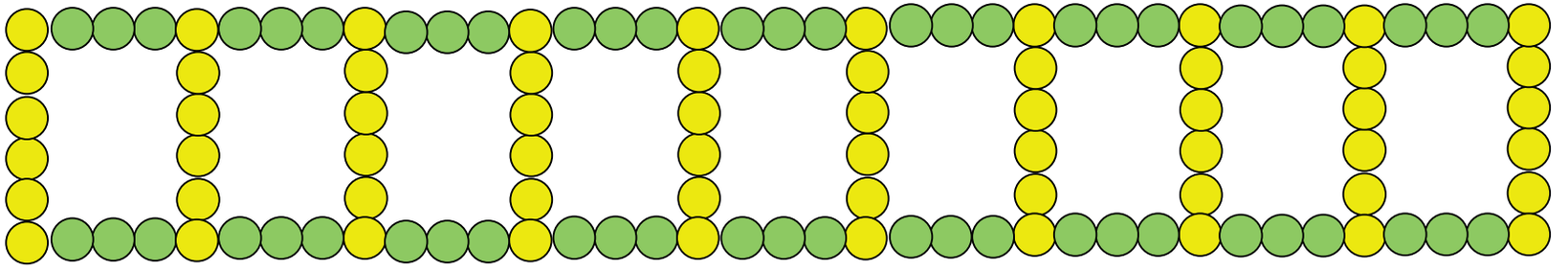
(b)



(c)



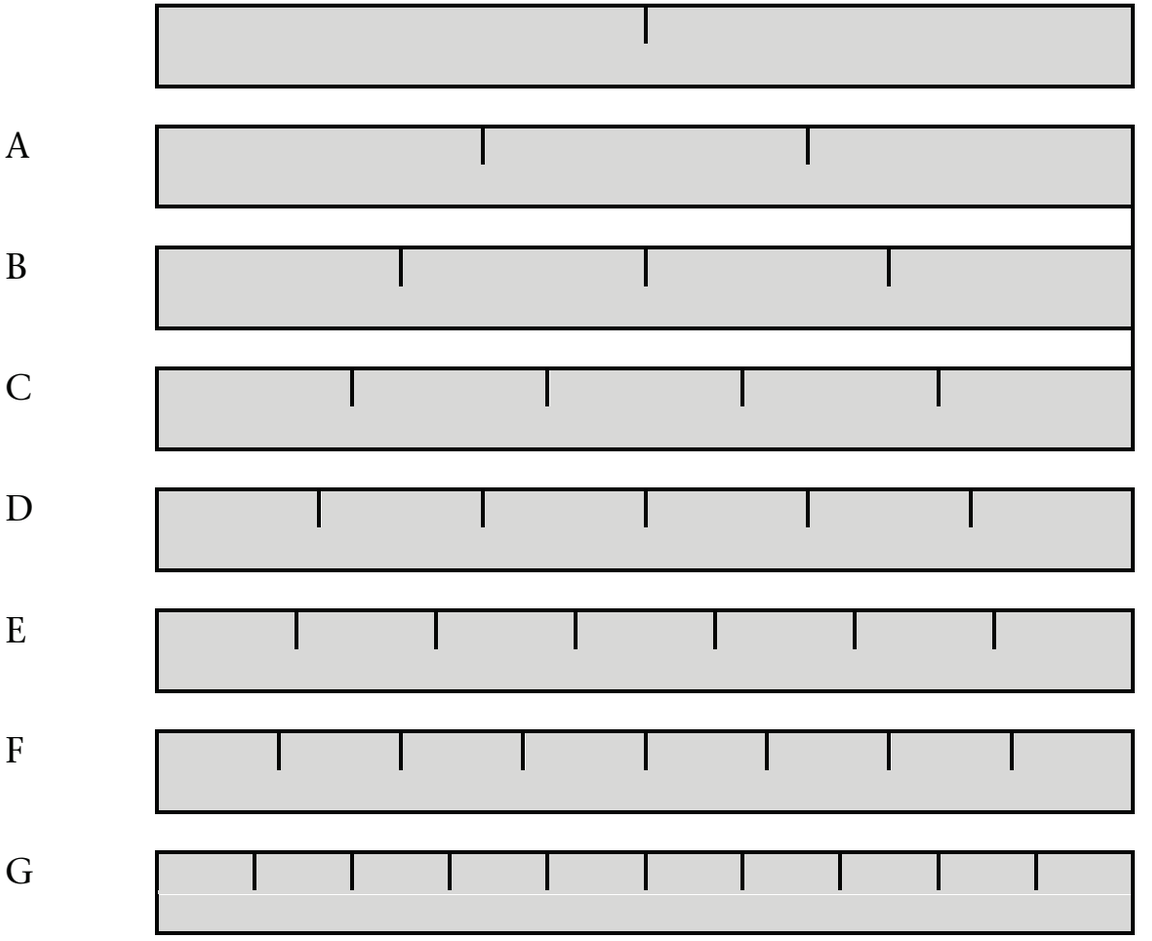
(d)



Term 3

Term 3 Unit 1: Section 1.2, question 1 (TG p. 241; LB p. 214)

What fractions are shown on each of the measuring sticks?



halves

A .....

B .....

C .....

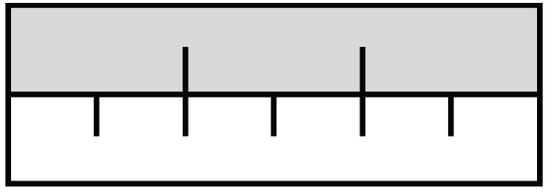
D .....

E .....

F .....

G .....

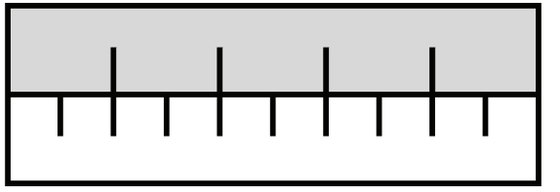
Term 3 Unit 1: Section 1.3, question 2 (TG p. 243; LB p. 216)



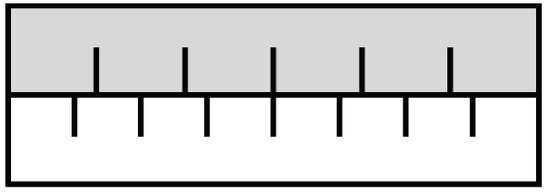
thirds

sixths

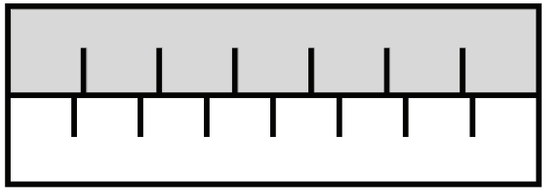
D



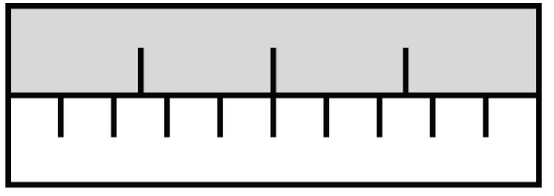
A



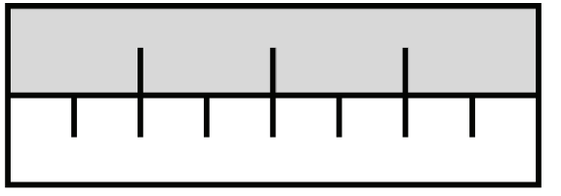
E



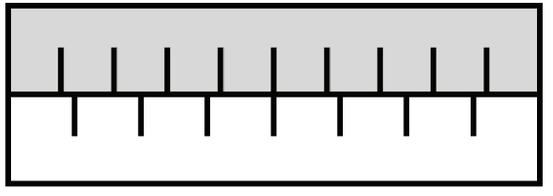
B



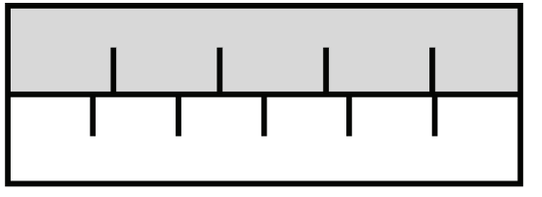
F



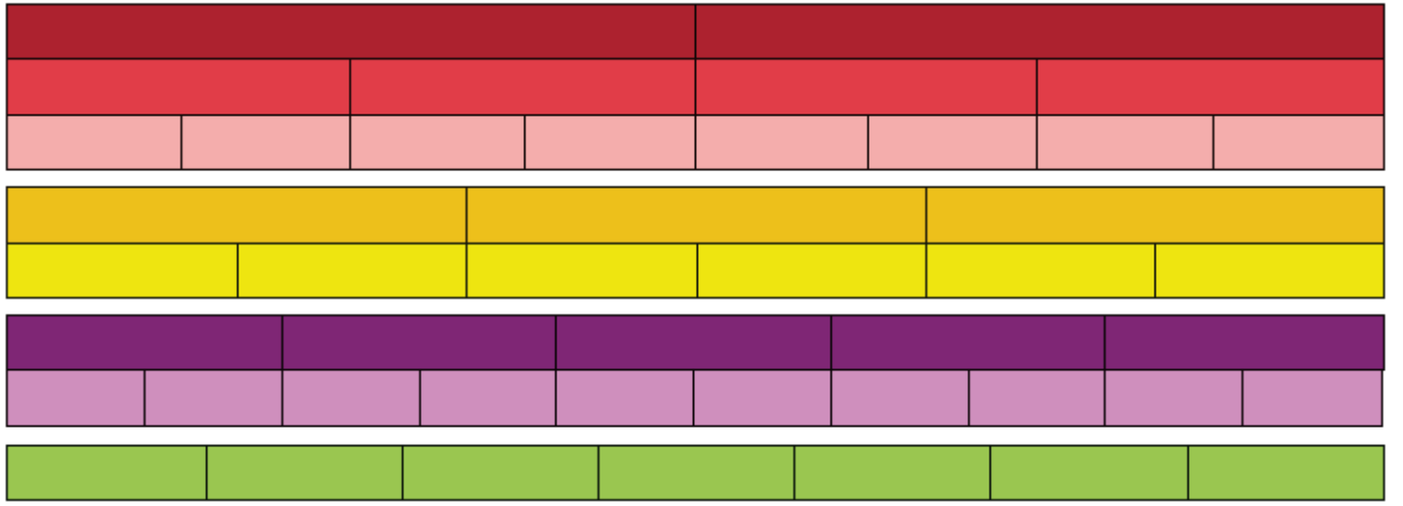
C

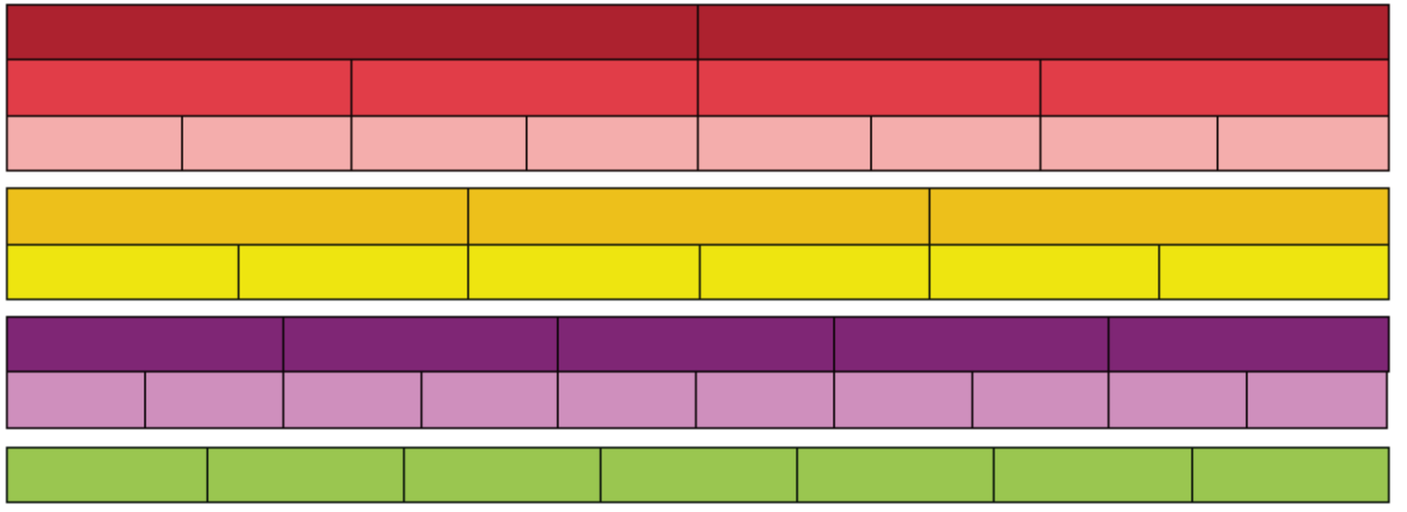


G



Term 3 Unit 1: Section 1.6, question 2 (TG p. 251, LB p. 223)





Term 3 Unit 3: Section 3.2, question 1 (TG p. 272; LB p. 240)

(a)



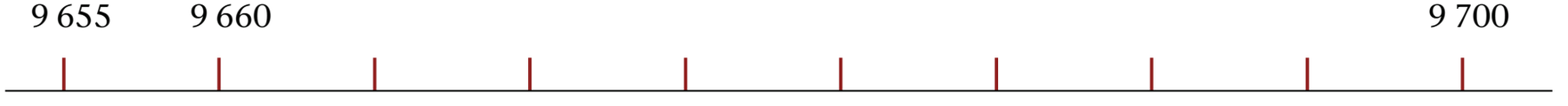
(b)



(c)



(d)



Term 3 Unit 9: Section 9.1, question 6 (TG p. 309; LB p. 269)

Table A: The distance covered after different times

Time	0	10 min	20 min	30 min	40 min	50 min
Annie	0	1 867		5 504		
Ellen	0	1 768		5 345		
Gap	0	99				

Table B: The distances covered in different 10-minute periods

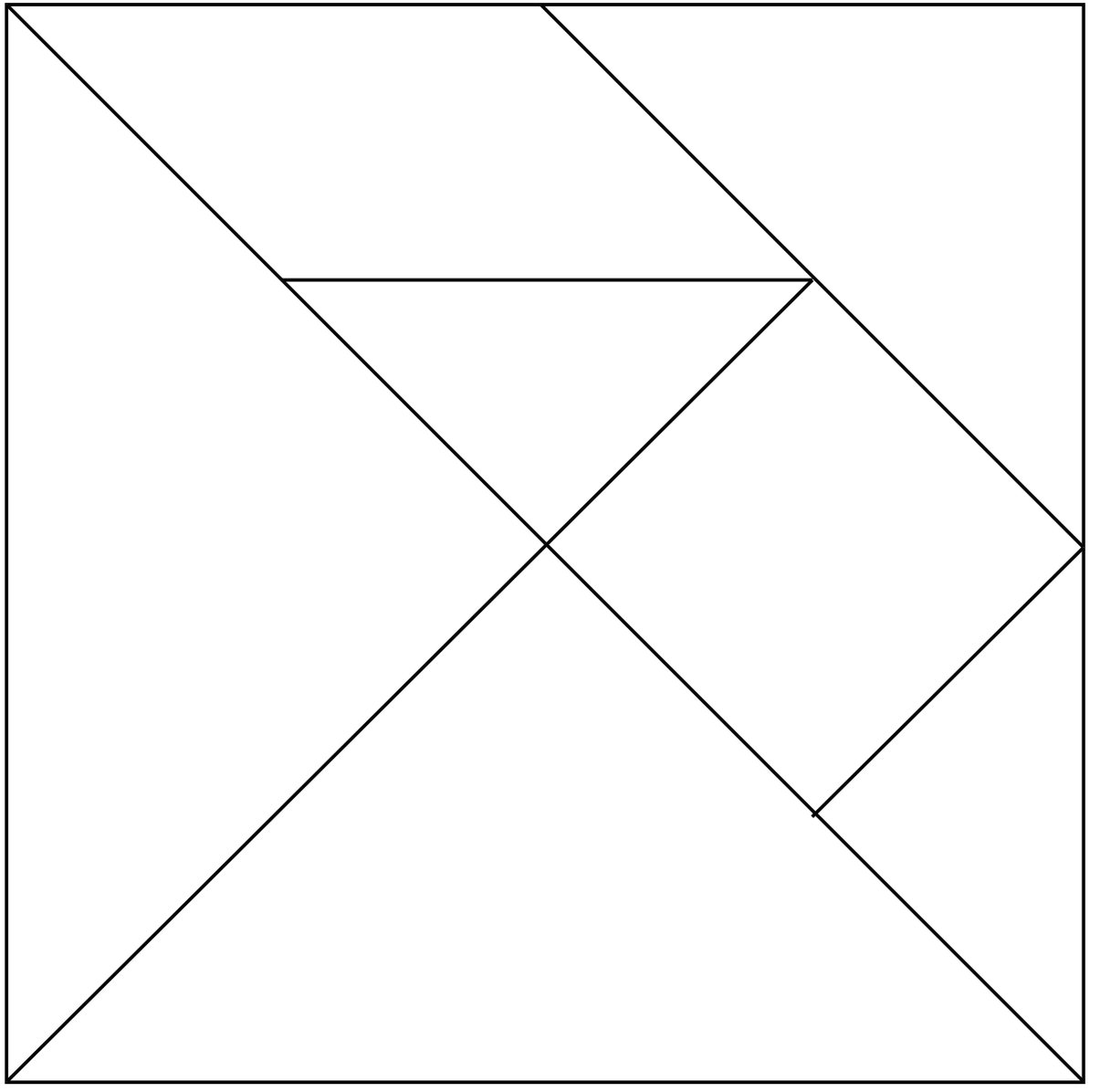
Period	First 10 min	Second 10 min	Third 10 min	Fourth 10 min	Fifth 10 min
Annie	1 867	1 835			
Ellen	1 768	1 778			
Difference					

Term 3 Unit 9: Section 9.1, question 10 (TG p. 310; LB p. 270)

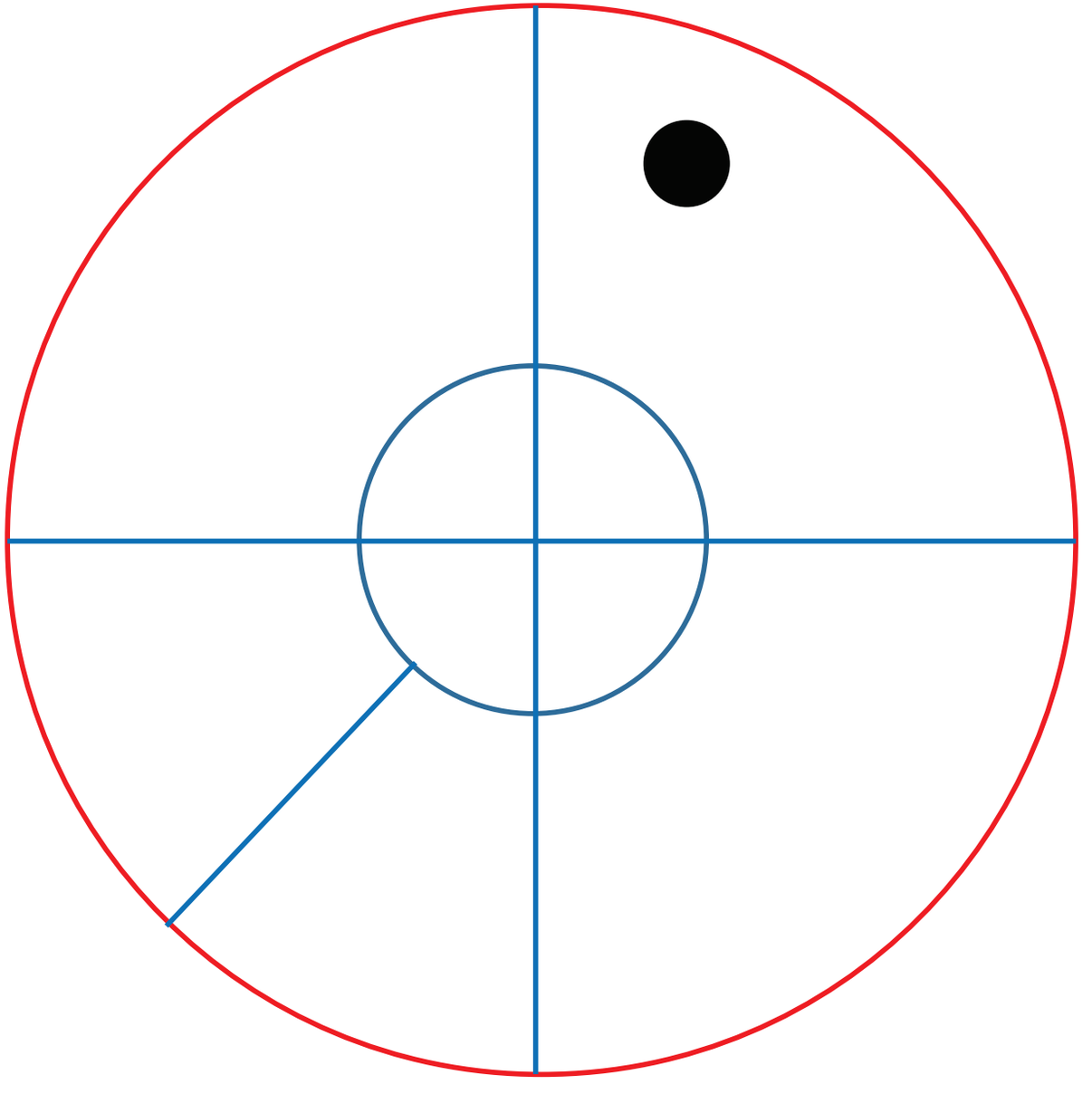
Table C: The differences between the distances covered in different 10-minutes periods

Periods	1st and 2nd periods	2nd and 3rd periods	3rd and 4th periods	4th and 5th periods
Annie				
Ellen				

Term 3 Unit 12: Section 12.1, question 1 (TG p. 329; LB p. 286)



Term 3 Unit 12: Section 12.2, question 3 (TG p. 333; LB p. 288)



Term 4

Term 4 Unit 2: Section 2.2, question 5 (TG p. 344; LB p. 296)

Tank A

Period of time	10:00 to 11:00	11:00 to 12:00	12:00 to 13:00	13.00 to 14:00	14:00 to 15:00
Water lost (in ℓ)					

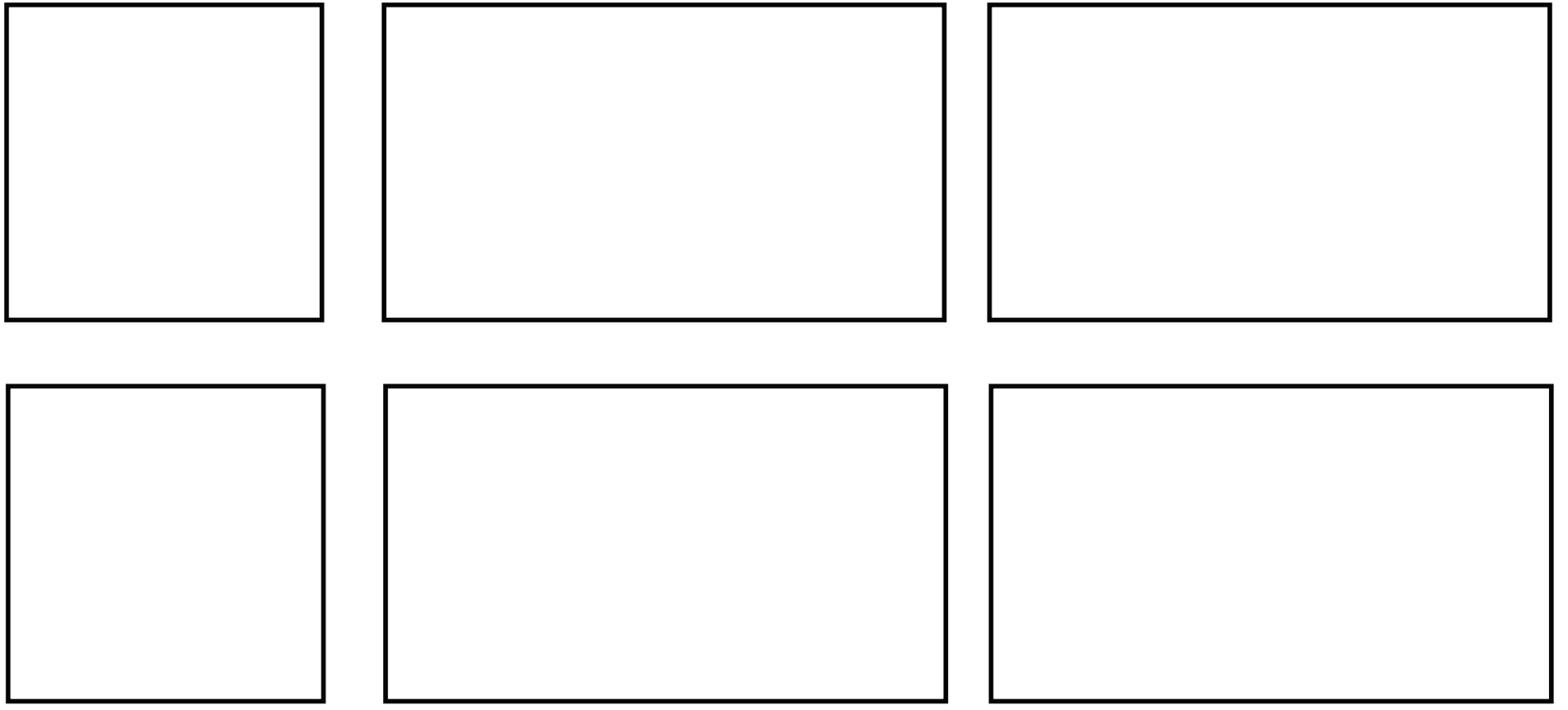
Tank B

Period of time	10:00 to 11:00	11:00 to 12:00	12:00 to 13:00	13.00 to 14:00	14:00 to 15:00
Water lost (in ℓ)					

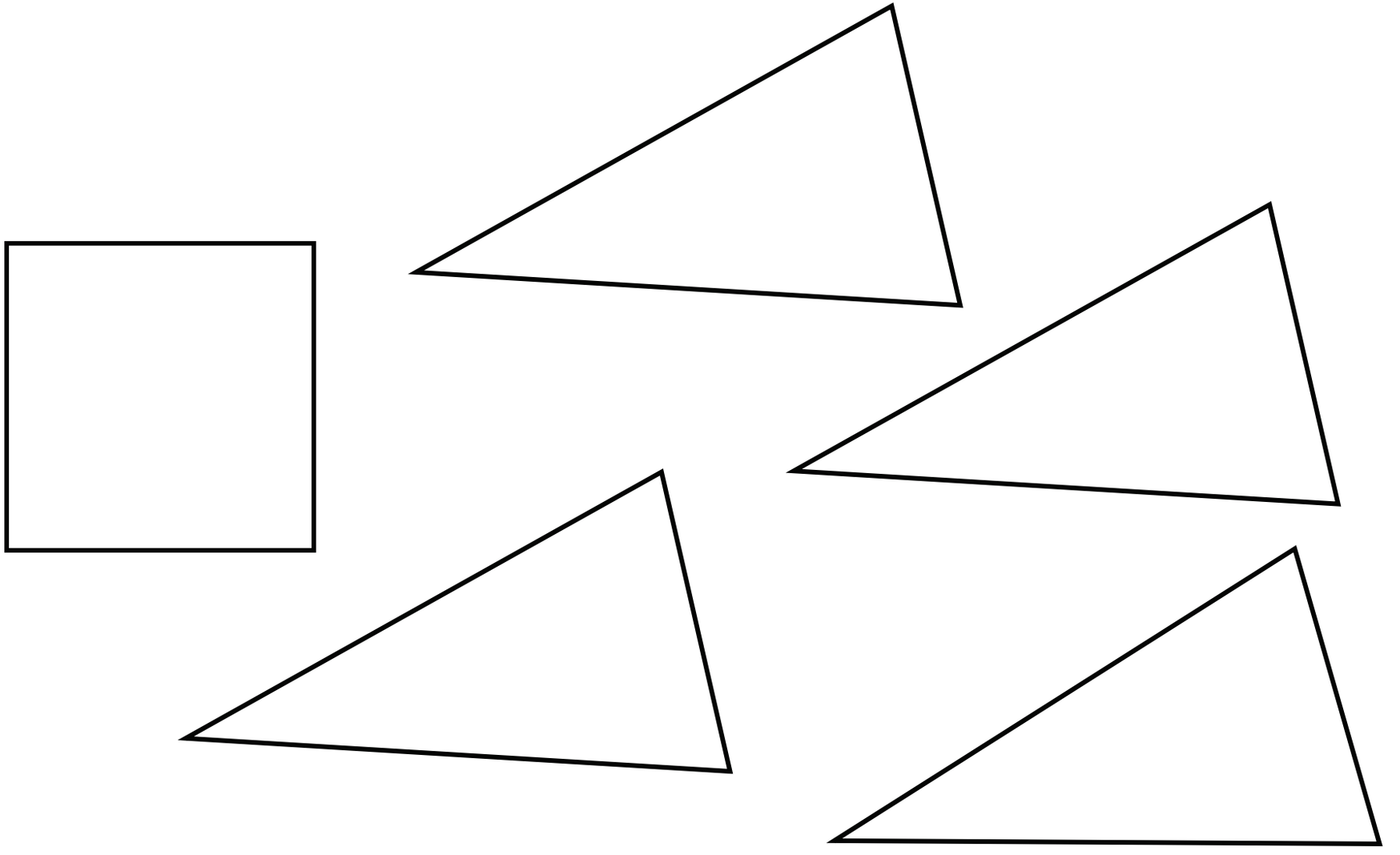
Tank C

Period of time	10:00 to 11:00	11:00 to 12:00	12:00 to 13:00	13.00 to 14:00	14:00 to 15:00
Water lost (in ℓ)					

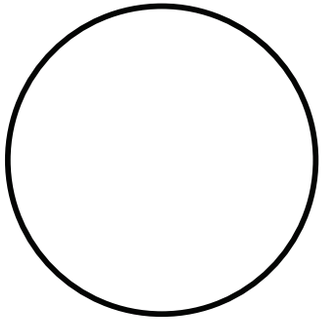
Term 4 Unit 4: Section 4.1, question 2 (TG p. 356; LB p. 306)

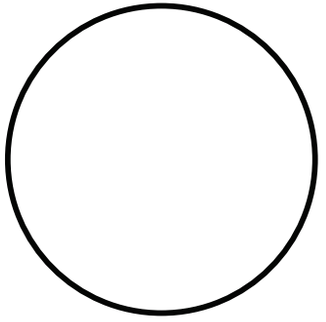


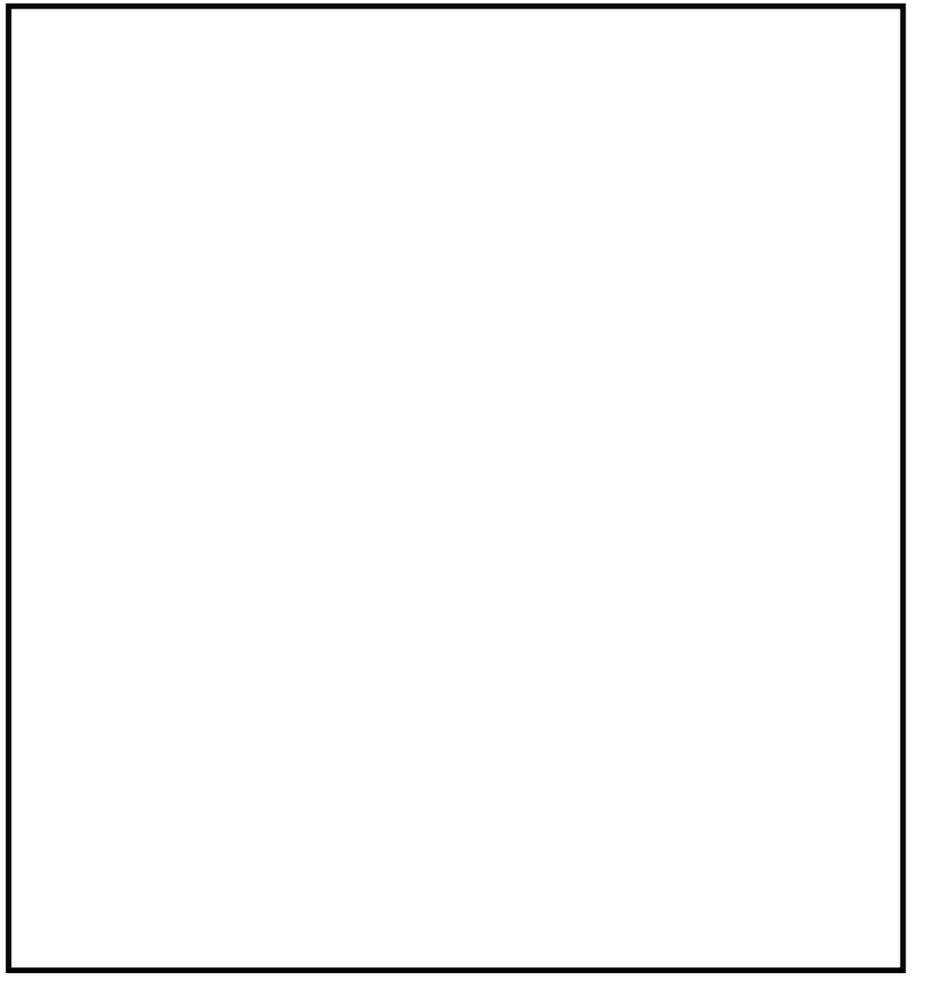
Term 4 Unit 4: Section 4.1, question 3 (TG p. 357; LB p. 307)



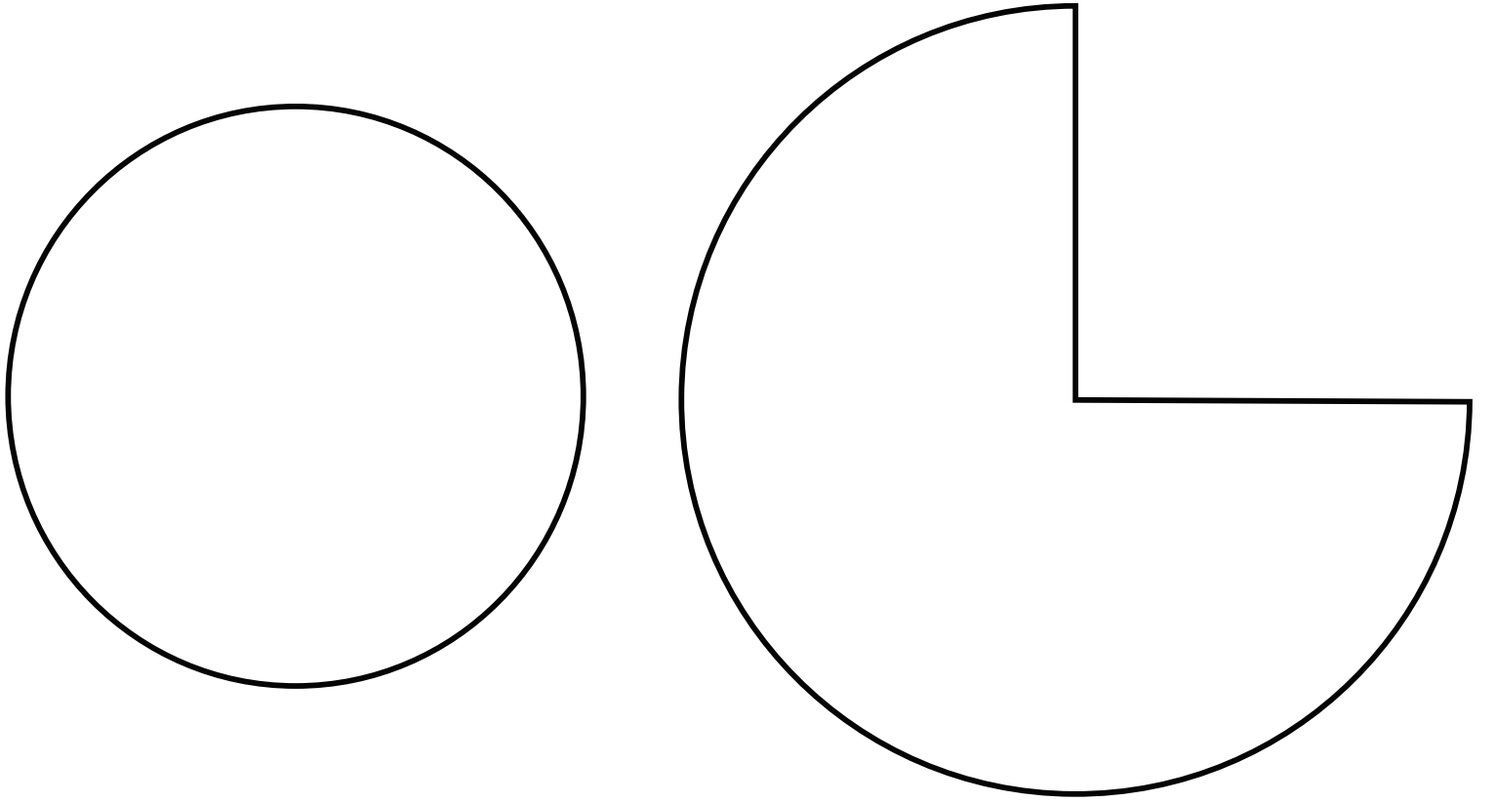
Term 4 Unit 4: Section 4.1, question 4 (TG p. 357; LB p. 307)



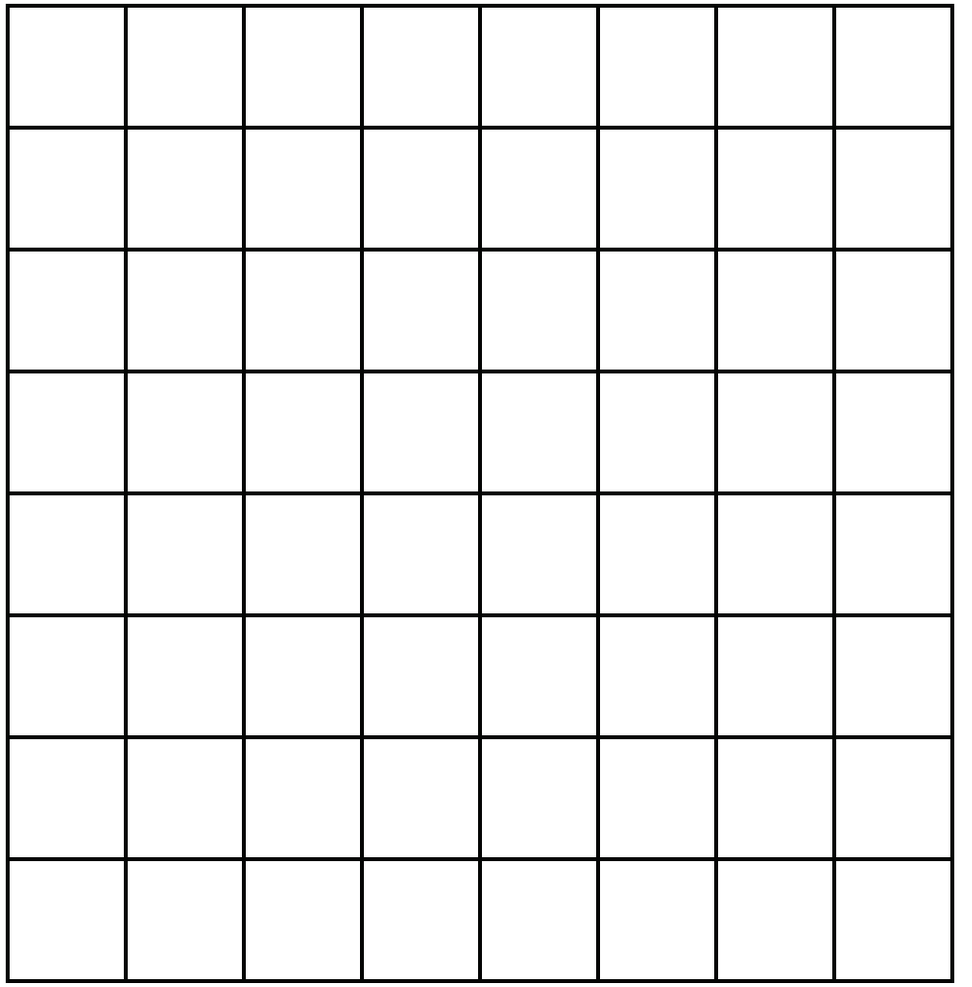




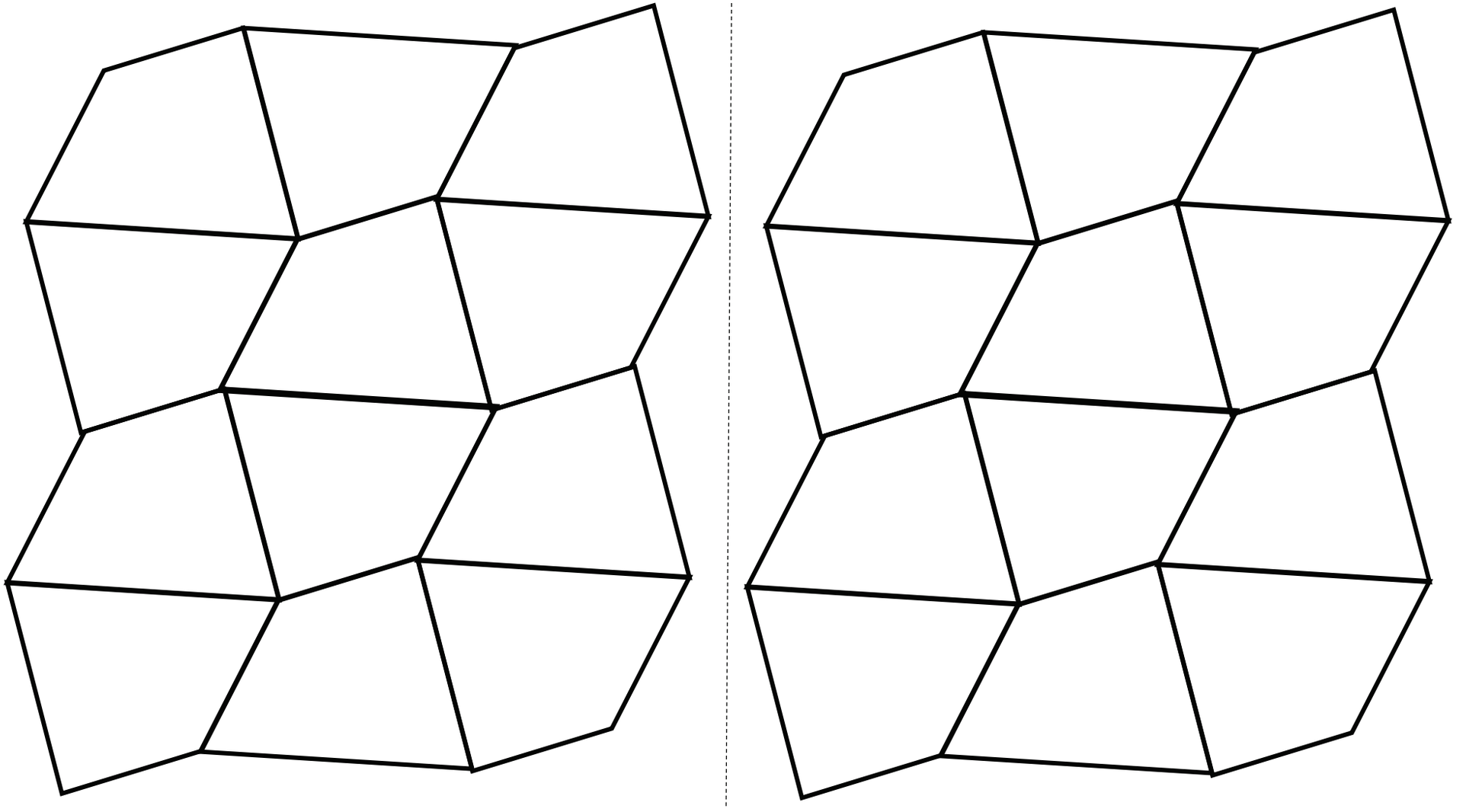
Term 4 Unit 4: Section 4.1, question 5 (TG p. 358; LB p. 308)



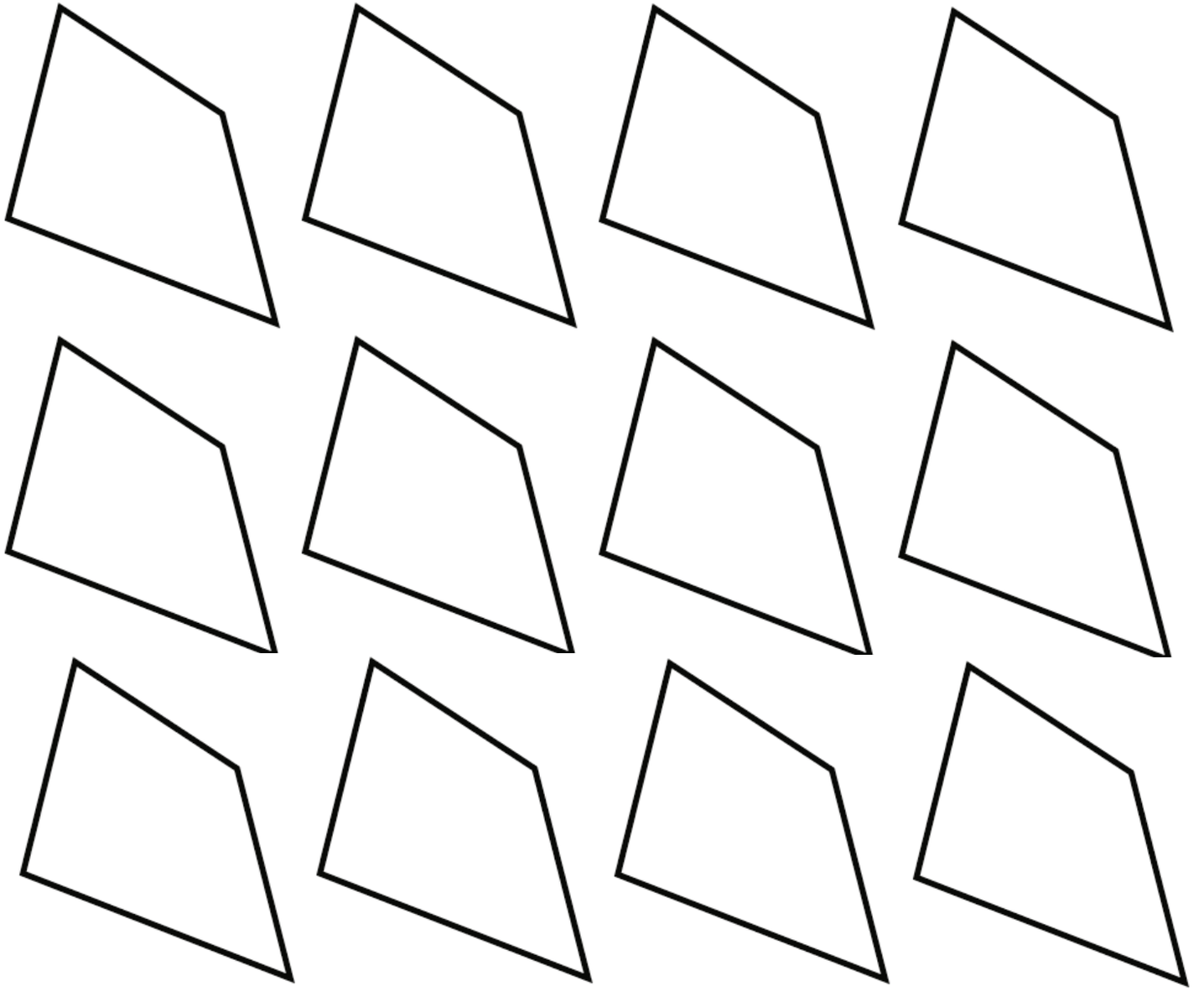
Term 4 Unit 8: Section 8.2, question 1 (TG p. 388; LB p. 332)

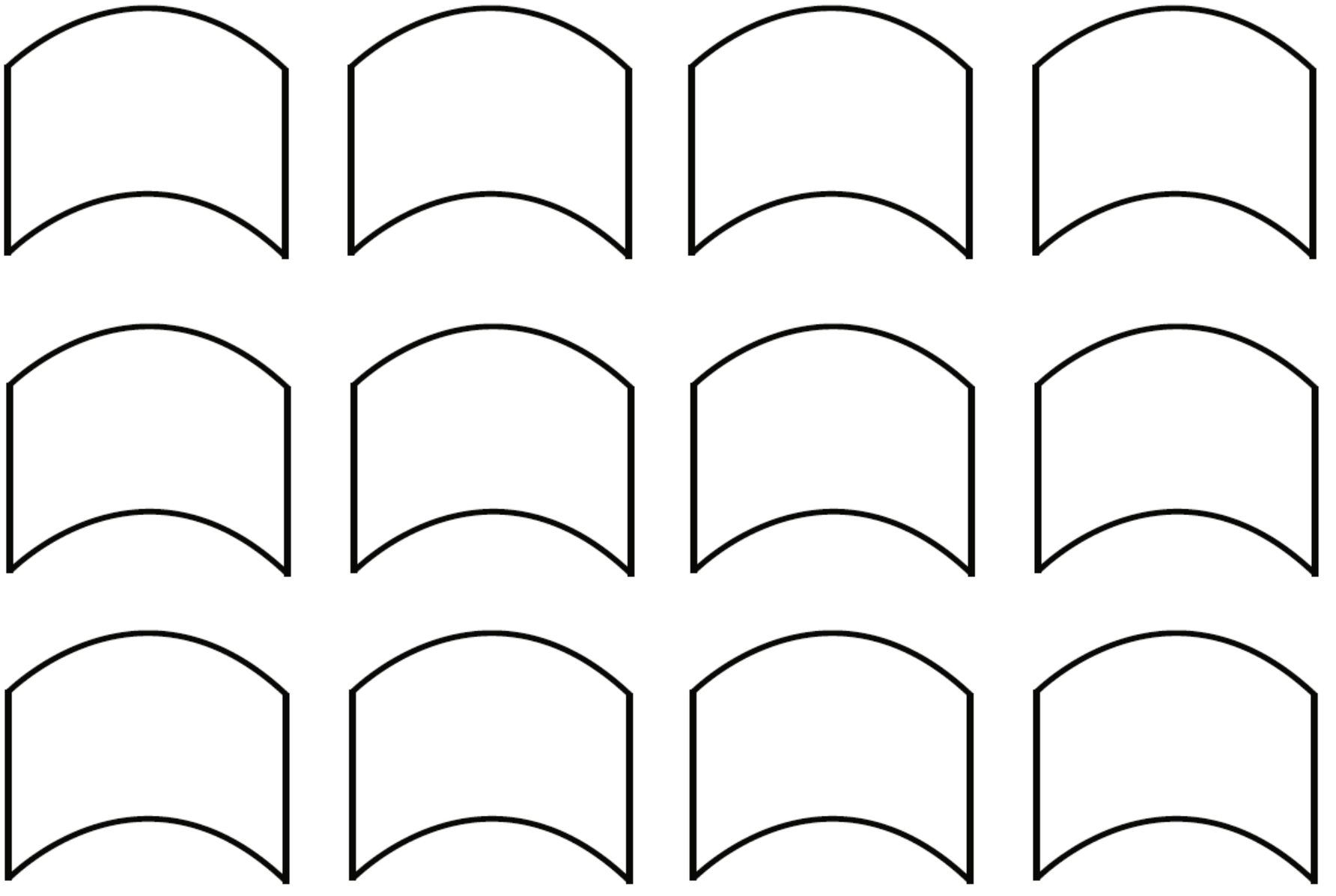


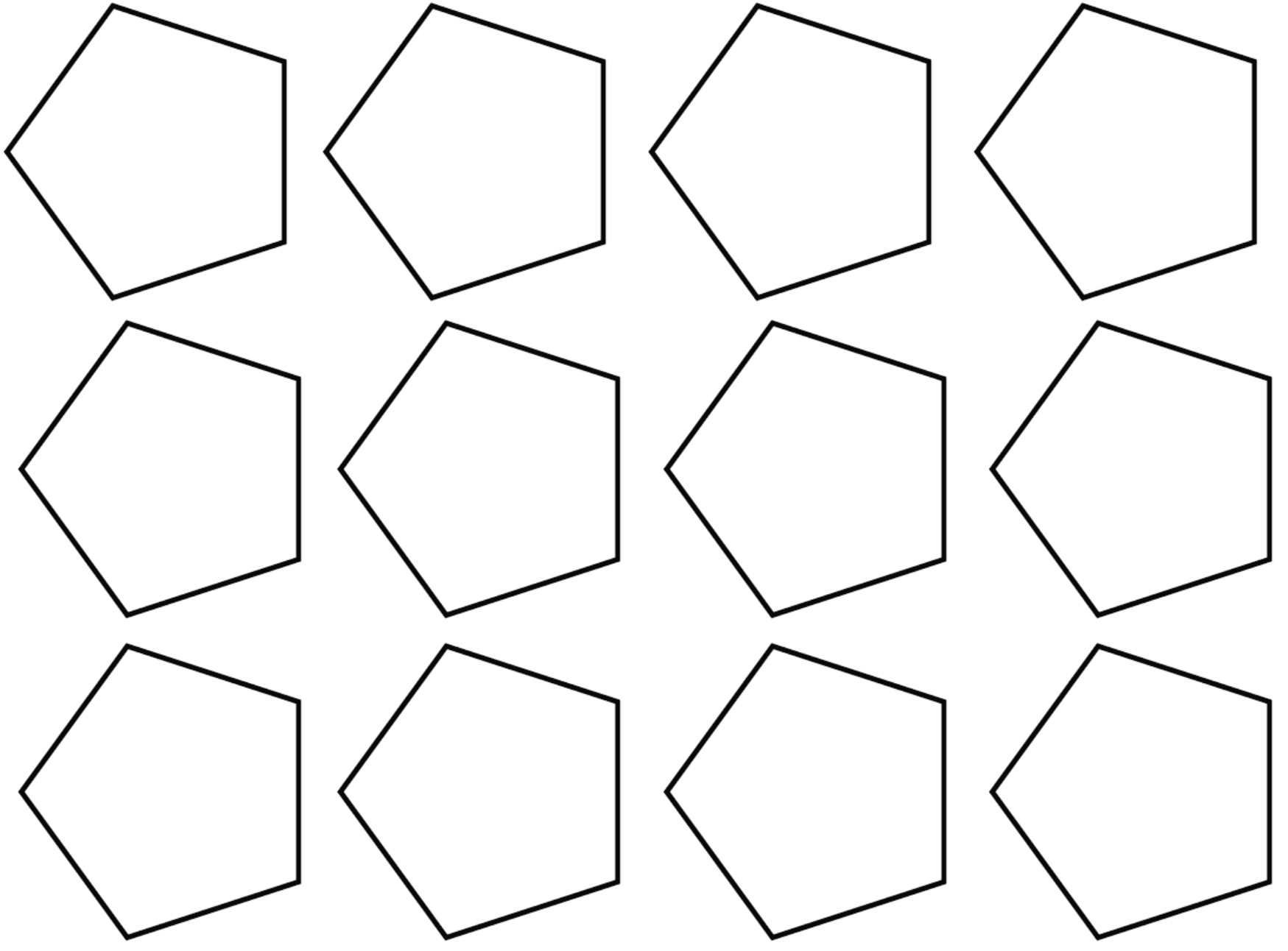
Term 4 Unit 9: Section 9.1, question 4 (TG p. 393; LB p. 336)



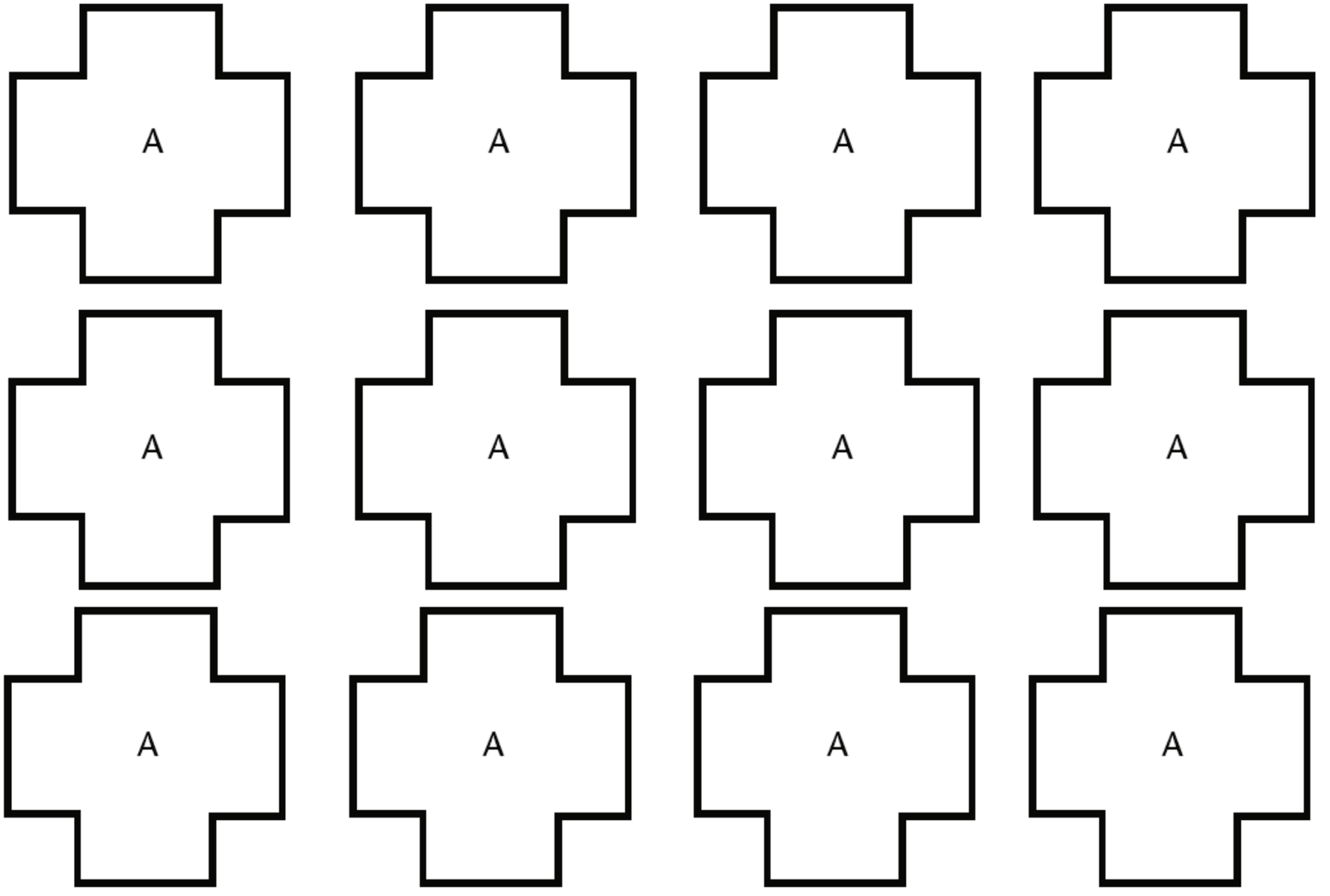
Term 4 Unit 9: Section 9.2, question 1 (TG p. 395; LB p. 338)

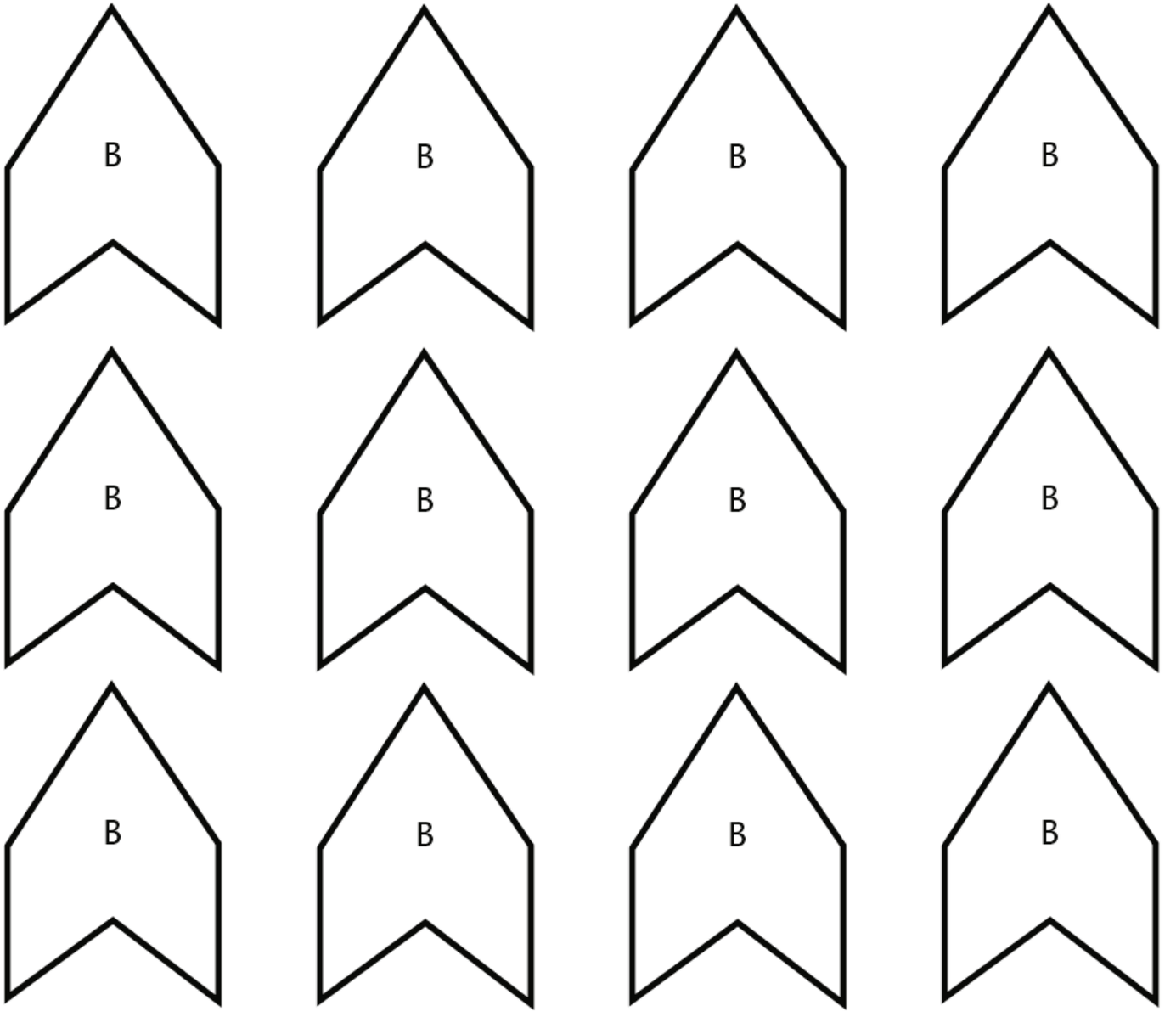






Term 4 Unit 9: Section 9.2, question 2 (TG p. 395; LB p. 338)

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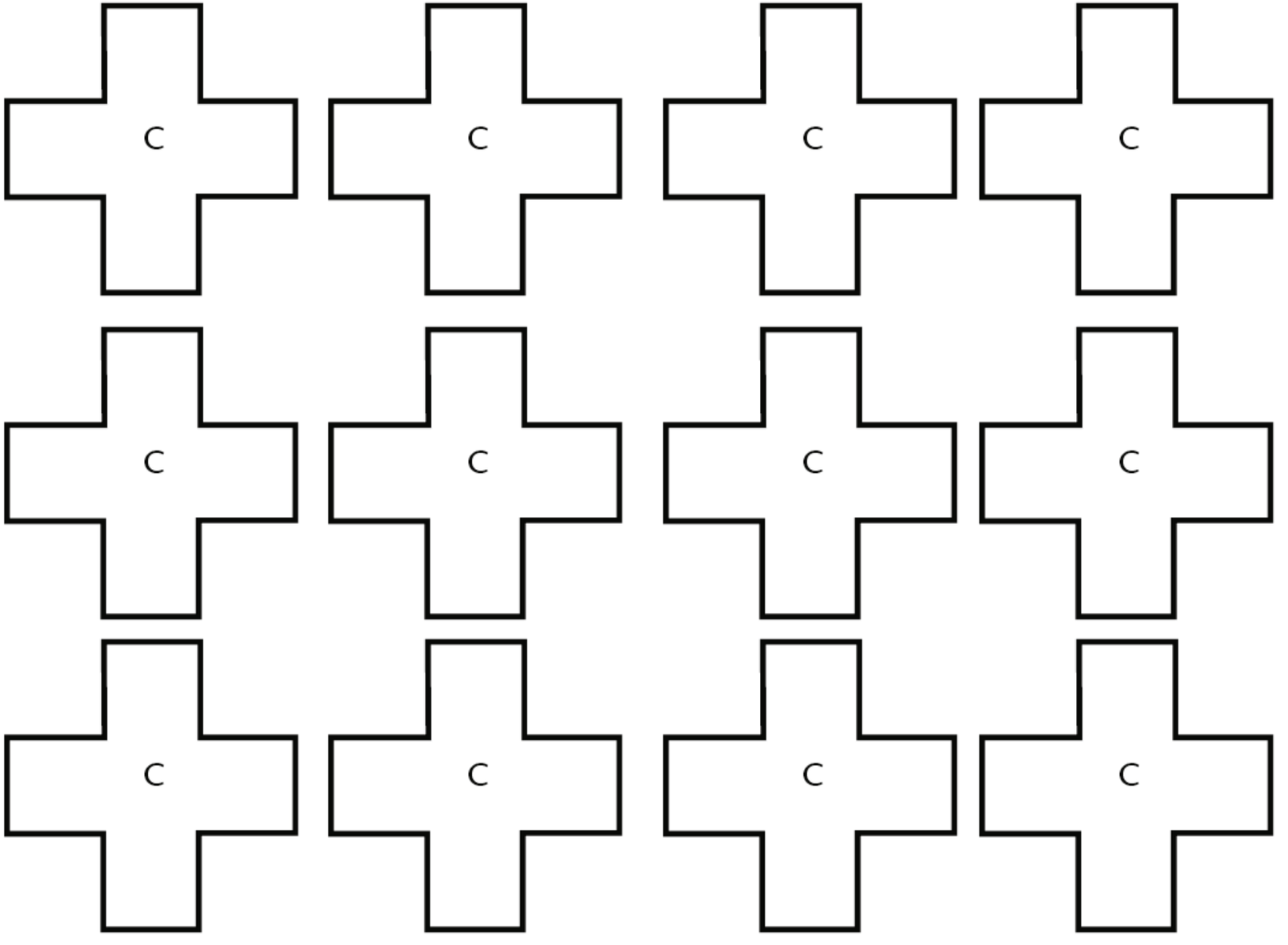
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